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A Method of Determining the Control Parameters in the Energy-Saving Control Problem

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Abstract: In the study, researchers discuss the methodology and computational algorithm for determining the parameters of control function in the energy-saving control problem. The algorithm implementation involves the use of GRID-computing.

Key words: Multidimensional object management, the Cayley-Hamilton theorem, inverse matrix, GRID-computing, function

INTRODUCTION

When solving problems of energy-saving control for multidimensional objects, it is a big challenge to determine all possible types of optimal control functions, find correlations for the computation of their d_i parameters and make sure that the conditions for the solutions of the control problem for the specific numeric input data really exist.

As shown by Yu *et al.* (2008), the determination of vector D parameters d_i $(i=\overline{1, n})$ where n is the number of parameters) for the optimal control of an object, characterized by given constant matrices $A(a_{i,j})_{n\times n}$ and $B = diag(b_1, ..., b_n)$, in some cases is reduced to solving a system of linear equations of the form:

$$\begin{aligned} &d_1\sum_{i=1}^{n} b_i\varphi_{1,i,1}+\ldots+d_n\sum_{i=1}^{n} b_i\varphi_{1,i,n} = l_1 \\ &d_1\sum_{i=1}^{n} b_i\varphi_{2,i,1}+\ldots+d_n\sum_{i=1}^{n} b_i\varphi_{2,i,n} = l_2 \\ &\vdots \\ &d_1\sum_{i=1}^{n} b_i\varphi_{n,i,1}+\ldots+d_n\sum_{i=1}^{n} b_i\varphi_{n,i,n} = l_n \end{aligned} \tag{1}$$

Where:

$$\varphi_{j,i,k} = \int_0^2 f_{j,i}(A(2-\tau))f_{i,k}(-A^T\tau)d\tau$$

$e^M = (f_{i,j}(M))_{n\times n}$ is a matrix exponential (M is a matrix argument), $L = colon(l_1, ..., l_n)$ is a vector of synthesizing variables whose components are determined by the equation:

$$l_i = z_{ei} - \sum_{j=1}^{n} f_{i,j}(2A)z_{0j}$$

where, $Z_0 = colon(z_{01}, ..., z_{0n})$ is a predetermined vector of the initial state of an object, $Z_e = colon(z_{e1}, ..., z_{en})$ is pre-determined vector of the finite state of an object.

It should be noted that vector D generally comprises two types of parameters: coefficients preceding time functions and values of switching time moments. At these moments, control functions come to the boundary. Dimensionality of the vector D and its composition depends on the form of the optimal control.

As seen from the system (Eq. 1), determination of vector D for the specific data is related to significant computational difficulties. Therefore, to solve this problem it is advisable to use GRID-computing.

COMPUTING THE MATRIX EXPONENTIAL

As shown above, one of the key steps in determining the parameters of the control function is to compute the matrix exponential thus minimizing the amount of computation. Computation of an approximate value is as follows:

$$e^{At} = E+\sum_{i=1}^{\infty}\frac{(At)^i}{i!} = \sum_{i=0}^{\infty}\frac{(At)^i}{i!}$$

Where:
E = A unit matrix
t = Time related to the need to compute the high powers of the matrix A

We obtain a formula to compute the matrix exponential using the n-1 power of the matrix A. Let, the characteristic equation of the matrix A has the form:

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$$\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} -, ..., -p_n = 0 \quad (2)$$

By the Cayley-Hamilton theorem (Gantmaher, 1959) the matrix A satisfies the matrix equation similar to Eq. 2:

$$A^n - p_1A^{n-1} - p_2A^{n-2} -, ..., -p_nE = 0$$

from which:

$$A^n = p_1A^{n-1} + p_2A^{n-2} +, ..., +p_nE \quad (3)$$

Following the method of Faddeev (1959), the coefficients of the characteristic equation are determined by the following recurrence relation:

$$p_k = \frac{s_k - p_1s_{k-1} -, ..., -p_{k-1}s_1}{k}$$

where, $s_k = \mathrm{Sp}\,A^k$ is the trace of the matrix A^k (the sum of the elements on the main diagonal), $p_1 = \mathrm{Sp}\,A$, $k = \overline{1, n}$. Next, we introduce the notation. If m = 0 then $q_{0,k} = p_k$. Otherwise (if $m \in N$, N is a set of natural numbers):

$$q_{m,k} = p_kq_{m-1,1} + q_{m-1,k+1}, q_{m-1,n+1} = 0$$

We multiply the both sides of the relation (Eq. 3) by the matrix A given the introduced notations. We obtain:

$$A^{n+1} = q_{0,1}A^n + q_{0,2}A^{n-1} +,...,+ q_{0,n}A = (p_1q_{0,1} + q_{0,2})A^{n-1} + (p_2q_{0,1} + q_{0,3})A^{n-2} + (p_3q_{0,1} + q_{0,4})A^{n-3} +,...,+ (p_{n-1}q_{0,1} + q_{0,n})A + p_nq_{0,1}E \quad (4)$$

Equation 4 can be rewritten as:

$$A^{n+1} = q_{1,1}A^{n-1} + q_{1,2}A^{n-2} +,...,+ q_{1,n}E \quad (5)$$

Next, we multiply both sides of Eq. 5 by the matrix A, inserting (Eq. 3) into the obtained relation:

$$A^{n+2} = q_{1,1}A^n + q_{1,2}A^{n-1} +,...,+ q_{1,n}A = (p_1q_{1,1} + q_{1,2})A^{n-1} + (p_2q_{1,1} + q_{1,3})A^{n-2} + (p_3q_{1,1} + q_{1,4})A^{n-3} +,...,+ (p_{n-1}q_{1,1} + q_{1,n})A + p_nq_{1,1}E \quad (6)$$

Then, from the Eq. 6 using sequential multiplication of both sides by the matrix A, it follows that:

$$A^{n+m} = q_{m,1}A^{n-1} + q_{m,2}A^{n-2} +,...,+ q_{m,n}E = \sum_{k=0}^{n-1} A^k q_{m,n-k}$$

Now, we consider the matrix exponential as:

$$e^{At} = \sum_{k=0}^{n-1} A^k \frac{t^k}{k!} + \sum_{m=0}^{\infty} \frac{t^{n+m}}{(n+m)!} \sum_{k=0}^{n-1} A^k q_{m,n-k} = \sum_{k=0}^{n-1} A^k \frac{t^k}{k!} + \sum_{k=0}^{n-1} A^k \sum_{m=0}^{\infty} q_{m,n-k} \frac{t^{n+m}}{(n+m)!}$$

Hence, we obtain:

$$e^{At} = \sum_{k=0}^{n-1} A^k \left[\frac{t^k}{k!} + \sum_{m=0}^{\infty} \frac{q_{m,n-k}}{(m+n)!} t^{m+n}\right] \equiv \sum_{k=0}^{n-1} A^k \left[\frac{t^k}{k!} + \sum_{m=0}^{\infty} r_{m,k} t^{m+n}\right]$$

MATRIX NOTATION OF THE ORIGINAL SYSTEM OF LINEAR EQUATIONS

The system (Eq. 1) can be rewritten as:

$$\left[\int_0^2 e^{A(2-\tau)} B e^{-A^T\tau} d\tau\right] D = L$$

or using the Eq. 2 shown by Bellman (1997) as:

$$\left[e^{2A} \int_0^2 e^{A(-\tau)} B e^{A^T(-\tau)} d\tau\right] D = L$$

To reduce the amount of computation in the future, we remove the integral. We assume that $\omega = -\tau$. We denote coefficients corresponding to the matrix A^T by $r^*_{m,i}$. We obtain:

$$\left[e^{2A}\sum_{k=0}^{n-1}A^kB\sum_{i=0}^{n-1}\left(A^T\right)^i\int_{-2}^{0}\left(\frac{\omega^k}{k!}+\sum_{m=0}^{\infty}r_{m,k}\omega^{m+n}\right)\cdot\left(\frac{\omega^i}{i!}+\sum_{m=0}^{\infty}r^*_{m,i}\omega^{m+n}\right)d\omega\right]D=L \tag{7}$$

We remove the parentheses:

$$\left(\frac{\omega^k}{k!}+\sum_{m=0}^{\infty}r_{m,k}\omega^{m+n}\right)\left(\frac{\omega^i}{i!}+\sum_{m=0}^{\infty}r^*_{m,i}\omega^{m+n}\right)=\frac{\omega^{k+i}}{k!i!}+\frac{1}{i!}\sum_{m=0}^{\infty}r_{m,k}\omega^{m+n+i}+\frac{1}{k!}\sum_{m=0}^{\infty}r^*_{m,i}\omega^{m+n+k}+\omega^{2n}\sum_{m=0}^{\infty}r_{m,k}\omega^m\sum_{m=0}^{\infty}r^*_{m,i}\omega^m$$

The Cauchy product of the exponential series (Fikhtengolts, 1966) takes the form:

$$\omega^{2n}\sum_{m=0}^{\infty}r_{m,k}\omega^m\sum_{m=0}^{\infty}r^*_{m,i}\omega^m=\sum_{m=0}^{\infty}\sum_{j=0}^{m}r_{j,k}r^*_{m-j,i}\omega^{m+2n}$$

Then, the integral:

$$\int_{-2}^{0}\left(\frac{\omega^k}{k!}+\sum_{m=0}^{\infty}r_{m,k}\omega^{m+n}\right)\left(\frac{\omega^i}{i!}+\sum_{m=0}^{\infty}r^*_{m,i}\omega^{m+n}\right)d\omega=2\left(\frac{(-2)^{k+i}}{k!i!(k+i+1)}+\frac{(-2)^{n+i}}{i!}\sum_{m=0}^{\infty}\frac{(-2)^m r_{m,k}}{m+n+i+1}+\frac{(-2)^{n+k}}{k!}\sum_{m=0}^{\infty}\frac{(-2)^m r^*_{m,i}}{m+n+k+1}+(-2)^{2n}\sum_{m=0}^{\infty}\frac{(-2)^m\sum_{j=0}^{m}r_{j,k}r^*_{m-j,i}}{m+2n+1}\right) \tag{8}$$

$$L=Z_e-e^{2A}Z_0$$

COMPUTATION OF THE LINEAR EQUATIONS COEFFICIENTS IN A DISTRIBUTED COMPUTING ENVIRONMENT

To obtain the coefficient values of the system (Eq. 1) following Eq. 7, we need to find n-1 of the power of the matrices A and A^T (A^1 = A and $(A^T)^1 = A^T$ shall not be determined). We note that the computational procedure for the powers of the matrix A does not depend on those for the matrix A^T. Therefore, at the first computational phase such computations can be implemented in a distributed computing environment. The results of GRID-computing (matrix powers, coefficients p_k and p_k^*) are recorded in an online database that is accessible to all computing processes. A parallel algorithm for computation of the matrix powers can be constructed by the doubling scheme described by Voevodin and Voevodin (2002)

It follows from Eq. 8 that the next phase involves independent approximate computation of n.n.3 = $3n^2$ (by k and i) of the sums of numerical series. We use the data obtained in the previous computational phase (coefficients p_k and p_k^* determining $r_{m,k}$ and $r^*_{m,i}$, respectively).

FINDING THE CONTROL PARAMETERS

Rewrite the system (Eq. 1) in the general form:

$$HD=L \tag{9}$$

wherein the matrix H is computed as described above:

As seen from the Eq. 5, computation of the matrix e^{2A} occurs in the formation of the matrix H. We assume that the determinant $|H|\neq 0$. Otherwise, the system (Eq. 9) may have no solutions or have an infinite number of solutions. The latter can permit selecting the control parameters so as to satisfy the conditions for the existence of the solution of the control problem. In this case, it is advisable to use symbolic computation for constructing a set of solutions of the system (Eq. 9). However, the existence of the matrices A and B and the vector L, under which the given fact holds is highly questionable (this is confirmed by the fact that the matrix exponential is always non-degenerate (Bellman, 1997) and is computed approximately). In this study, we consider the case when the system (Eq. 9) has a unique solution.

In the numerical solution of the system (Eq. 9) by the exact methods, there are several sources of inaccuracy of the obtained solution. The first source is related to rounding of real numbers in the computing process.

The second source is that the matrix H and the vector L are computed approximately, causing errors in the solution. They can lead to the fact that the determinant of the matrix H will be close to zero that is the matrix H is ill-conditioned (small changes in the elements of the matrix correspond to significant changes in its inverse matrix). In this case, in order to achieve a given accuracy of computation of the control parameters, after finding the solution of the system (Eq. 9) it is necessary to increase the accuracy of computation of the matrix H and re-make computations. After that, it is necessary compare how different the newly obtained solution of the system (Eq. 9) is from the one used before the accuracy was increased. In this case, it may be necessary to use libraries for high-computing with virtually unlimited range of real numbers (MPFR C library, 2015). To characterize the matrix in terms of its conditionality several researchers proposed various quantitative characteristics, for example, the Todd number (Faddeev, 1959).

We will apply the exact method the inverse matrix method for finding solutions of the system (Eq. 9) using GRID-computing wherein the amount of divisions by a number is much smaller than that in the Gauss Method where the amount of multiplication and division operations is close to (Samarskiy and Gulin, 1989). In some cases, this may reduce systematic error of the resulting solution. The inverse matrix will be calculated by the following algorithm.

We write the following equation for the matrix H, similar to Eq. 3:

$$H^{n}\text{-}u_1H^{n-1}\text{-}u_2H^{n-2}\text{-}, ..., \text{-}u_nE = 0 \quad (10)$$

where, the coefficients u_k are determined by the traces of powers of the matrix H. By multiplying both sides of the Eq. 10 by the matrix H^{-1}, we obtain:

$$u_nH^{-1} = H^{n-1}\text{-}u_1H^{n-2}\text{-}u_2H^{n-3}\text{-},...,\text{-}u_{n-2}H\text{-}u_{n-1}E$$

from which:

$$H^{-1} = \frac{H^{n-1}\text{-}u_1H^{n-2}\text{-},...,\text{-}u_{n-2}H\text{-}u_{n-1}E}{u_n} \quad (11)$$

Thus, the parallel algorithm for determining the matrix H^{-1} also includes finding the powers of the matrix H using the doubling scheme.

As it follows from the Eq. 11 the existence of the inverse matrix is determined not only by $|H|\neq 0$ but also by $u_n\neq 0$.

Thus, the applied method of computation in a distributed computing environment can significantly reduce the time of determining the control parameters and improve the accuracy of the results through, the use of high-precision computing.

CONCLUSION

The aim of the study is to develop methods of numerical computing to determine parameters d_i of a control function in a distributed computing environment. We note that the control process can be measured against the computational time. Therefore, it is necessary to determine control function parameters within a reasonable time limit. Then, following the system (Eq. 1), the numerical computing method will be as follows: the optimal way to compute the matrix exponential which determines the coefficients of the system (Eq. 1), the removal of the integral sign in the coefficients $\varphi_{j, i, k}$ and solution of the system (Eq. 1) taking into account the conditionality of matrix coefficients.

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