

The Study of Stress-Strain Reinforced Concrete Beam at Bending. The Analysis of Picture in the Presence of Prestressed Reinforcement

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Abstract: The and discusses the stress-strain state of reinforced concrete product at the action of concentrated bending load in the case of this load approximation to the supporting beam power. The study also provides the analysis of stress and displacement for the cases of prestressed reinforcement and the reinforcement without prestressing by applying a tensile load to a reinforced concrete beam.

Key words: Reinforced concrete beam, simulation in ANSYS, the prestressing of reinforcement, reinforced concrete beam curvature

INTRODUCTION

Concrete is a composite material with less tensile strength than steel but with a number of useful properties, which make it indispensable in construction. In its turn, the reinforcement of concrete structures by the means of longitudinal and transverse steel reinforcement rods allows to increase the service life and therefore, the reliability of concrete structures which is certainly of interest in this area of research (Mobasher *et al.*, 2015; Gasratova and Stareva, 2015; Hemamalini and Gopinathan, 2015). During the study and the modeling of reinforced concrete structures, a number of difficulties appear associated primarily with the physical and mechanical properties of a material which makes it difficult to perform accurate calculations. As was mentioned, the addition of steel inclusions increases the durability of buildings but it is necessary to study the influence of various factors on the stress-strain state of the structure in order to optimize the model to be able not only to extend the service life of a structure, avoiding a variety of destructions (Grekov, 2002a, b, 2011; Grekov and Morozov, 2006; Pronina, 2010, 2011, 2013a, b; Sedova and Pronina, 2015a-c; Pronina, 2015a, b; Al-Rousan, 2015; Sedova *et al.*, 2014) but also to save resources and reduce the construction costs.

Until recently, the study of reinforced concrete structure behavior was carried out exclusively with the use of analytical calculations which did not provide a full picture. Furthermore, analytical calculations almost do not allow to predict the behavior of the structure over time. Of course, the development of the finite element method produced an incredible breakthrough in the study of reinforced concrete structures, allowed to analyze the influence of different loads on the model of an

interesting product more accurately and visually (Almassri *et al.*, 2015; Musmar *et al.*, 2014; Vasudevan and Kothandaraman, 2016). The finite element method provided the opportunity to assess not only the stress-strain state of a body but also to see the development of the picture in time.

Concrete strain diagram: Many researches are devoted to the material analysis, to which the linear theory of elasticity is applied (Gasratova, 2014; Kolpak *et al.*, 2015; Kolpak and Maltseva, 2015). The study of material behavior with nonlinear properties, makes an interest nowadays but is not a sufficiently elaborated subject (Dahl and Pronina, 2006).

Since, concrete is the material with nonlinear physical and mechanical properties, the classical Hooke's law takes place only in the case of small deformations (ϵ) and stresses (σ), then the relationship between them can be described by a linear dependence. Therefore, during the simulation of concrete products it is not recommended to take a linear Hooke's law as the basis as it can lead to incorrect results. It is necessary to set the concrete deformation diagram for a reliable analysis. This diagram will determine the relationship between σ and ϵ . Nikulin (2013) in order to describe the concrete compression diagrams at the center loading proposes to use the fractional-rational function of the following form:

$$\sigma_b = E_{b01}\epsilon_b(1 + D_{b01}\epsilon_b) / (1 + C_{b01}\epsilon_b) \quad (1)$$

Where:

$E_{b01}, D_{b01}, C_{b01}$ = The initial elasticity modulus and concrete deformation nonlinearity parameters obtained experimentally and theoretically

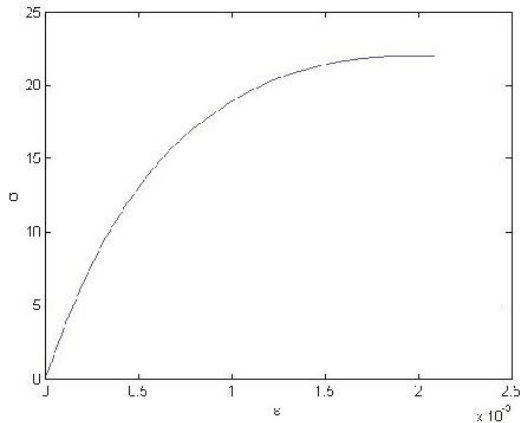


Fig. 1: Concrete deformation diagram

σ_b, ϵ_b = The current values of stresses and compression deformations

For class B30 concrete these parameters are assumed to be the following ones $E_{b01} = 38519.0 \text{ MПа}$; $D_{b01} = -146.34$; $C_{b01} = 738.49$. The diagram based on (Eq. 1) has the form shown on Fig. 1.

MATERIALS AND METHODS

Study of concrete beam stress-strain state at bending: In order to determine the bearing capacity of a reinforced concrete beam without reinforcement is considered in the compressed area with the following dimensions: length $a = 300 \text{ cm}$, width $b = 40 \text{ cm}$, height $h = 40 \text{ cm}$. The concrete type is B30, the elasticity module makes $E_b = 32.5 \times 10^9 \text{ Pa}$, Poisson's ratio $\nu_b = 0.18$. Heavy concrete makes $\rho = 2280 \text{ kg m}^{-3}$. The beam is reinforced by A400 (A-III) reinforcement with the elasticity modulus $E_a = 210 \times 10^9 \text{ Pa}$, Poisson's ratio $\nu_a = 0.3$, by two rods with the diameter $d = 2 \text{ cm}$, located in an extended area. The fixing is rigid one. The model with the described parameters is shown in Fig. 2.

In this case, since the fixing is rigid one, a beam span is assumed to be equal to the length of the beam: $l = 300 \text{ cm}$. The estimated reinforcement tensile strength is taken equal to $R_a = 3600 \text{ kg cm}^{-2}$. The calculated concrete compressive strength is taken equal to $R_b = 17 \text{ MPa} = 173 \text{ kg cm}^{-2}$. The reinforcement area $A_s = 6.28 \text{ cm}^2$.

The protective layer of concrete is taken equal to $a_1 = 5 \text{ cm}$, thus the relative height is obtained as follows: $h_0 = 40 - 5 - 1 = 34 \text{ cm}$. The height of the concrete compressed zone is determined according to equation.

$$y = R_a A_s / R_b b \approx 3.27$$

Let's check whether this value is acceptable satisfying the following inequality: $y/h_0 \leq \xi_R$ where ξ_R , the

boundary value of the concrete compressed zone relative height for the reinforcement of A400 class equal to $\xi_R = 0.531$.

$$\frac{y}{h_0} = 0.096 \leq 0.531$$

The value of the moment is obtained from the following condition

$$M < R_b \text{ by } (h_0 - 0.5y)$$

In the context of the problem being solved, the following term should be satisfied:

$$M < 173 \times 40 \times 3.27 (34 - 0.5 \times 3.27) = 732368.17 \text{ kg cm}$$

Consequently, the maximum value of the moment is taken as $M = 7323 \text{ kg m}$. The bending moment for the concentrated force case is found according to the following Equation:

$$M = Ql/8$$

The expression for the concentrated load becomes the following one:

$$Q = 8M/l = 8 \times 8482/3 = 19538.7 \text{ kg}$$

For a given load value, the values of the longitudinal stresses σ_{zz} are determined in the points, corresponding to the maximum torques according to the load scheme shown on Fig. 3. The voltage values in the points corresponding to the maximum torques are found according to the following formula:

$$\sigma_{zz} = \frac{M}{W_x}$$

Where W_x the axial section modulus for the considered problem, equal to $W_x = b^3/6$. Thus, the voltage in the central section and in the beam edges (fixing points):

$$\sigma_{zz} = \frac{M}{W_x} = 686907 \text{ Pa}$$

which corresponds to the stress distribution pattern by applying a concentrated load equal to obtained Q value, if there is no consideration of concrete deformation diagram (Fig. 4). In the case of non-linear concrete properties consideration the distribution of stresses and the deflection are shown on Fig. 5. The model is designed

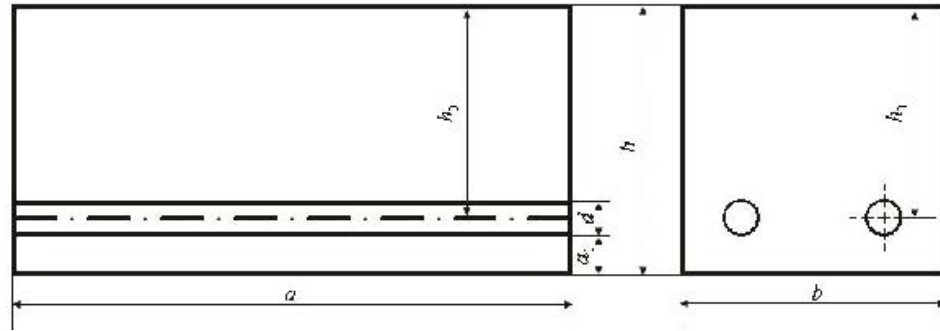


Fig. 2: Reinforced concrete beam with in the tension area

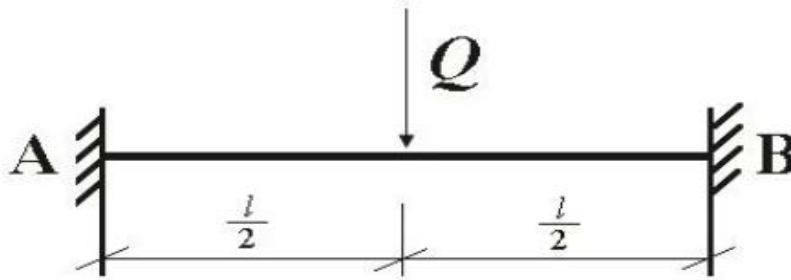


Fig. 3: Load scheme

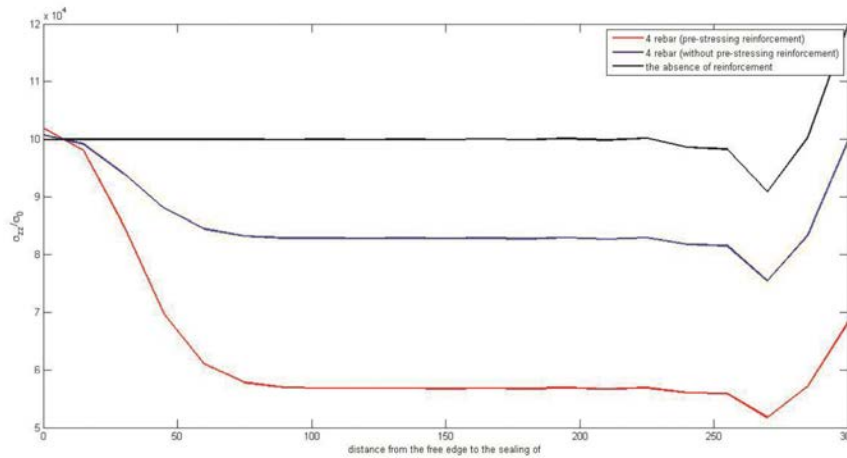


Fig. 4: Calculation according to the linear elasticity theory: a) The values of the stresses σ_{zz} ; b) deflection

using ANSYS package. During the simulation of concrete solid 65 material is selected (Gasratova and Stareva, 2015), involving the cracking at maximum load excess.

$$\sigma_{zz} = \frac{M}{W_x} = 686907 \text{ Pa}$$

when you consider the compressive stresses σ_{zz} within the terms of a concentrated load application found by analytical calculations according to linear elasticity theory, we can conclude that in the case of concrete nonlinear properties consideration the axial stress $\sigma_{zz} = 659669 \text{ Pa}$ in the points, corresponding to the

maximum values of the bending moment, differs significantly (a relative difference makes 4%) from the analytical results. If the diagram of concrete deformation is not taken into account, the relative difference between the analytical $\sigma_{zz} = 686907 \text{ Pa}$ and the numerical $\sigma_{zz} = 683098 \text{ Pa}$ stress values in described points does not exceed 0.6%.

In addition, it should be noted that the stress distribution patterns in the described cases are quite different which suggests that the neglect of concrete deformation diagram at the simulation cannot provide a sufficient idea about the stress-strain state of a reinforced concrete product.

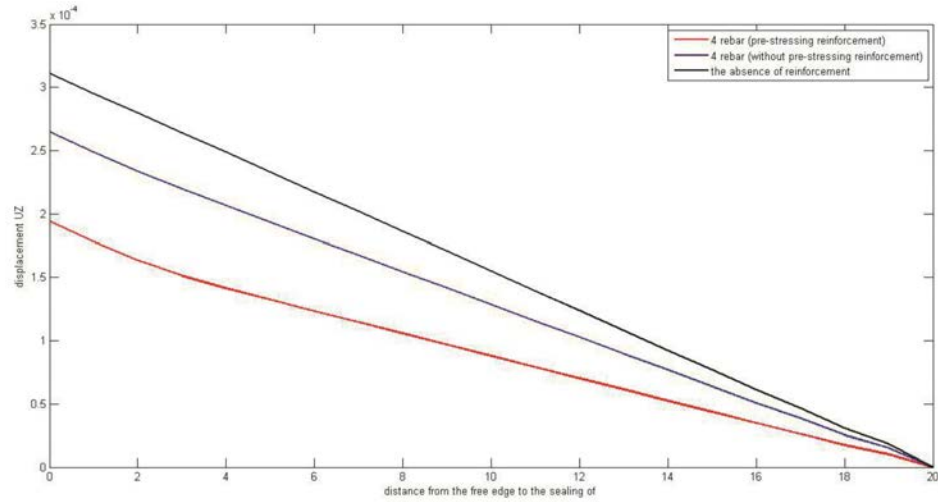


Fig. 5: Calculation according to non-linear elasticity theory: a) The values of stresses σ_{zz} ; b) deflection

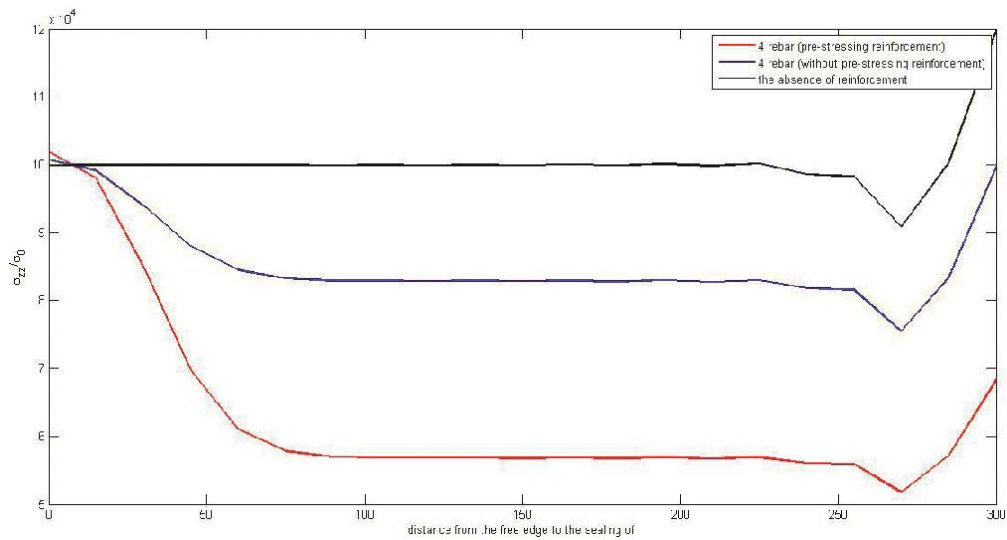


Fig. 6: Stresses σ_{zz}/σ_0 . The dependence on pre-stress in the reinforcement

RESULTS AND DISCUSSION

The effect of reinforcement prestress on the stress-strain state of reinforced concrete product: In order to harden the concrete in the tension zone designers often use such technique as reinforcement prestressing. This process proceeds as follows: a reinforcement is stretched, poured with concrete and released. Thus, concrete acquires initial compression, making it more susceptible to tensile stresses.

The impact of reinforcement prestressing on the stresses and the displacements in a design provides a certain interest. In order to determine this dependence the axial tension of a reinforced concrete beam is

used in the case of pre-tensioning absence and at the prestressing of reinforcement bars.

A beam has the following geometric dimensions: width $a = 40$ cm, height $b = 40$ cm, length $l = 300$ cm. The concrete class is B30, the modulus of elasticity makes $E_b = 32.5 \times 10^9$ Pa, Poisson's ratio makes $\nu_b = 0.18$. Hard concrete makes $\rho = 2280$ kg m^{-3} . The beam is reinforced by the reinforcement of A400 (A-III) type with the elasticity modulus $E_a = 210 \times 10^9$ Pa, Poisson's ratio makes $\nu_a = 0.3$, four rods with the diameter $d = 2$ cm. The right side of the beam is rigidly fixed, the tensile pressure $\sigma_0 = 100000$ Pa is applied to the left one. Reinforcement prestressing: $\sigma = 1000000$ Pa. The results for the stresses σ_{zz} and the movements along the axis OZ are presented on Fig. 6 and 7, respectively.

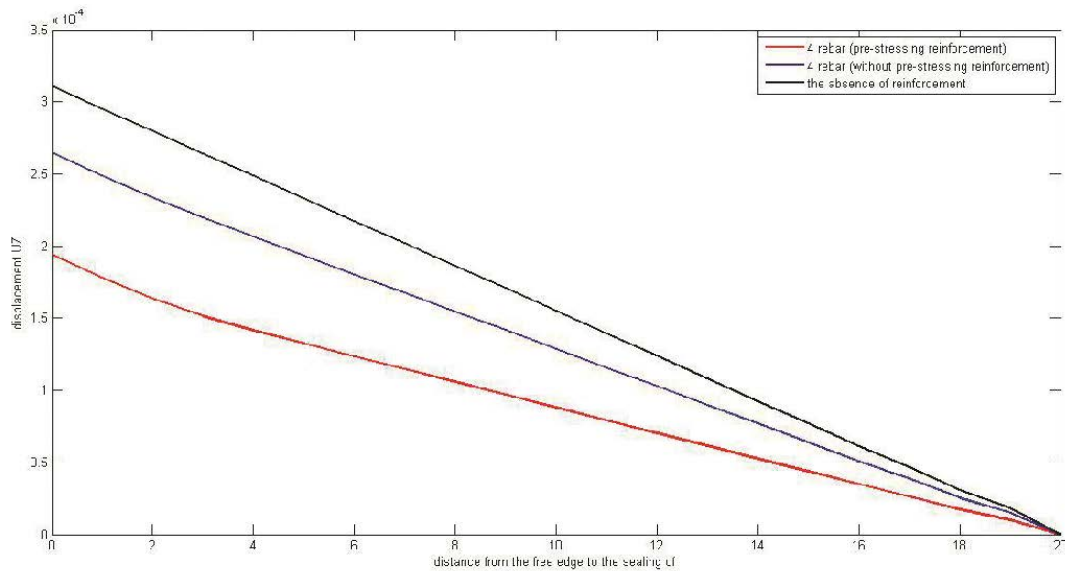


Fig. 7: Movements along the axis *OZ*. The dependence on reinforcement prestressing

The relative difference of movements in the case of a reinforcement prestressing and without it makes 26%. For stresses this value makes 30%, respectively. Thus, we may conclude that the prestressing of reinforcement significantly influences the stress-strain state of a reinforced concrete structure under tension.

CONCLUSION

The performed study allowed to obtain the following conclusions:

- The consideration of nonlinear material properties at the simulation plays an important role at the evaluation of a product stress-strain state. The refusal from concrete deformation diagram at modeling may confuse a researcher concerning the real picture of an object state
- The presence of prestress in a reinforcement may significantly reduce the tension (up to 30%) in the tension zone of concrete..

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