Journal of Engineering and Applied Sciences 11 (1): 119-124, 2016
ISSN: 1816-949X

Optimization of Reliability Coefficient of Engineering Systems Using a Specific Model for Genetic Algorithms

[1]Mohammad Mohammadi Najafabadi and [2]Sara Mansouri
[1]Department of Computer Science, Payame Noor University (PNU), P.O. Box 19395-3697, Tehran, Iran
[2]Department of Teaching English Language, Faculty of Humanities, Islamic Azad University, Najafabad Branch, Najafabad, Isfahan

Abstract: In a broad sense, the reliability of a system is one of the important measures to implement an engineering system. With the development of science and technology and engineering systems' getting complex and thereby the effect of their unreliable behavior in different fields and the tendency to evaluate the coefficient of reliability of engineering systems and the need to improve this coefficient has become very important. To obtain or manufacture products with high reliability, various methods are used to optimize these systems. One of the most widely used methods is genetic algorithms that have high potential in the optimization of engineering systems. In this study, a special model for the optimization of these problems is presented.

Key words: Reliability problem, optimization methods, linear programming, genetic algorithms, engineering

INTRODUCTION

In any modern society, engineers and technical managers are responsible for planning, designing, construction and operation from the simplest product to the most complex product systems. Products and systems failure causes disorder at different levels and can even be seen as a severe threat to society and the environment. Thus, consumers, generally people, expect products and systems to be valid, reliable and safe. So, as a key question it is argued: how much is the system can be reliable during its future life and how much it is safe?

Regarding history, methods of assessing reliability primarily formed in association with aerospace and military applications. However, it was immediately used and paid attention to by the nuclear industry that is under intense pressure to ensure the safety of nuclear reactors to supply electrical energy or continuous process industries such as steel and chemical industry that every hour of their stop can cause big financial losses and casualties and environmental pollution because of flaws.

In a broad sense, reliability is one of the measures of system implementation. As systems have become more complex, the effects of their unreliable behavior in different fields have become more severe and tend to assess the reliability of systems. The need to improve the reliability of the system has become very important. For this purpose, researchers and engineers have tried to optimize system reliability by using reliability optimization methods and by development of computer science the use of these methods have further spread (Chern, 1992; Amari, 2012). Genetic algorithm is one of the optimization methods modeled based on nature while using computer. Recently, engineers and researchers use this method more. In this study, genetic algorithms are used to optimize a problem of reliability whose objective function and constraints are fuzzy.

INTRODUCING MATHEMATICAL MODEL OF COEFFICIENT OF RELIABILITY

The reliability of a system can be defined as the probability that a system acts successfully during a specified time and under specified conditions and the desired results are achieved. Reliability has parallel and series structures or a combination of these two modes, so it is needed to explain the structure of these two modes.

Structure of the series mode: From the standpoint of reliability, an n member set is called a series string whenever the success of the entire system is dependent on the success of all members. In this case, the whole system is successful if and only if all its members are successful. Diagram of such a system is shown in Fig. 1.

Corresponding Author: Mohammad Mohammadi Najafabadi, Department of Computer Science, Payame Noor University (PNU), P.O. Box 19395-3697, Tehran, Iran

Suppose that x_i is an incident whose ith unit has been successful and x_i is an incident whose ith unit has not been successful in this case, system reliability coefficient is defined as follows:

$$R = P(x_1.x_2, \ldots, x_n) = P(x_1).P(x_2|x_1).\ P(x_3|x_2, x_1)\ldots P(x_n|x_1, x_2, \ldots, x_{n-1})$$

In this case, if the events were independent, we would have:

$$R = P(x_1.x_2, \ldots, x_n) = P(x_1)P(x_2)\ldots P(x_n) = \prod_{i=1}^{n} P(x_i) = \prod_{i=1}^{n} R_i$$

In addition, unreliability coefficient of the system is defined as follows:

$$Q = P(\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_n) = 1 - P(x_1.x_2\ldots.x_n) = 1 - R$$

If the events were independent we would have:

$$Q = 1 - R = 1 - \prod_{i=1}^{n} P(x_i) = 1 - \prod_{i=1}^{n} (1 - Q_i)$$

Parallel structure: From the standpoint of reliability, an n member set is called parallel when the system is successful and if at least one of the members is successful whose is shown in Fig. 2. Now, suppose that x_i is an event whose ith unit is successful and suppose that x_i is an event whose ith unit is not successful then, the coefficient of reliability of the system is defined as follows:

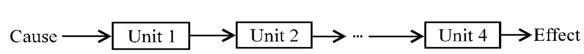

Fig. 1: Series structure of additional units

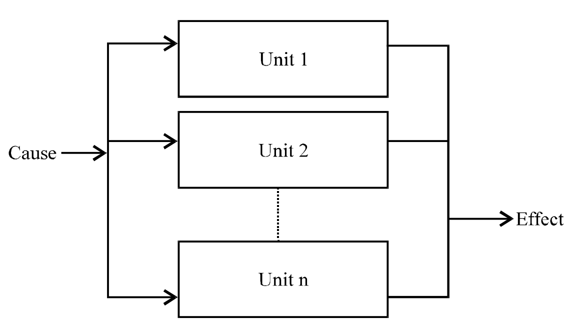

Fig. 2: Parallel structure of additional

$$R = P(x_1 + x_2 + \ldots + x_n) = 1 - P(\bar{x}_1 \cdot \bar{x}_2 \cdot \ldots \cdot \bar{x}_n)$$
$$= 1 - P(\bar{x}_1) \cdot P(\bar{x}_2|\bar{x}_1) \cdot P(\bar{x}_3|\bar{x}_1, \bar{x}_2) \ldots P(\bar{x}_n|\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{n-1})$$

If the events are independent, then we have:

$$R = 1 - \prod_{i=1}^{n} P(\bar{x}_i) = 1 - \prod_{i=1}^{n} Q_i$$

Therefore, unreliability coefficient of the system is defined as follows:

$$Q = \prod_{i=1}^{n} Q_i = \prod_{i=1}^{n} (1 - R_i)$$

Symbols and definitions: Now for stating the formula and mathematical model of the reliability of a system it is first necessary to define some symbols and definitions (Fyffe *et al*., 1968).

Symbol 1:

$$\binom{n}{k}_F$$

is the symbol to state a system whose operator fails when k number of its n extra units lead to failure.

Symbol 2:

$$\binom{n}{k}_G$$

symbol is used for a system whose performance is good and desirable when k number of its n extra units have performed well. Now suppose that system (m) has N sub-systems and its each. Sub-system i has m_i members, so we have:

$$m = [m_1, m_1, \ldots, m_1]$$

Moreover, the number of members of sub-system i (m_i) has μ_i upper bound. For each sub-system i there are two types of failure modes defined as follows.

Class O failure model: Sub-system i has O model class, whenever the performance of the sub-system is as:

$$\binom{m_i}{1}_F$$

or in other words, the performance of the sub-system leads to errors when the performance of one of the members is defected.

Class A failure mode: Sub-system i has A model when the performance of the sub-system is as:

$$\binom{m_i}{1}_G$$

or in other words, the performance of system is good when one of its members has a desirable performance. s_i: the total number of failure models in i sub-system whether class O or class A. h_i: the number of failure models in the class O in sub-system i (u = 1, 2, ..., h_i). s_i-h_i: the number of class a failure ins ubsystem i ($\mu = h_i+1, h_i+2, ..., s_i$). q_{iu}: is the probability of failure of model u for each unit in the sub-system i. $Q_u^o(m_i)$: the probability of failure in the sub-system i that has m_i units for the failure model u of the class O:

$$Q_u^o(m_i) = 1-(1-q_{iu})^{mi} \qquad u = 1, 2, \ldots, h_i \tag{1}$$

$Q_u^A(m_i)$: the probability of failure in the sub-system i that has m_i unit for the failure model u of class A:

$$Q_u^o(m_i) = (q_{iu})^{m_i} \qquad u = h_i+1, h_i+2, \ldots, s_i \tag{2}$$

$Q^o(m_i)$: the probability of failure in the sub-system i that has m_i units regarding the failure patterns of O:

$$Q^o(m_i) = \sum_{u=1}^{h_i} Q_u^o(m_i) \tag{3}$$

$Q^A(m_i)$: the probability of failure in the sub-system i that has m_i units regarding the failure patterns of A:

$$Q^A(m_i) = \sum_{u=h_i+1}^{s_i} Q_u^A(m_i) \tag{4}$$

$Q_i(m_i)$: the probability of failure in the sub-system i that has m_i units:

$$Q_i(m_i) = Q^o(m_i) + Q^A(m_i) \tag{5}$$

R (m): the coefficient of reliability of the entire system. Given the definition of the required terms, reliability-coefficient optimization model of the system along with several failure models is as follows:

$$\max \quad R(m) = \prod_{i=1}^{N}(1-Q_i(m_i))$$

$$\text{s.t.} \quad G_t(m) = \sum_{i=1}^{N} g_{ti}(m_i) \le b_t$$

$$1 \le m_i \le u_i,\ m_i\text{: integer},\ \ i = 1, 2, \ldots, N$$

Where:

- b_t = The amount of resources available from the source t (t = 1, 2, ..., T)
- $g_{ti}(m_i)$ = The amount of surplus resources for i sub-system with $G_t(m)$ surplus units and finally
- (s_i-h_i) = The total need of the whole system from the source t when the total share of system elements is equal to m

In this model, the purpose is to determine the surplus units of each sub-system (m_i) as the coefficient of reliability of the entire system reaches the maximum extent possible given the limitations of the system ($m = m_1, m_2, ..., m_N$).

AN INTRODUCTION ON GENETIC ALGORITHMS

Genetic algorithms are a branch of social optimization techniques in which biological systems and their principles are used. Although, these algorithms provide simple models of biological processes in normal circumstances in practice have shown a lot of ability and efficiency. The basic idea of providing genetic algorithms is using limited population of elements, each of which precisely determines a point of the search space where algorithm of population is defined by chromosomes (Coit, 2003). After this stage, chromosomes population (people) enters the evolutionary process in nature. At first, randomly a population of chromosomes is generated then recombination, mutation and selection operators appear then depending on the need other operators of evolution are applied on this chromosome and a new generation of chromosomes can be achieved and then for the new generation optimality criteria are (Coit, 2001) checked. If optimality criteria are met for this new generation, algorithm stops and the best chromosome is chosen as the optimum solution for transport model. Otherwise, different genetic operators are applied on chromosomes within the population, new generations are generated and until meeting the optimality criteria, the process continues. Genetic algorithm is superior to other methods of optimization algorithms and because of this tend to use them is increasing. Since, the beginning of the development of genetic algorithms, the experts tried to make these algorithms simpler and more understandable and to provide models of genetic algorithms that have less operation and great capabilities in the fields of optimization and the outputs of these algorithms consider the best answer to the problem.

GENETIC ALGORITHM MODEL SPECIFIC TO OPTIMIZATION OF RELIABILITY ENGINEERING SYSTEMS

Suppose the optimization of reliability problem with three non-linear constraints with parallel surplus units in its sub-systems where each sub-system has A failure model. Its mathematical model is as follows:

$$\max \quad R(m) = \prod_{i=1}^{3}\left[1\text{-}[1\text{-}(1\text{-}q_{i1})^{m_i+1}]\text{-}\sum_{u=2}^{4}(q_{iu})^{m_i+1}\right]$$

$$\text{s.t.} \quad G_1(m)=(m_1+3)^2+(m_2)^2+(m_3)^2 \le 51$$

$$G_2(m) = 20\sum_{i=1}^{3}(m_i+\exp(\text{-}m_i)) \ge 120$$

$$G_3(m) = 20\sum_{i=1}^{3}\left(m_i \times \exp\left(-\frac{m_i}{4}\right)\right) \ge 65$$

$$1 \le m_1 \le 4, \qquad 1 \le m_2 \& m_3 \le 7$$

$$m_i \ge 0,\ m_i\text{: integer} \quad i = 1, 2, 3$$

where ($m = m_1, m_2, ..., m_3$) and all sub-systems have four failure patterns (s_i = 4) of which one model is class O (h_i = 1) and three other models are of class A. Table 1 of the probability of each failure mode for each sub-system i (i = 1, 2, 3) as follows:

Representation of chromosomes: In prevalent genetic algorithms, chromosomes are commonly represented as binary strings. For this problem, the integer values for binary string are displayed by m_i. The length of the string depends on upper bound $m_i(u_i)$ of additional units. For example, if the upper bound $m_i(u_i)$ is equal to 4 we need three binary bits to represent m_i. In this example, upper bounds of additional units in each sub-system is u_1 = 4, u_2 = 7, u_3 = 7, in the case, each m_i variable needs three binary places. In this case, the total bit required is equal to 9. If m_1 = 2, m_2 = 3 and m_3 = 3 chromosomes are shown as follows:

Table 1: The probability of error in each sub-system due to the error patterns

Sub-system i	Failure model	The probability of failure
	O	0.04
1	A	0.05
	A	0.10
	A	0.18
	O	0.08
2	A	0.02
	A	0.15
	A	0.12
	O	0.04
3	A	0.05
	A	0.20
	A	0.10

$$V = [x_{33}, x_{32}, x_{31}, x_{23}, x_{22}, x_{21}, x_{13}, x_{12}, x_{11}]$$
$$= [0 \ \ 1 \ \ 1 \ \ 0 \ \ 1 \ \ 1 \ \ 0 \ \ 1 \ \ 0]$$

So, that x_{ij} is a symbol of jth binary bit for variable m_i.

Initial population: Initial population of the chromosomes is randomly generated. Each chromosome contains a 9-bit binary. One random approach of selecting chromosomes may make chromosomes unacceptable in two terms. It may violate the system limitation state or upper limit. So, to avoid these two cases and to produce legal chromosome, the following methods is used (Coit and Lio, 2000):

```
Procedure: Initialization
Begin
For i = 1 to pop-Size do
Produce a random chromosome vi;
    If (vi is not feasible) then
i = i-1;
end
  end
end
```

We form a population of 5 chromosomes (pop-Size = 5):

$$V_1 = [1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1],\ V_2 = [1\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 1],$$
$$V_3 = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0],\ V_4 = [0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1],$$
$$V_5 = [1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$$

Its correct values are as follows:

$$V_1 = [\overset{m_3}{6}\ \overset{m_2}{2}\ \overset{m_1}{1}],\ V_2 = [4\ 1\ 3],\ V_3 = [5\ 1\ 2],$$
$$V_4 = [3\ 2\ 3],\ V_5 = [4\ 2\ 1]$$

Chromosomal analysis: The following formula is used to assess the legal and illegal chromosomes:

$$\text{eval}(V_k) = \begin{cases} R(m) & G_t(m) \le b_t\ \forall t \text{ and } 1 \le m_i \le u_i\ \forall i \\ -M & \text{otherwise} \end{cases}$$

Where:
V_k = The kth chromosome
M = A positive integer

The fitness values (system reliability) of chromosomes can be obtained as follows:

- eval (v_1) = 0.543625
- eval (v_2) = 0.632703
- eval(v_3) = 0.610062
- eval (v_4) = 0.629119
- eval (v_5) = 0.589642

Recombination: Here, single-point crossover is used. Assume that the crossover probability is $p_c = 0.4$ and the collections of random numbers are between zero and one as follows:

0.550279 0.379650 0.243294 0.494583 0.771811

If the random number of chromosomes is <0.4, consequently its equivalent chromosomes are selected. Given the numbers above, chromosomes V_2 and V_3 are selected for recombination. Now a random number in the interval is chosen (Coit, 2003). For example:

$$V_2 = [1\ 0\ 0\ |0\ 0\ 1\ 0\ 1\ 1] \rightarrow O_1 = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1]$$
$$V_3 = [1\ 0\ 1|0\ 0\ 1\ 0\ 1\ 0] \qquad O_2 = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1]$$

Its correct values are:

$$O_1 = [\overset{m_3}{4}\ \overset{m_2}{1}\ \overset{m_1}{2}] \rightarrow \text{eval}\ (O_1) = 0.607591$$
$$O_2 = [5\ 1\ 3] \qquad \text{eval}\ (O_2) = 0.635277$$

Mutation: Suppose that the probability of mutations in chromosomes from one generation is $p_m = 0.1$ this means that 10% of the entire chromosome of each generation undergoes mutations. Moreover, the genes of chromosomal are in binary, so mutation happens on zero and one. In this syudy, the binary mutation operator is used. The general trend is that a chromosome is randomly selected then a random number between one and the total number of genes in chromosome is selected, if the linked gene is zero it changes to one and if it is one it changes to zero:

Before mutation 0 1 1 1 0 0 0 1 1 0 1 0

⇓

After mutation 0 1 1 0 0 0 1 1 0 1 0

Selection: A new operator presented in this study is the result of combining the wheel as well as chromosome ranking technique (Coit, 2001). In this operator, at first, a rank is given to the chromosome of each generation where ranking process is contrary to common ranking. That is that the worst chromosomes is the first and the best one is the last. Based on the following formula, the fitness related to chromosome of a given generation is obtained where sp is a random number between the numbers one and two and pos is the rank if chromosome in the generation:

$$\text{Fittness(pos)} = 2\text{-sp} + 2\times(\text{sp-1})\times\frac{(\text{pos-1})}{(\text{N-1})}$$

Now, suppose our first generation of chromosomes is equal to n then rank of the worst answer is equal to one and the same way the rank of the best answer with is equal to n and according to the above formula its fitness level is obtained: such as $f_1, f_2, ..., f_n$. It is now acted as follows. At first:

$$F = \sum_{i=1}^{n} f_i$$

is obtained, then the probability of each answer is given as follows:

$$P_i = \frac{f_i}{F}$$

and based on that the cumulative probability of each of the solutions is calculated as follows:

$$q_1 = p_1,\ q_i = q_{i-1}+p_i,\ i = 2, 3, \ldots, n$$

Then, they are pictured a segment from zero to one. Now, the same as the number of selections needed, random number in the interval (Amari, 2012) are selected answers similar to these numbers are selected. Obviously, stronger answers have greater fitness and the likelihood to be chosen for them is over other solutions in a generation and this is based on Darwin's principle of natural selection that strong animals remain and the weak are doomed to die out. Now, suppose n new chromosomes are selected by the operator transportation model many of which may be repetitive. Now, after applying the selection operator, recombination operator affects the results of this generation to make new chromosomes.

ALGORITHM IMPLEMENTATION PROCESS

Based on the feasible area of generation, zero or the first generation is created at random and obtain its fitness. Then optimality criteria is assessed for this generation. If the criteria of optimality are met, algorithm stops and the best chromosome of that generation is selected as the best answer to the problem otherwise the selection operator is applied on this generation of chromosomes and n number of new chromosomes are selected. After this step, the chromosomal recombination operator is applied on this generation and n chromosome is converted to 2n chromosome which makes diversity in the chromosomes. After this step, the mutation operator is applied so that n chromosome changes into n new chromosome based on the mutation trend. So, after this stage 4n new chromosome is achieved. However, because the number of the members of a generation must remain stable so the selection operator is used and n number of chromosomes is chosen from 4n chromosome and this chromosome forms the new generation such as the above-mentioned process, optimality criteria for this generation is examined. If optimality criteria are met, the best chromosome is selected, otherwise evolutionary operators are applied on this generation and a new generation is created. We continue this process until the optimality criteria are met and the algorithm stops and the best response is achieved. The algorithm is designed based on the specific characteristics of the transport model and is capable of being applied to other issues as well (Coit and Smith, 1996). In general, algorithm can be designed step by step as following:

- Step one: creating the answers at random as the initial generation
- Step two: getting the value of the objective function to determine the suitability of this generation
- Step three: assessing the optimality criterion and if optimal, go to step seven
- Step four: applying recombination operator
- Step five: applying mutation
- Step six: applying operator selection and go to the second step
- Seventh step: getting the optimum solution

RESULTS

Today with the development of science and technology and engineering systems' getting complex resulting in a non-reliable behavior, the slightest unreliable behavior of these systems causes irreparable damage to life and property and the environment like military and nuclear industries of electrical industries. Thus, the importance of optimizing and improving the reliability factor of these systems that are costly and this technology is of paramount importance is obvious. To optimize system reliability, various methods are used in this study, a particular model of genetic algorithm for optimization of the structure is used that is simpler than other methods and has great ability to efficiently optimize the reliability. Moreover, the answers resulting from this method, compared with two optimization method using the lagrange multipliers as well as linear programming optimization method are closer to optimal solution and have better applicability in the system.

CONCLUSION

At first, it is necessary to examine reliability structures, then the general trend and operators used from genetic algorithms are mentioned and then the optimal solution of the objective function is obtained. Moreover, the optimization model of reliability is checked by several failure models.

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