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Effects of Misspecification on the Economic-Statistical Design of the Coefficient of Variation Chart

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Abstract: The Coefficient of Variation (CV) chart monitors changes in the ratio of the standard deviation against the mean, instead of monitoring changes in the mean and/or standard deviation. The economic-statistical design of the CV chart allows practitioners to select optimal chart parameters which minimize the cost subject to constraints in the run length. However, a large number of parameters need to be estimated. This study looks at the effects when these parameters are incorrectly estimated. It was found that a minimal increase in cost is shown when the parameters are not estimated accurately, except for the incorrect specification of the shift size and the out-of-control quality cost which shows a slightly higher increase in cost. Thus, only a rough approximation is needed for most of the parameters.

Key words: Coefficient of variation chart, economic-statistical design, incorrect specification, increase in cost, rough approximation

INTRODUCTION

Control charts are one of the important tools used in process monitoring. Most control charts in the literature monitors changes in the mean and/or standard deviation. However, there are many in-control processes where the mean is expected to fluctuate and the standard deviation changes with the mean. In such processes where the mean and standard deviation do not remain constant, it is not feasible to use the traditional $\bar{X}$ and S or R charts. However, if such processes have a standard deviation which is a linear function of the process mean, changes in the process variability can be detected by monitoring the sample Coefficient of Variation (CV).

Kang *et al*. (2007) is the pioneer who proposed the first control chart monitoring the CV. Subsequently, several extensions are done to improve the performance of CV charts. Castagliola *et al*. (2011) proposed an Exponentially Weighted Moving Average (EWMA) chart for the CV while Calzada and Scariano (2013) proposed a synthetic chart for the CV. CV charts with varying chart parameters are also proposed. Castagliola *et al*. (2013) proposed the variable sampling interval CV chart while Castagliola *et al*. (2015) and Yeong *et al*. (2015a) proposed the variable sample size CV chart. Control charts monitoring the CV for multivariate data are proposed by Yeong *et al*. (2015b). These papers show that control charts monitoring the CV are receiving increasing attention, and charts with better detection ability are frequently proposed. Thus, practitioners have a wide variety of control charts to enable them to monitor the CV.

However, all the papers mentioned in the preceding paragraph only studied the statistical performance of the CV charts. In the existing literature, studies related to the economic properties of the CV chart cannot be found.

To maintain cost competitiveness in industries, Duncan (1956) proposed the economic design of the $\bar{X}$ chart. Subsequently, Lorenzen and Vance (1986) generalized the economic design so that it can be adopted on a wide variety of control charts. In the economic design, a systematic approach is provided in which it enables the practitioners to select the chart parameters which result in minimum cost. However, the economic design results in poor statistical properties. This leads Saniga (1989) to propose the economic-statistical design. In the economic-statistical design, statistical constraints are incorporated into the economic design, so that the statistical performance of the chart can be improved. In this study, constraints in terms of the in-control Average Run Length (ARL_0) and out-of-control Average Run Length (ARL_1) are adopted. It is desirable to have a large ARL_0, as this indicates fewer false alarms and a small ARL_1, as this indicates a quick detection of an out-of-control condition.

Extensive studies have been conducted on economic and economic-statistical designs, where some of the recent ones are Yeong *et al*. (2015b), Amiri *et al*. (2015),

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Noorossana *et al*. (2014), Chiu (2015), Guo *et al*. (2014), Franco *et al*. (2014), Saghaei *et al*. (2014), Lupo (2014) and Niaki *et al*. (2014).

A large number of parameters need to be estimated to implement the economic-statistical design. However, these parameters may not be estimated accurately in actual scenarios. Thus, this paper studies the effects of incorrectly estimating the parameters for the economic-statistical design of the CV chart.

MATERIALS AND METHODS

Coefficient of Variation (CV) chart: A brief overview of the CV chart proposed by Kang *et al*. (2007) is provided. The chart gives an out-of-control signal when the sample CV $\hat{\gamma}$ falls outside the upper and lower control limits. The sample CV, $\hat{\gamma} = S_i/\bar{X}_i$ where $\bar{X}$ and S_i are the sample mean and sample standard deviation of a sample of n independent and identically distributed normal random variables. The c.d.f. (cumulative distribution function) of $\hat{\gamma}$ is defined as follows:

$$F_{\hat{\gamma}}(x \mid n, \gamma) = 1 - F_t\left(\frac{\sqrt{n}}{x} \middle| n-1, \frac{\sqrt{n}}{\gamma}\right) \quad (1)$$

where, F_t is the c.d.f of the non-central t-distribution with n-1 degrees of freedom and non centrality parameter $\sqrt{n}/\gamma$. The Lower and Upper Control limits (LCL and UCL) can be computed as follows:

$$LCL = \mu_0(\hat{\gamma}) - k\sigma_0(\hat{\gamma}) \quad (2)$$

And:

$$UCL = \mu_0(\hat{\gamma}) + ks_0(\hat{\gamma}) \quad (3)$$

where, k is the control limit coefficient which affects the width of the in-control region and is determined by the practitioner.

In Eq. 2 and 3 $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$ are the mean and standard deviation of $\hat{\gamma}$ when the process is in-control, i.e, $\gamma_i = \gamma_0$ Since there is no closed form for $\mu_0(\hat{\gamma})$ and, $s_0(\hat{\gamma})$ approximations proposed by Yeong *et al*. (2015a) are adopted in this study, i.e.:

$$\mu_0(\hat{\gamma}) » \gamma_0 \left[\begin{aligned} &1 + \frac{1}{n}\left(\gamma_0^2 - \frac{1}{4}\right) + \frac{1}{n^2}\left(3\gamma_0^4 - \frac{\gamma_0^2}{4} - \frac{7}{32}\right) \\ &+ \frac{1}{n^3}\left(15\gamma_0^6 - \frac{3\gamma_0^4}{4} - \frac{7\gamma_0^2}{32} - \frac{19}{28}\right) \end{aligned} \right] \quad (4)$$

And:

$$s_0(\hat{\gamma}) » \gamma_0 \sqrt{\begin{aligned} &\frac{1}{n}\left(\gamma_0^2 + \frac{1}{2}\right) + \frac{1}{n^2}\left(8\gamma_0^4 + \gamma_0^2 + \frac{3}{8}\right) \\ &+ \frac{1}{n^3}\left(69\gamma_0^6 + \frac{7\gamma_0^4}{2} + \frac{3\gamma_0^2}{4} + \frac{3}{16}\right) \end{aligned}}. \quad (5)$$

The shift size τ is defined as $\tau = \gamma_i\gamma_0$ where γ_0/γ_1 are the in-control and out-of-control CV. The ARL_1 is computed as $ARL_1 = 1/P$ where:

$$P = 1 + F_{\hat{\gamma}}(LCL|n, \gamma_1) - F_{\hat{\gamma}}(UCL|n, \gamma_1)$$

The ARL_0 can be obtained by letting $\gamma_0 = \gamma_1$.

Economic-statistical design of the cv chart: This study adopts the general cost function of Lorenzen and Vance (1986) to determine the optimal values of the CV chart's parameters. The expected cost per unit time, C is obtained as follows:

$$C = \frac{\dfrac{C_0}{\lambda} + C_1B + \dfrac{b+cn}{h}\left(\dfrac{1}{\lambda} + B\right) + \dfrac{sY}{ARL_0} + W}{\dfrac{1}{\lambda} + \dfrac{(1-\varphi_1)sT_0}{ARL_0} + EH} \quad (6)$$

where, $B = (ARL_1\text{-}0.5)h+F$, $F = ne+\phi_1T_1+\phi_2T_2$, $EH = (ARL_1\text{-}0.5)h+G$, $G = ne+T_1+T_2$ and $s = (\lambda h)^{-1}\text{-}0.5$. The derivation of Eq. 6 is shown by Lorenzen and Vance (1986). The parameters in Eq. 6 are defined as follows:

Where:

ARL_0 = Average run length while in control
ARL_1 = Average run length while out-of-control
b = Fixed cost per sample
c = Cost per unit sampled
C = Expected cost per hour
C_0 = Expected quality cost per hour while in control
C_1 = Expected quality cost per hour while out-of-control
e = Expected time to sample and interpret one unit
h = Sampling interval
n = Sample size
s = Expected number of samples taken before an assignable cause occur
T_0 = Expected search time for a false alarm
T_1 = Expected time to find the assignable cause
T_2 = Expected time to repair the process
W = Cost of finding and fixing an assignable cause
γ = Cost of false alarm
φ_1 = 1 if production continues during search
= 0 if production stops during search

φ_2 = 1 if production continues during repair
= 0 if production stops during repair
λ = Expected failure time

The economic-statistical design selects the sample size (n), control limit coefficient for the CV chart (k) and the sampling interval (h) which minimizes C in Eq. 6, subject to constraints in the ARL_0 and ARL_1. This study describes the methodology to obtain the optimal n, k and h. For a given (n, k), the optimal h is obtained from the following Eq. 7:

$$h = \frac{-r_2 + \sqrt{r_2^2 - 4r_1r_3}}{2r_1} \quad (7)$$

Where:

$$r_1 = \frac{1}{2\lambda ARL_0}\left[\begin{array}{l}(ARL_1 - 0.5)(\lambda(Y + C_1T_0(-1+\varphi_1)) \\ \left(-2ARL_0\,(C_0 + \lambda(ARL_1 - 0.5)b + \right. \\ \left.(ARL_1 - 0.5)cn + W) + C_1(-1 + F\lambda - G\lambda))\right)\end{array}\right]$$

$$r_2 = -\frac{2(ARL_1 - 0.5)\left(\begin{array}{l}Y + C_1T_0(-1+\varphi_1) + \\ ARL_0(b+cn)(1+F\lambda)\end{array}\right)}{\lambda ARL_0}$$

And:

$$r_3 = -\frac{1}{2\lambda^2 ARL_0}\left[\begin{array}{l}2Y + 2C_0T_0(-1+\varphi_1) - bT_0\lambda - 2(ARL_1 - 0.5)bT_0\lambda - \\ 2C_1FT_o\lambda - cnT_0\lambda - 2(ARL_1 - 0.5)cnT_0\lambda - 2T_0W\lambda + \\ 2GY\lambda + bT_0\varphi_1\lambda + 2(ARL_1 - 0.5)bT_0\varphi_1\lambda + 2C_1FT_0\varphi_1\lambda + \\ cnT_0\varphi_1\lambda + 2(ARL_1 - 0.5)cnT_0\varphi_1\lambda + 2T_0W\varphi_1\lambda - bFT_0\lambda^2 - \\ cFnT_0\lambda^2 + bFT_0\varphi_1\lambda^2 + cFnT_0\varphi_1\lambda^2 + \\ 2ARL_0(b+cn)(1+F\lambda)(1+G\lambda)\end{array}\right]$$

The derivation of Eq. 7 is shown in Yeong *et al.* (2012). To obtain the approximate optimal values n*, k* and h*, the following numerical procedure is adopted:

- Start with the combination (n, k) = (2, 0.01)
- Calculate ARL_1 and ARL_0
- If ARL_0<250 or ARL_1, let C be a very large value. Else, calculate h and C from Eq. 6 and 7, respectively
- Increase k by 0.01, and maintain the values of n
- Repeat steps 2-4 until k = 3
- Reset k to 0.01, and increase n by 1
- Repeat steps 2-6 until n = 30

The chart parameters n*, k* and h* which give the overall approximate minimum cost per hour C* are the approximate optimal chart parameters for the CV chart. The upper limit of k is fixed as 3 because the minimum cost is always achieved before k = 3 while the upper limit of n is fixed as 30 as a sample size larger that 30 is usually not desirable. The purpose of step 3 is to impose a large penalty cost to the cost function when one or both of the ARL constraints are not satisfied. This is to avoid selecting the chart parameters which do not satisfy the ARL constraints.

RESULTS AND DISCUSSION

To illustrate the economic-statistical design of the CV chart, suppose we have the following cost and process parameters: γ_0 = 0.20, λ = 0.20, τ = 1.50, C_0 = \$114.24, C_1 = \$949.20, Y = W = \$977.40, b = \$0, C = \$4.22, e = T_0 = T_1 = 0.083, T_2 = 0.75, φ_1 = 1 and φ_2 = 0. We obtain the minimum cost per hour, C* as \$239.53 with the optimal chart parameters n* = 8, k* = 3.02 and h* = 0.79. The ARL_0 and ARL_1 are 254.42 and 4.75, respectively.

For the example in the preceding paragraph, we assume that all the input parameters are estimated accurately. However, in actual scenarios, these input parameters may be incorrectly specified. Thus, in this section, we would like to investigate the impact of misspecification of the input parameters toward the optimal cost. To study the effects of incorrect specification of each input parameter, the numerical examples in Table 1 and 2 are adopted. Note that the bolded values in Table 1 and 2 are the values which are different from the example in the first paragraph of this study. Table 1 shows the values of λ, τ, C_0, C_1, Y, W and b for example No. 1-29 while Table 2 shows the values of c, e, T_0, T_1, T_2, φ_1 and φ_2 for Example No 1-29.

We refer to the optimal chart parameters obtained in Paragraph 1 of this study, i.e., n* = 8, k* = 3.02 and h* = 0.79, as the benchmark design. To study the effects of incorrect specification, we compute the cost for Examples 1-29 of Table 1 and 2 based on the benchmark design, then compare it with the cost computed from optimal chart parameters based on correct values of the input parameters, i.e., the values shown in Table 1 and 2. We denote the cost based on the benchmark design as C, while the cost based on correct values of the input parameters is denoted as C*. For example, by referring to Example No 1, C is computed based on the chart parameters n* = 8, k* = 3.02 and h* = 0.79 which are the optimal chart parameters which minimizes the cost based on λ = 0.02 while C* is computed based on n* = 10, k* =

Table 1: Input parameters for the numerical examples

λ	τ	C_0 ($)	C_0 ($)	Y ($)	W ($)	b ($)
0.01	1.50	114.24	949.2	977.4	977.4	0
0.04	1.50	114.24	949.2	977.4	977.4	0
0.02	1.25	114.24	949.2	977.4	977.4	0
0.02	1.75	114.24	949.2	977.4	977.4	0
0.02	2.00	114.24	949.2	977.4	977.4	0
0.02	2.50	114.24	949.2	977.4	977.4	0
0.02	1.50	57.12	949.2	977.4	977.4	0
0.02	1.50	228.48	949.2	977.4	977.4	0
0.02	1.50	114.24	474.6	977.4	977.4	0
0.02	1.50	114.24	1898.4	977.4	977.4	0
0.02	1.50	114.24	949.2	488.7	977.4	0
0.02	1.50	114.24	949.2	1954.8	977.4	0
0.02	1.50	114.24	949.2	977.4	488.7	0
0.02	1.50	114.24	949.2	977.4	1954.8	0
0.02	1.50	114.24	949.2	977.4	977.4	5
0.02	1.50	114.24	949.2	977.4	977.4	10
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0
0.02	1.50	114.24	949.2	977.4	977.4	0

Table 2: Input parameters for the numerical examples

C ($)	e	T_0	T_1	T	φ_1	φ_2
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
4.22	0.083	0.083	0.083	0.750	1	0
2.11	0.083	0.083	0.083	0.750	1	0
8.44	0.083	0.083	0.083	0.750	1	0
4.22	0.042	0.083	0.083	0.750	1	0
4.22	0.166	0.083	0.083	0.750	1	0
4.22	0.083	0.042	0.083	0.750	1	0
4.22	0.083	0.166	0.083	0.750	1	0
4.22	0.083	0.083	0.042	0.750	1	0
4.22	0.083	0.083	0.166	0.750	1	0
4.22	0.083	0.083	0.083	0.375	1	0
4.22	0.083	0.083	0.083	1.500	1	0
4.22	0.083	0.083	0.083	0.750	0	0
4.22	0.083	0.083	0.083	0.750	0	1
4.22	0.083	0.083	0.083	0.750	1	1

2.98 and h^* = 1.37 which are the optimal chart parameters which minimizes the cost based on λ = 0.01. In this case, we will obtain C as $202.48 and C* as $198.79. Thus, incorrectly specifying λ = 0.01 as λ = 0.02 results in an increase of 1.82% from the optimal cost. The reason the effect of misspecification is studied is to examine the effect in cost when the chart parameters are determined based on incorrect values of the input parameters and to study the sensitivity of the optimal design towards changes in the input parameters. Table 3 shows the values for C, C* and the percentage increase of C from C*.

Table 3: Effects of incorrect specification

C ($)	C* ($)	Percentage increase of C from C* (%)
202.48	198.79	1.82
304.59	298.93	1.86
352.45	335.50	4.81
213.98	209.45	2.12
205.31	195.13	4.96
199.62	181.70	8.98
187.47	187.40	0.04
343.66	343.37	0.08
204.01	197.38	3.25
310.57	298.33	3.94
237.33	236.39	0.40
243.93	243.26	0.27
230.62	230.62	0.00
257.35	257.34	0.00
245.78	244.65	0.46
252.02	248.35	1.46
218.46	215.28	1.46
281.68	274.28	2.63
235.01	233.79	0.52
248.53	245.34	1.28
239.53	239.53	0.00
239.53	239.53	0.00
238.97	238.97	0.00
240.67	240.67	0.00
241.18	241.18	0.00
236.30	236.30	0.00
237.94	237.94	0.00
251.50	251.48	0.01
253.09	253.07	0.01

From Table 3, the percentage increase of C from C* are generally < 3%, except for the misspecification of τ and C_1. From Table 3, example No. 6 shows the largest increase of 8.98%. The optimal chart parameters in Example No 6 are computed based on τ = 1.50 when the actual value of τ is 2.50. From Table 3, we can see that the misspecification for W, T_0, T_1, T_2 φ_1 and φ_1 have no effect on the optimal cost.

Since, the increase in cost due to incorrect specification is generally not high, only a rough approximation is needed for most of the input parameters. However, a better approximation is needed for τ and C_1 as the percentage increase is slightly higher.

CONCLUSION

The economic-statistical design provides a systematic approach for practitioners to select the economically optimal chart parameters, while at the same time satisfying statistical constraints. However, the economic-statistical design requires a number of parameters to be estimated. These parameters may be incorrectly estimated in actual scenarios. Thus, this study studies the effects of incorrect estimation of the input parameters towards the optimal cost of the CV chart. It was shown that the increase in cost due to incorrect estimation is not large, thus, only a rough estimation of the input parameters are needed.

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