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Comparing the Modified Newton-Raphson Analysis and Sequential Linear Approximation in the Analysis of Large Fluctuations of the Beam

Sajjad Zangeneh, Seyed Javad Mohammadi Baygi and Nader Beheshti
Islamic Azad University, Damghan Branch, Damghan, Iran

Abstract: There has been a study on a comparison of large fluctuation's analysis in thin beams between two methods. These methods include sequential linear approximation and The modified Newton-Raphson analysis. These methods have been testet on a thin clamped long beam in finite elements model and the fluctuations of the beam changes made by the applied power to the tail are obtained. But out of this scope the sequential linear approximation method, despite having relatively good computational efficiency is less precise than Newton-Raphson method in terms of time and analysis stability.

Key words: Deformation, fixed the cracked beam, despite, less precise, computational

INTRODUCTION

Using linear methods that we'll deal with in the strength of materials is accompanied with different assumptions which are rarely accomplished in performane. Hence, using these methods sometimes decreases the accuracy in calculation and design. One of the instances in which strength of material's linear methods can make considerable error is long and thin beams analysis. There are various problems in engineering that solving them depends on carefull analysis of the beam's fluctuations especially in large transformations.

A good example is the analysis of long, thin composite and plastic beam's transformation or analysis of an aircraft's long, thin wing's transformations and tensions in aerodynamic loadings. One of the functional cases used in nonlinear analysis of non-metalic plait beams is interlaminar fracture test or adhesive test in laminated composites. In these problems, strain energy release rate that show the laminate's adhesion strength is a function of the fluctuation of applying power arms. Nevertheless calculating the laminate's separation in finite elements method is time-consuming and costly. So finding a method with suitable calculation extent and enough accuracy can have a variouty of usages (Williams, 1989; Crisfield, 1991). In reference (Webster, 1980) there is a review of nonlinear problems in solid and structural mechanics and existing methods.

The present study compares two prevailing methods of the beam's nonlinear analysis by means of finite elements method and its aim is to compare the accuracy and efficiency of two methods of the beam's nonlinear analysis which include sequential linear approximation and the modified newton-raohson method (Cook *et al*., 2001). ANSYS and NISA II finite elements softwares is used for analyzing by the modified Newton-Raohson method. And for sequential linear approximation method the public finite elements software FE77 is used. The effect of the beam length, the beam material, cross-section and the load value in the beam tail's transformation is computed by both methods and the diagram for eachone is drawn. It is considered that by increasing the beam length, dcreasing the beam's moment of interia, increasing the load and decreasing the the coefficient of elasticity, the difference of two methods will gradually increase until the results of sequential approximation will be considerably different from the modified Newton-Raphson analysis.

MATERIALS AND METHODS

Theory: Dividing the problems as linear or non-linear depends on their physics. Some problems can be explained by good approximation with linear relations that leads to analysis simplicity and computational efficiency increases. There are various methods for nonlinear analysis. In this study, two prevailing methods including the modified Newton-Raphson analysis (existing in NISA II and ANSYS Software) and sequential linear approximation method are accounted and are briefly explained. First, we consider the beam which has large transformations. We suppose that k nonlinear stiffness is a combination of k_0 instant stiffness and an stiffness dependant on transformation, k_n:

$$(k_0 + k_n) \times u = P \quad (1)$$

That:

$$k_n = f(u) \text{ or } f(P) \quad (2)$$

To explain the modified Newton-Raphson analysis, first the Newton-Raphson method is explaied. This

Corresponding Author: Sajjad Zangeneh, Islamic Azad University, Damghan Branch, Damghan, Iran

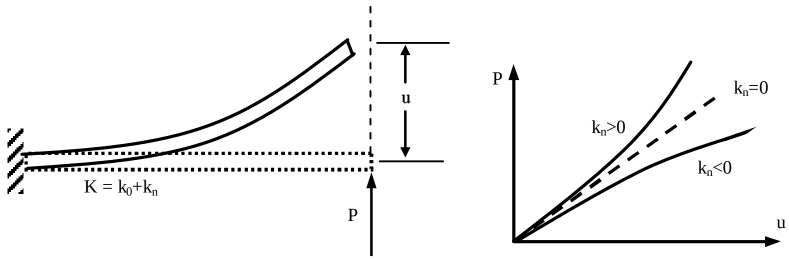

Fig. 1: Non-linear beam with large transformations (k_n>0 for large transformations and k_n<0 for plastic transformation or a combination of large and plastic transformations)

method which is shown schematically in Fig. 1 is based on the correction of spring stiffness to minimize the power changes or displacement in two consecutive repetitions. According to Fig. 2, we suppose that load P_a is applied to the beam and the related transformation is u_a, based on the Eq. 1 we'll have:

$$(k_0 + k_{na}) \times u_a = P_a \quad (3)$$

that k_{na} = f(u_A) in it. Now we increase the load to P_B and will compute the resultant displacement (u_B). We write the Taylor for p = f(u) around u_A:

$$f(u_A + \Delta u) = f(u_A) + (\frac{dP}{du})_A . \Delta u_1 \quad (4)$$

$$\frac{dP}{du} = \frac{d}{du}(k_0 u + k_N u) = k_0 + \frac{d}{du}(k_N u) = k_t \quad (5)$$

That k_t is known as tangential stiffness in it. Now we should determine u_1 as it is valid in f(u_A+Δu_1) P_B. So with having f (u_A) P_A and k_t in A point, Eq. 4 we'll have:

$$P_B = P_A + (k_t)_A \times \Delta u_1$$

Or:

$$(k_t)_A \times \Delta u_1 = P_B - P_A \quad (6)$$

That P_B-P_A is the non-balance load and comprises the difference between the applied load P_B and the P_A = (k_0+k_{NA}).u_A power in the beam when it's first fluctuation is u_A. After finding Δu_1, we'll correct the estimated displacement to u_1 = u_A+Δu_1. For the next repetition, we can find the new stiffness matrix, $(k_t)_1$,using Eq. 5 with the supposition of u = u_1. And we'll also find new non-balance load (P_B-P_1) using P_1 from relation (Eq. 1) when u = u_1. The estimated displacement value is as u_2 = u_1+Δu_2 that Δu_2 will be found by $(k_t)_1$.Δu_2 = P_B-P_1.

The only difference between the modified Newton-Raphson method and Newton-Raphson method

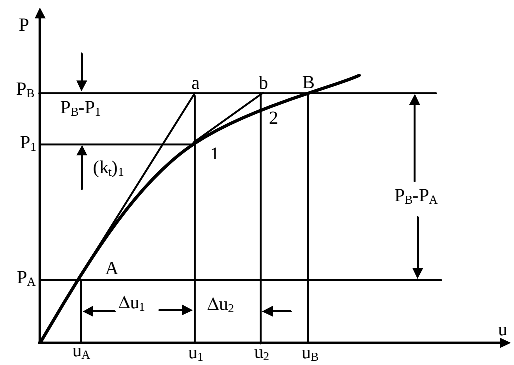

Fig. 2: Newton-Raphson's method in non-linear problems

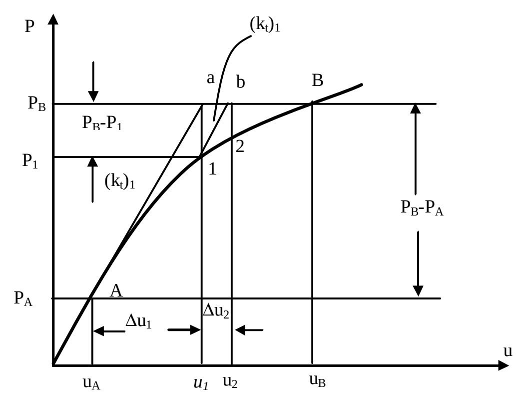

Fig. 3: The corrected Newton-Raphson method in non-linear problems anlysis

is that k_t's stiffness in the modified method is corrected just once in as many repetitions as possible, therefore in large models and problems, excessive repeats are prevented. This method is shown schematically in Fig. 3.

In the sequential linear approximation method, the beam's nonlinear stiffnesss is computed as a series of linear approximations. In this method, first we analyze the beam's finite element model with a part of the applied power and will find the shifts in the model's points. Then by summing the point shifts with nodal coordinates of the model elements, we'll have a new finite element model of the beam. After that, another part of the power is applied to this new model and linear analysis is repeated. This process will be repeated until the whole power is applied to the beam. This method is so easy for testing in each finite elements code and has no divergence problem. We can also complete this analysis by using a linear finite elements code but in some problems it may be necessary to have several repetitions to achieve high accuracy. For more explaining the method, we'll rewrite the relation Eq. 1 as p = F (u) and with defining the k_t = dP/du and considering the additive load ΔP for sequential repetitions that begin from p = 0 and u = 0 we'll have:

$$\begin{aligned} u_1 &= 0+(k_t)_0^{-1}.\Delta p_1; \quad (k_t)_0 = k_t; \quad u = 0 \\ u_2 &= u_1+(k_t)_1^{-1}.\Delta p_2; \quad (k_t)_1 = k_t; \quad u = u_1 \\ u_3 &= u_2+(k_t)_2^{-1}.\Delta p_3; \quad (k_t)_2 = k_t; \quad u = u_2 \\ &\vdots \\ u_{i+1} &= u_i+(k_t)_i^{-1}.\Delta p_{i+1} \end{aligned} \tag{7}$$

These stages are also shown in Fig. 4:

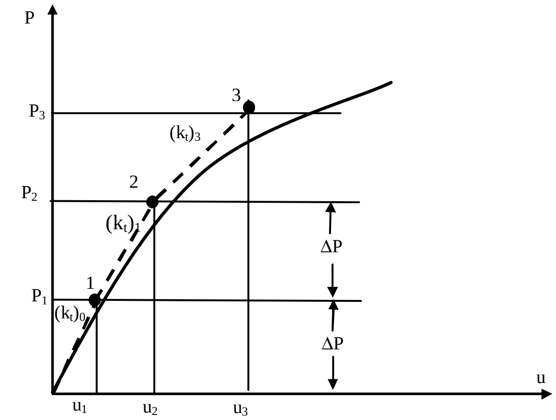

Fig. 4: The sequential linear approximation method

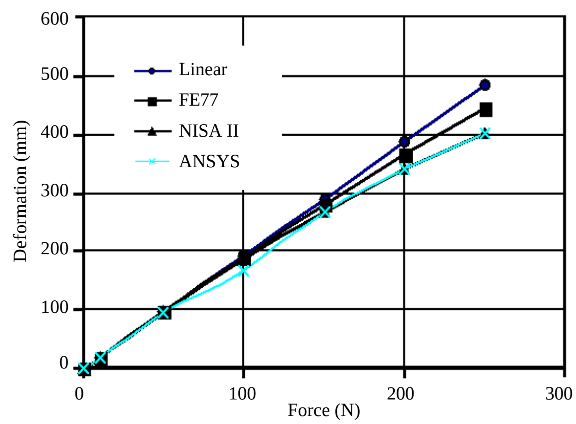

Fig. 5: Displacement versus force changes diagram

RESULTS AND DISCUSSION

Numerical comparison between the modified Newton-Raphson method and the sequential linear approximation: In this study, we'll compare the modified Newton-Raphson method and the sequential linear approximation for analyzing large fluctuations in a long, thin clamped beam. We will use the ANSYS and NISA-II finite element software for the modified Newton-Raphson method and the features of FE77 linear and general finite element software for analyzing the sequential linear approximation method. The FE77 Software has the feature of repeated analysis of finite element in one loop and by equally dividing the applied load, after each time implementing the finite elements linear software, the model's coordinates is summed with point shifts and so a new model is gained. In the mentioned sequential linear approximation analyzes in this section (exept for the specified items) the applied load to the beam was completed in 10 steps.

The desied beam has 1m length and 1×1 mm cross-section and is an steel beam with E = 207 Gpa modulus and 0.3 Poisson's ratio and load P is applied to it. The finite elements model is made of an invariable network with eight-node membrane elements that reduced integration has been used in it.

The results of the transformation of the beam's tail, using the linear analysis (Crandall *et al.*, 1978) and the desired non-linear analyzes are shown in Table 1 and Fig. 5.

Now we'll calculate the transformations of the beams with ideal length that are under constant power p = 250 N by means of three introduced softwares (all the beam's features are the same as last part). The results are shown in Table 2 and Fig. 6. Changes of the fluctuations of the beam versus the moment of inertia of their plane area changes for the linear method, sequential linear approximation and the modified Newton-Raohson method are given in Table 3 and Fig. 7. In this analysis, the beam's length is 1 m, its' breed is steel and is under 250 N charge. Now we calculate transformations of the beam with different materials. In this modeling, the beam's length is 1m and the applied power is 100 N and the beam cross-section is considered 10×10 mm. The results are given in Table 4 and Fig 8.

As we see in the charts by increasing the power applied to the beam and increasing it's length and also decreasing modulus or moment of interia of plane area, the difference of the two software's-NISA II and FE77 responses gradually increases. This means the increase of

Table 1: Examining the effect of power on transformation of the beam's tail

The applied load (N)	The linear analysis responses (mm)	The sequential linear analysis responses FE77 (mm)	The modified Newton-Raphson analysis responses NISA II (mm)	The modified Newton-Raphson analysis responses ANSYS (mm)
0	0.000	0.000	0.00	0.000
10	19.316	19.311	19.27	19.318
50	96.580	96.067	95.55	95.714
100	193.160	190.340	186.30	186.320
150	289.740	280.950	268.20	268.230
200	386.330	366.560	340.10	339.840
250	482.910	444.300	401.80	401.250

Table 2: The effect of beam length on the transformation of the beam tail

The beam length (m)	The linear analysis responses (mm)	The sequential linear analysis responses FE77 (mm)	The modified Newton-Raphson analysis responses NISA II (mm)	The modified Newton-Raphson analysis'responses ANSYS (mm)
0.2	0.0039	0.0039	0.0039	0.00380
0.3	0.0130	0.0130	0.0130	0.00130
0.4	0.0308	0.0308	0.0307	0.03070
0.5	0.0602	0.0599	0.0595	0.05950
0.6	0.1040	0.1030	0.1014	0.10130
0.7	0.1650	0.1621	0.1570	0.15700
0.8	0.2464	0.2388	0.2268	0.22670
0.9	0.3507	0.3336	0.3096	0.30880
1.0	0.4802	0.4443	0.4018	0.40120
1.1	0.6397	0.5627	0.5033	0.50130
1.2	0.8293	0.6661	0.6079	0.60670

Table 3: The effect of moment of interia on the transformation of beam tail

The moment of interia 10^{-10} (m)^4	The linear analysis responses (mm)	The sequential linear analysis responses FE77 (mm)	The modified Newton-Raphson analysi's responses NISA II (mm)	The modified Newton-Raphson analysis responses ANSYS (mm)
6	0.6668	0.5491	0.4976	0.4953
8	0.5008	0.4591	0.4144	0.4128
9	0.4455	0.4219	0.3809	0.3796
10	0.4011	0.3802	0.3518	0.3513
12	0.3344	0.3221	0.3048	0.3035
14	0.2867	0.2789	0.2668	0.2664
15	0.2676	0.2612	0.2512	0.2509

Table 4: The effect of the beam modulus on the beam tail's transformation

Young's modulus 10^9* (N/m^2)	The linear analysis responses (mm)	The sequential linear analysis responses FE77 (mm)	The modified Newton-Raphson analysis' responses NISA II (mm)	The modified Newton-Raphson analysis' responses ANSYS (mm)
50	0.7621	0.6495	0.5450	0.5447
70	0.5975	0.5040	0.4491	0.4492
100	0.3979	0.3780	0.3486	0.3494
150	0.2655	0.2596	0.2489	0.2495
200	0.1992	0.1968	0.1919	0.1923

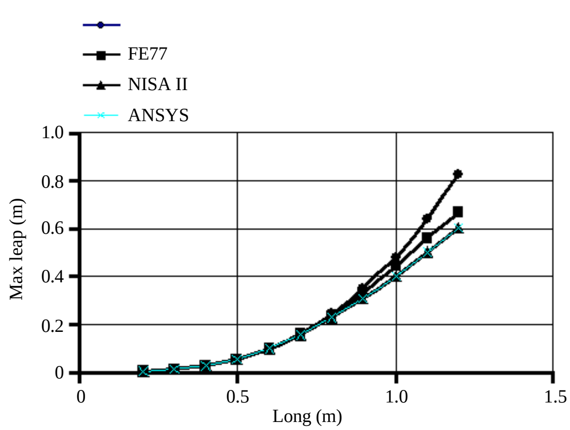

Fig. 6: Changes of fluctuations of maximum beam versus beam length

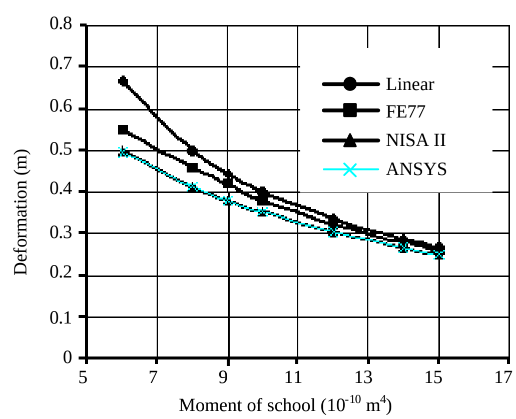

Fig. 7: Fluctuations of maximum beam changes versus the beam's moment of area

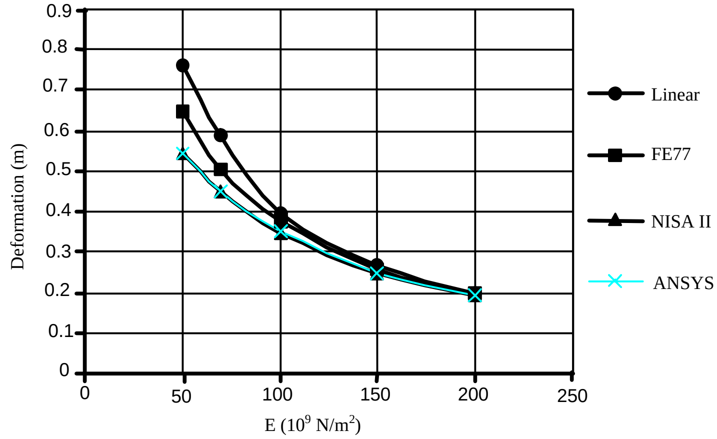

Fig. 8: Fluctuations of beam changes versus modulus changes

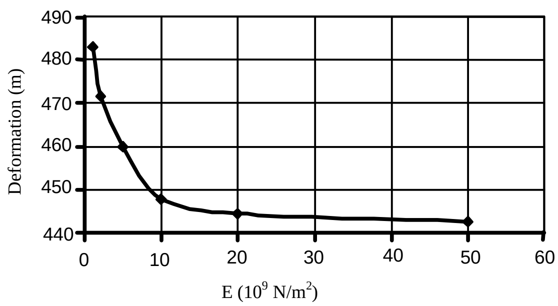

Fig. 9: The sequential linear approximation method's convergence

Table 5: The effect of the number of repetitions on the analysis of linear approximation method

The number of repetitions	The sequential analysis responses (m) FE77
1	810/4
2	718/4
5	597/4
10	474/4
20	443/4
50	442/4

the difference of the responses obtained from non-linear modified Newton-Raphson method and sequential linear approximation method. To examine the effect of the number of repetitions on the responses of sequential linear approximation, we have checked the method's convergence by modeling the beam with 250N load and different number of repetitions.

The results are given in Table 5 and Fig 9. As we observe, according to Fig. 9, by increasing the number of repetitions, the analysi's response gets closer to the final response and in this problem choosing 20 repeatitions is a reasonable choice.

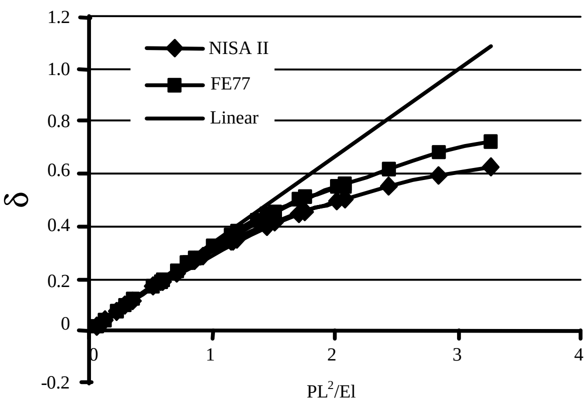

Fig. 10: The results of dimentionless analysis for the linear method, sequential linear and the modified Newton-Raphson

Dimensionless value's analysis: Now we make the obtained values from the two numerical methods dimensionless and draw a diagram for them. We explain these dimensionless values as PL^2/EI and δ/L by the use of Buckingham δ theorem that δ indicates the transformation of the free tail of the beam. The results are presented in Fig. 10. Now by means of this diagram we can compute the δ/L values in a certain PL^2/EI in the modified linear and nonlinear Newton-Raphson analyzes and sequential linear approximation method and then compare the δ value in three above-mentioned methods.

According to the presented dimentionless diagram, we can see that. For dimensionless values of $PL^2/EI<0.6$, we can use linear methods with good approximation that could help saving time and cost. For dimensionless values of $<PL^2/EI<1.5$ 0.6, we can use sequential linear approximation with good approximation and 10% error and $0.6<PL^2/EI<1.0$ with 5% error for non-linear analyzes. For dimensionless $PL^2/EI>1.5$ values, we need to use the modified Newton-Raphson method in order not to bear so many errors.

CONCLUSION

The results of experimental tests and analyzing two desired nonlinear methods indicate that despite good computational efficiency of sequential linear aproximation compared to the modified Newton-Raphson method and lack of computational divergence in this method, it's accuracy for $PL^2/EI>1.5$ to determine the fluctuation is unacceptable. Coefficient of PL^2/EI is obtained from making the analysis results dimensionless and can be regarded as an index for the beam nonlinearity. In low values of this coeficient ($PL^2/EI<0.6$) we can basically use strength of material's linear method. Also the sequential linear method in the scope of $0.6<PL^2/EI<1.5$ is mostly accurate to compute the fluctuation of the beam. In $PL^2/EI>1.5$ values, using nonlinear Newton-Raphson mathod can lead to accurate results. It is also observed that by increasig the amount of power and the beam length and also decreasing the moment of interia and modulus, the difference obtained from the modified Newton-Raphson method and sequential linear approximation method will increase.

REFERENCES

Cook, R.D., D.S. Malkus, M.E. Plesha and R.J. Witt, 2001. Concepts and Applications of Finite Element Analysis. 4th Edn., John Wiley and Sons Inc., New York, ISBN-10: 0471356050, pp: 784.

Crandall, S.H., N.C. Dahi and T.J. Lardner, 1978. An Introduction to the Mechanics of Solids. 2nd Edn., McGraw Hill, New York, USA.

Crisfield, M.A., 1991. Nonlinear Finite Element Analysis od Solids and Structures. Vol. 1, John Wiely & Sons, New York, USA.

Webster, R.L., 1980. On the static analysis of structures with strong geometric nonlinearity. Comput. Struct., 11: 137-145.

Williams, J.G., 1989. Fracture mechanics of anisotropic materials. Appl. Fract. Mech. Compos. Mater., 6: 3-38.