# Specificity of Analysis and Synthesis of Energy Efficient Control 

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#### Abstract

Considered the model which characterizes similarity degree of different types of optimal energy-saving functions and allows to create the subsets of that functions.


Key words: Energy-intensive facilities, optimal control, mathematical model, subsets, energy-saving functions

## INTRODUCTION

The main problem during analysis and synthesis of energy-saving control for energy-intensive facilities is a huge number of possible types of optimal control functions and hence complexity of mathematical support of microcontrollers (Muromtsev, 2004; Kamil and Fakolujo, 2012). Mathematical support contains complex relationships for domains borders of different types of optimal control functions, systems of equations and algorithms for calculations of control parameters.

For solving this problem, we can use cognitive graphic that shows result of analysis of optimal control by using synthesizing variables method for energy-intensive objects.

Significant reduction of mathematical operations quantity leads to good opportunity for developer to choose technical implementation of energy-efficiency control systems based on microprocessors.

## MAIN PART

Let's consider the problem state of energy-efficient control for multidimensional object as next example of optimal control problem (Muromtsev et al., 2002; Muromtsev, 2005). Suppose dynamic model of the object in matrix form describes as:

$$
\begin{equation*}
\dot{\bar{z}}=\mathrm{A} \overline{\mathrm{z}}(\mathrm{t})+\mathrm{B} \overline{\mathrm{u}}(\mathrm{t}), \quad \mathrm{t} \in\left[\mathrm{t}_{0}, \mathrm{t}_{\mathrm{K}}\right] \tag{1}
\end{equation*}
$$

and we need, during time interval $\left[\mathrm{t}_{0}, \mathrm{t}_{\mathrm{K}}\right]$, transfer object from initial state $\bar{z}^{0}$ to final state $\bar{z}^{k}$, i.e.:

$$
\begin{equation*}
\overline{\mathrm{z}}\left(\mathrm{t}_{0}\right)=\overline{\mathrm{z}}^{0}=\left(\mathrm{z}_{1}^{0}, \mathrm{z}_{2}^{0}, \ldots, \mathrm{z}_{\mathrm{n}}^{0}\right)^{\mathrm{T}} \rightarrow \overline{\mathrm{z}}^{\mathrm{K}}=\left(\mathrm{z}_{1}^{\mathrm{K}}, \mathrm{z}_{2}^{\mathrm{K}}, \ldots, \mathrm{z}_{\mathrm{n}}^{\mathrm{K}}\right)^{\mathrm{T}} \tag{2}
\end{equation*}
$$

with constraints on components of control vector $\bar{u}$ at each time:

$$
\begin{equation*}
\forall \mathrm{t} \in\left[\mathrm{t}_{0}, \mathrm{t}_{\mathrm{K}}\right]: \mathrm{u}_{\mathrm{i}}(\mathrm{t}) \in\left[\mathrm{u}_{\mathrm{i}}^{\mathrm{H}}, \mathrm{u}_{\mathrm{i}}^{\mathrm{B}}\right], \mathrm{i}=\overline{1, \mathrm{~m}} \tag{3}
\end{equation*}
$$

with minimum functional which show total energy consumption:

$$
\begin{equation*}
J_{\ni}=\int_{t_{0}}^{t_{\mathrm{K}}} \overline{\mathrm{u}}^{\mathrm{T}}(\mathrm{t}) \mathrm{C} \overline{\mathrm{u}}(\mathrm{t}) \mathrm{dt} \rightarrow \min _{\overline{\mathrm{u}}} \tag{4}
\end{equation*}
$$

where, $\overline{\mathrm{z}}: \mathrm{n}$-vector of phase coordinates; $\overline{\mathrm{u}} \mathrm{m}$-vector of control; $A=\left\|a_{i j}\right\|_{\text {lox }}, B=\left\|b_{i}\right\|_{\text {max }}$ : matrix of object parameters; $C=\left\|c_{i f}\right\|_{\text {wa }}:$ matrix of weight coefficients, taking into account cost of energy from different control channels.

In general case, we can get not only constraints (Eq. 2 and 3) but also constraints of velocity or acceleration of control actions, phase coordinates, energy limits, etc.

For numerical solution of problem (Eq. 1-4), i.e., determination of optimal control function type, $\bar{u}_{j}^{*}(t)=\left(u_{j 1}^{*}(t), u_{j 2}^{*}(t), \ldots, u_{j n}^{*}\right)^{T}$ and its parameters $d$, there is the initial data array:

$$
\begin{equation*}
\mathrm{R}=\left(\mathrm{A}, \mathrm{~B}, \mathrm{C}, \overline{\mathrm{u}}_{1}^{\mathrm{H}}, \overline{\mathrm{u}}_{2}^{\mathrm{B}}, \ldots, \mathrm{u}_{\mathrm{m}}^{\mathrm{H}}, \mathrm{u}_{\mathrm{m}}^{\mathrm{B}}, \overline{\mathrm{z}}^{0}, \overline{\mathrm{z}}^{\mathrm{K}}, \mathrm{t}_{0}, \mathrm{t}_{\mathrm{K}}\right) \tag{5}
\end{equation*}
$$

Array of parameters d in general case contains two types of parameters-coefficients of time functions and time values (switching time) when control functions goes to saturation mode (or from saturation mode). Dimension and structure of d array depends from optimal control function type.

Definition 1: Analysis of energy-saving control as optimal control problem (Eq. 1-4) consists the following. For any values of initial data array $R$ define: set of possible types of optimal control functions $V_{u}$ and set of their abbreviations $V_{x}$ such that between $V_{u}$ and $V_{x}$ performed one correspondence, i.e.:

$$
V_{u}=\left\{\bar{u}_{j}^{*}(t), j=0,1, \ldots, N\right\} \leftrightarrow V_{z}\left\{x_{j}, j=0,1, \ldots, N\right\}
$$

where, $\left|\mathrm{V}_{\mathrm{u}}\right|=\left|\mathrm{V}_{\mathrm{x}}\right|=\mathrm{N}+1$ : number of different types of optimal control functions; $\mathrm{x}_{\mathrm{j}}$ : abbreviation (code) of optimal control function $\bar{u}_{j}^{*}(t)$. $V_{u}$ is defined as cartesian product of sets of optimal control function types for control vector $\bar{u}$ components, i.e.:

$$
\mathrm{V}_{\mathrm{u}}=\mathrm{V}_{1} \times \mathrm{V}_{2} \times \ldots \times \mathrm{V}_{\mathrm{m}}
$$

where, $\mathrm{V}_{\mathrm{i}}$ set of function types for ith component $\left(u_{i}\right), i=\overline{1, m}$. For forming $V_{i}$ set using the maximum principle (Pontryagin et al., 1969).

Inside sets of function types $V_{u}$ and $V_{i}$ we can allocate subsets or classes of optimal control functions with different numbers of parameters and other properties.

Definition 2: Sets of function types for component $u_{i}(t)$, formed by functions $u_{i, j}\left(t ; d_{j}\right)$ with number of parameters $\mathrm{d}_{\mathrm{j}}$ non less than n , will call complete in terms of parameters and denoted as $V_{i}^{\Pi}$. Vector $\bar{u}(t) \in V_{u}^{\Pi}$, if all function types of all its components $u_{i}(t) \in V_{i}^{\Pi}$. Obviously that for set $V_{u}^{\Pi}$ of functions $\bar{u}(t)$, characterized by "complete" number of parameters, occurs:

$$
\mathrm{V}_{\mathrm{u}}^{\Pi}=\mathrm{V}_{1}^{\Pi} \times \mathrm{V}_{2}^{\Pi} \times \ldots \times \mathrm{V}_{\mathrm{m}}^{\Pi}
$$

Functions $u_{i}(t) \in V_{i}^{n}$ have next features. Each kind of $u_{i, j}(t) \in V_{i}^{\Pi}$ in space $R$ of values for $R$ array corresponds to a certain continuum region $R_{j}$. Each point of $R_{j}$ region corresponds to the function $u_{i j}(\mathrm{t})$ with defined values of parameters $\mathrm{d}_{\mathrm{j}}$ which calculated by the same equation systems.

Model $\mathrm{M}_{\mathrm{x}}$, characterized similarity degree for different function types, allows to formed subset of optimal control function types $\mathrm{V}\left(\mathrm{x}_{\mathrm{j}}\right) \subset \mathrm{V}_{\mathrm{x}}$ which contains functions similar to $x_{j}$ and obtained by insignificant changing of component values for $R\left(x_{j}\right)$ array which corresponds to the optimal control function type $\mathrm{x}_{\mathrm{j}}$, i.e.:

$$
\mathrm{M}_{\mathrm{z}}: \mathrm{R}\left(\mathrm{x}_{\mathrm{j}}\right) \rightarrow \mathrm{V}\left(\mathrm{x}_{\mathrm{j}}\right)
$$

The model is based on cognitive maps in the form of graph. Graph nodes corresponds to sets of optimal control function types $\mathrm{V}_{\mathrm{i}}^{\Pi}$ and graphs verges characterized type of $\mathrm{u}_{\mathrm{i}, \mathrm{j}}\left(\mathrm{t} ; \mathrm{d}_{\mathrm{j}}\right)$ function during changing initial data array $R$.

Operator $\eta$ determines optimal control function type using model $M_{\mathrm{r}}$ and initial data array $R$, i.e.:

Operator $\psi$ for checking existence of solution for problem (Eq. 1-4) and initial data R, i.e.:

$$
s=\psi\left(R, M_{z}\right), s \in\{0 ; 1\}
$$

Where:
$\mathrm{s}=$ Auxiliary variable which can accept two values
$0=$ Solution did not exist
$1=$ Solution exist

Operator (algorithms, procedures) $\xi$ of automotive construction of equations systems for calculation parameters d for optimal control functions:

$$
\begin{equation*}
F_{j}=\xi\left(R, M_{z}\right), F_{j}(R, D)=0 ; j=0,1,2, \ldots, N \tag{6}
\end{equation*}
$$

Where:
$R, D=$ Sets of values initial data array
$\mathrm{R}=$ Array of parameters d , respectively
Algorithms (transformation) for solving Eq. 6, i.e.:

$$
\gamma: \mathrm{R} \times \mathrm{V}_{\mathrm{u}} \rightarrow \mathrm{D}
$$

Thus, technology of analysis for optimal control problem involves determining of sets of optimal control function types $\mathrm{V}_{\mathrm{u}}, \mathrm{V}_{\mathrm{x}}$ model $\mathrm{M}_{\mathrm{x}}$, operators $\eta, \psi, \xi$ and algorithms $\gamma$.

## CONCLUSION

Variety of opportunities for presentation results of cognitive graphics in address space of microprocessors hardware of control systems allows to synthesize adaptive algorithms of energy-saving control for energy-intensive objects with minimum costs of software and hardware and intelligence supply of energy-saving system.

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