

Reliability Estimation of Machine Parts with Complicated Geometry on a Base of Methods of Nonparametric Statistics

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Abstract: The study considers the durability estimation task of details with complicated geometrical shape working in random regime at operation. The analytical dependences for stress-strain condition calculation of machines part and units do not exist, therefore estimation of the stresses and displacements of such objects is possible to carry out only by computer simulation with the help of numerical methods: Finish Element Method (FEM). External loads to details (pressure and temperature) are random values and generally aren't described by known laws of distribution. Researchers have developed the original algorithm for estimation of probability of no-failure operation of details based on use of the apparatus of nonparametric statistics. Adjustment of nonparametric generators of random numbers is realized by methods of nonparametric statistics in accordance with real samples of pressure and temperature. As a result of realization of multiple-factor, computer experiment for calculation the stress-strain condition of detail under random loading the functions approximating the stress variation in dangerous points depending on the loads level are determined. On the basis of these functions, the estimation of probability of no-failure operation in all dangerous points of a detail is carried out. The algorithm developed by researchers is shown on the example of durability estimation (probability of no-failure operation) of the body of wedge valve KZ13010-100.

Key words: Durability, wedge valve, stress-strain condition, finish element method, multiple-factor computer experiment, nonparametric statistic

INTRODUCTION

The study considers the task of durability estimation of details with complicated geometry. Analytic dependences for stresses determination in dangerous places of such details are absent. Approximate methods do not guarantee the required precision of calculations.

Base stages of the durability estimation task: Details are subjected to the influence external both force and temperature loads in exploitation conditions. Loads are random variables, their functions of density of distribution can not be described with required error of the first kind in accordance with tests for concordance by methods of parametric statistics. The researchers propose new approach, including following base stages (Fig. 1):

- Determining of functions of density of distribution for random external loads by methods of nonparametric statistics (Botev *et al.*, 2010; Syzrantsev and Chernaya, 2014; Syzrantsev *et al.*, 2015; Syzrantseva, 2009b, c). Adjustment of nonparametric generators of random numbers on basis of these functions
- Planning of the multiple-factor experiment for realization the calculation of stress-strain condition of details
- Carrying out of computer experiment realizing the calculation of finite number of variants of details stress-strain conditions by numerical method of the elasticity theory (for example, by finish element Method (Oden, 2010; Syzrantsev *et al.*, 2003; Syzrantseva, 2009a; Wittbrodt *et al.*, 2012)
- Determining of the functions, approximating the stress variation in dangerous places on surface of researched detail depending on values of external loads
- Obtaining the representative samples of stresses in dangerous places of detail with the help of generators of random numbers (Syzrantsev and Chernaya, 2014; Syzrantseva, 2009a, b) and approximating functions
- Establishing the functions of density of distribution of stresses in detail dangerous points by methods of nonparametric statistics (Syzrantsev and Chernaya, 2014; Syzrantsev *et al.*, 2015; Syzrantseva, 2009b, c) in accordance with these representative samples

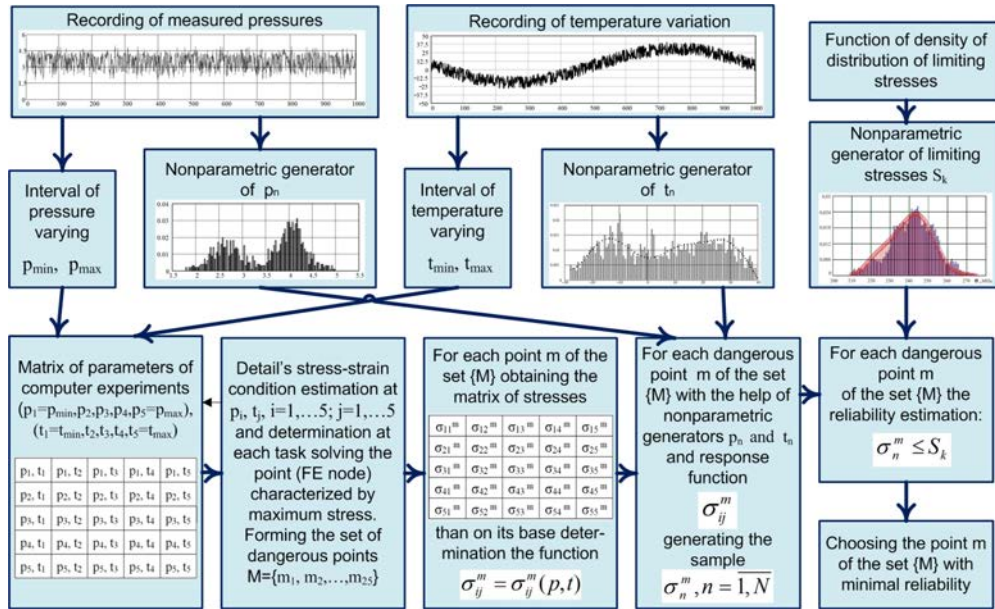


Fig. 1: Algorithm of task decision

- Calculating of probability of no-failure operation for each dangerous place on basis of the functions of density of distribution of real stresses and the functions of density of distribution of limit stresses
- Choice of detail place with minimal probability of no-failure operation, characterizing total durability of researched detail

MATERIALS AND METHODS

Realization of the method: We shall consider proposed approach by the example of decision of durability estimation task (probability of no-failure operation) for body of wedge valve KZ13010-100 (Fig. 2), loaded in real operational conditions by random values of pressure and temperature, presented correspondingly by samples $p_i, i = \overline{1, n_p}$ and $t_j, j = \overline{1, n_t}$.

Establishment of the functions of density of distribution of pressure and temperature: On the first stage, using point values measured values of pressure $p_i, i = \overline{1, n_p}$ and temperature $t_j, j = \overline{1, n_t}$, we shall establish by method of minimization of empirical risk (Syzyantseva, 2009b, c) the functions of density of distribution of pressure $f_{Np}(p)$ and temperature $f_{Nt}(t)$:

$$f_{Np}(p) = \sum_{i=1}^{N_p} \lambda_{pi} \times \varphi_i \left[\frac{(p - A_p)}{(B_p - A_p)} \right] \quad (1)$$

$$\varphi_i(\alpha) = \cos \left[(2i - 1) \frac{\pi}{2} \alpha \right]$$

$$f_{Nt}(t) = \sum_{j=1}^{N_t} \lambda_{tj} \times \varphi_j \left[\frac{(t - A_t)}{(B_t - A_t)} \right] \quad (2)$$

$$\varphi_j(\alpha) = \cos \left[(2j - 1) \frac{\pi}{2} \alpha \right]$$

Where:

- α = Element of $[0, 1]$
- λ_{pi} = Coefficients
- $I = \overline{1, N_p}$
- $J = \overline{1, N_t}$
- $A_p = \min_i \{p_i\}$
- $B_p = \max_i \{p_i\}$
- $A_t = \min_j \{t_j\}$
- $B_t = \max_j \{t_j\}$
- $N_{p,t}$ = No. of expansion terms

Then, we shall realize the adjustment of nonparametric generators of random values p_n and $t_n, n = \overline{1, N}$ on basis of Eq. 1 and 2 where N generated sample length.

Computer experiment planning: Development of computer aids has allowed to increase fundamentally, the validity and accuracy of calculations of the stress-strain

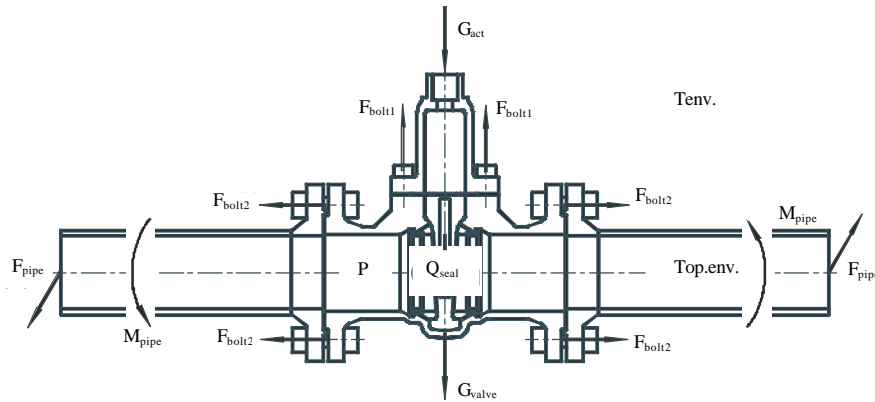


Fig. 2: Loading diagram for pipeline valves in operational conditions; P = Operation environment pressure (MPa); G_{valve} = Weight of valve (N); G_{act} = Weight of acuator (N); F_{bolt1} , F_{bolt2} = Forces in bolted connections of flange (N); Q_{seal} = Force in wedge sealing (in “close” position) (N); T_{env} = Temperature of environment, centigrade degrees; Top_{env} = Temperature of operation environment centigrade degrees; Top_{Env} = Temperature centigrade degrees; F_{pipe} = Forces arising on account of flange misalignment (N); M_{pipe} = Bending moment of pipe line (N*mm)

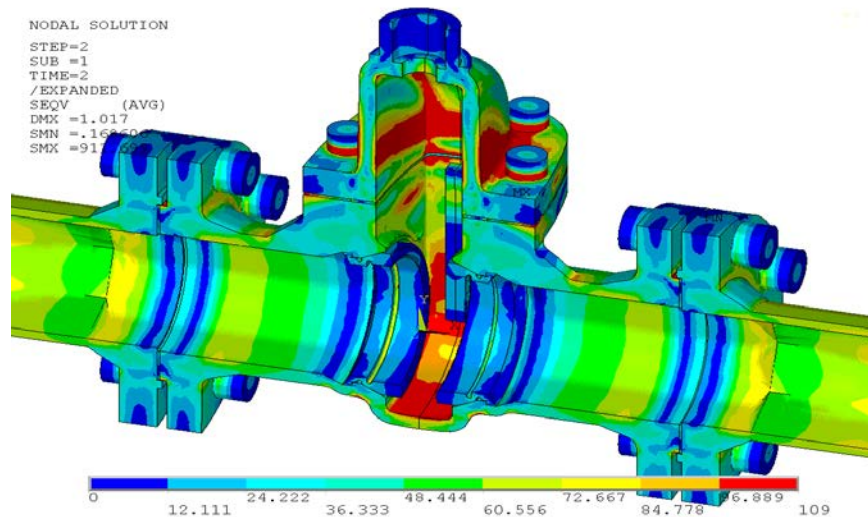


Fig. 3: Equivalent von Mises stresses distribution in details of wedge valve KZ13010-100 in accordance with applied temperature and external loads

condition of details with complicated geometry. There are developed a few software, the most commonly used is ANSYS (Syzrantseva, 2009a-c), realizing finish element method (Oden, 2010; Oshibkov *et al.*, 2015; Syzrantseva, 2009a, c). As an example, Fig. 3 and 4 illustrate the results of stresses (σ) and temperature (t) estimation in pipeline stop valve (Syzrantsev *et al.*, 2013) which appear at concrete pressure $p_i = \text{const}$ and external temperature $t_j = \text{const}$. However, ANSYS is too massive and can not be used as a subroutine in general program. Despite of performance of modern powerful computers, calculations by ANSYS are very laborious and prolonged. In other words, using ANSYS, we are able to execute some tens of

calculations but no some hundreds or some thousands as it is necessary for generating of representative sample of stresses $\sigma_{i,n} = \overline{1, N}$. Therefore, for calculation of probability of no-failure operation of wedge valve body, we shall use the technique, based on processing of computer experiment results which lead to obtaining the regression dependences $\sigma^{(m)}, \sigma^{(m)}(p, t)$, $m = \overline{1, M}$ for M dangerous points of a body. Realization of this technique provides carrying out two-factor experiment: first factor is pressure, second factor is temperature.

Interval of factors varying is defined by limit values of pressure and temperature taken from samples $p_{i,i} = \overline{1, n_p}$ and $t_{j,j} = \overline{1, n_t}$, $p_{\min} = \min\{p_i\}$, $p_{\max} = \max\{p_i\}$, $t_{\min} = \min\{t_j\}$,

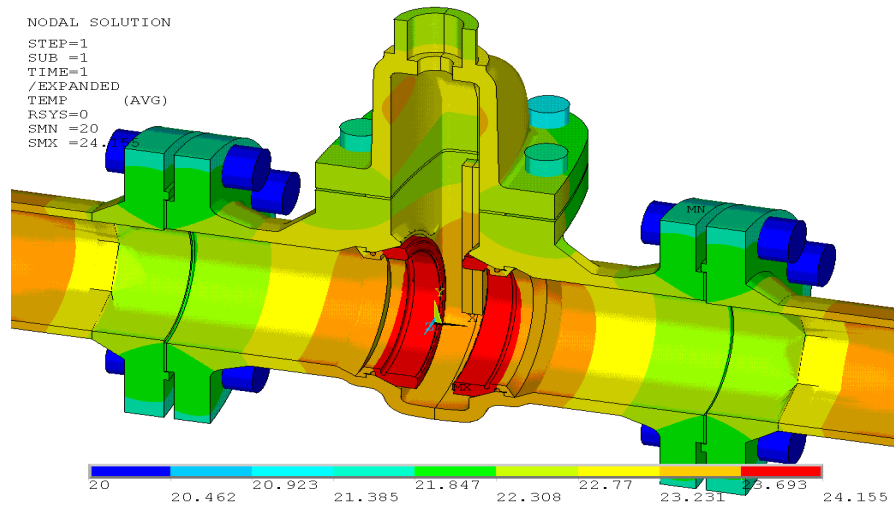


Fig. 4: Temperature distribution in details of wedge valve KZ13010-100 in accordance with applied temperature and external loads

$t_{\max} = \max\{t_i\}$. As dependences $\sigma^{(m)} = \sigma^{(m)}(p, t)$, $m = \overline{1, M}$, in general case are smooth and continuous, for approximation, we shall use polynomial function of a kind:

$$\sigma^{(m)} = a_0^{(m)} + a_1^{(m)} \times t + a_2^{(m)} \times t^2 \quad (3)$$

Where:

$$\begin{aligned} a_0^{(m)} &= b_{00}^{(m)} + b_{01}^{(m)} \times p + b_{02}^{(m)} \times p^2 \\ a_1^{(m)} &= b_{10}^{(m)} + b_{11}^{(m)} \times p + b_{12}^{(m)} \times p^2 \\ a_2^{(m)} &= b_{20}^{(m)} + b_{21}^{(m)} \times p + b_{22}^{(m)} \times p^2 \end{aligned}$$

Expanding Eq. 3, we shall obtain the expression:

$$\begin{aligned} \sigma^{(m)} &= b_{00}^{(m)} + b_{01}^{(m)} p + b_{10}^{(m)} t + b_{02}^{(m)} p^2 + b_{20}^{(m)} t^2 + \\ & b_{11}^{(m)} p t + b_{12}^{(m)} t p^2 + b_{21}^{(m)} t^2 p + b_{22}^{(m)} t^2 p^2 \end{aligned} \quad (4)$$

Stress-strain condition calculations: For determination of values of unknown coefficients $b_{ij}^{(m)}$, $i = \overline{0, 2}$; $j = \overline{0, 2}$ of Eq. 4, we shall realize the computer experiment which consist in carrying out several calculations of body stress-strain condition by finish element method (Syzrantseva, 2009a-c; Wittbrodt *et al.*, 2012) at fixed values pressure $p_i = \text{const}$ and temperature $t_i = \text{const}$, $i = \overline{1, L}$. At task decision, it is expediently to use the experiment planning methods. Follow these methods, we shall switch from dimensional values p_i and t_i , to nondimensional $\overline{p}_i (-1 \leq \overline{p}_i \leq +1)$ and $\overline{t}_i (-1 \leq \overline{t}_i \leq +1)$:

$$\overline{p}_i = \frac{2p_i - (p_{\max} + p_{\min})}{p_{\max} - p_{\min}} \quad (5)$$

And:

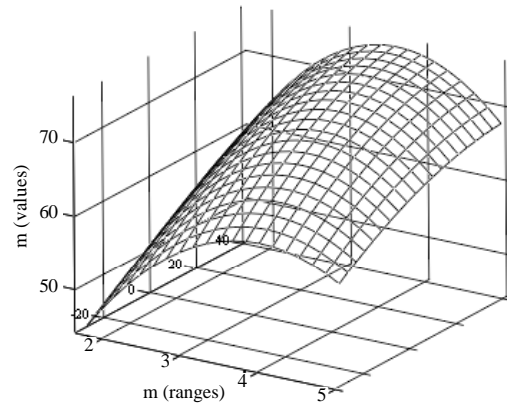


Fig. 5: Response function 4

$$\overline{t}_i = 2t_i - (t_{\max} + t_{\min}) / t_{\max} - t_{\min}$$

Calculation of Eq. 4 coefficients $b_{ij}^{(m)}$, $i = \overline{0, 2}$; $j = \overline{0, 2}$ is connected with necessity of using the results of 9 experiments as minimum. It demands carrying out bifactorial experiment at varying of parameters p_i and t_i on three levels. We shall define these levels by next values: $\overline{p}_i = -1; 0; +1$ and $\overline{t}_i = -1; 0; +1$. Consequently, we shall obtain the matrix of experiment planning, presented in Table 1. After body stress-strain condition calculations by finish element method at pressure and temperature values, corresponding to levels of varying, we shall establish 9 values of real stresses $\sigma_i^{(m)}$, $i = \overline{1, 9}$ for each dangerous point "m". Right column of Table 1 illustrates these stress values for one of body dangerous point "m" as an example (Fig. 5).

Table 1: Matrix of experiment planning

Exp. No.	\bar{t}_i	\bar{p}_i	$\sigma_i^{(m)}$	$\sigma_i^{(m)}$ (MPa)
1	-1	-1	$\sigma_1^{(m)}$	45
2	-1	0	$\sigma_2^{(m)}$	60
3	-1	+1	$\sigma_3^{(m)}$	58
4	0	-1	$\sigma_4^{(m)}$	53
5	0	0	$\sigma_5^{(m)}$	70
6	0	+1	$\sigma_6^{(m)}$	65
7	+1	-1	$\sigma_7^{(m)}$	60
8	+1	0	$\sigma_8^{(m)}$	75
9	+1	+1	$\sigma_9^{(m)}$	68

RESULTS AND DISCUSSION

Determining of the functions, approximating the stress changing in dangerous points: Substituting these values to left part of Eq. 4 and corresponding to this experiment values \bar{p} and \bar{t} from Table 1 to right part of Eq. 4 for each experiment, we shall obtain the linear equation system:

$$\begin{aligned}
 \sigma_1^{(m)} &= b_{00}^{(m)} - b_{01}^{(m)} - b_{10}^{(m)} + b_{02}^{(m)} + b_{20}^{(m)} + b_{11}^{(m)} - b_{12}^{(m)} - b_{21}^{(m)} + b_{22}^{(m)}; \sigma_2^{(m)} = b_{00}^{(m)} - b_{10}^{(m)} + b_{20}^{(m)}; \\
 \sigma_3^{(m)} &= b_{00}^{(m)} + b_{01}^{(m)} - b_{10}^{(m)} + b_{02}^{(m)} + b_{20}^{(m)} - b_{11}^{(m)} - b_{12}^{(m)} + b_{21}^{(m)} + b_{22}^{(m)}; \sigma_4^{(m)} = b_{00}^{(m)} - b_{01}^{(m)} + b_{02}^{(m)}; \\
 \sigma_5^{(m)} &= b_{00}^{(m)}; \sigma_6^{(m)} = b_{00}^{(m)} + b_{01}^{(m)} + b_{02}^{(m)}; \sigma_7^{(m)} = b_{00}^{(m)} - b_{01}^{(m)} + b_{10}^{(m)} + b_{02}^{(m)} + b_{20}^{(m)} - b_{11}^{(m)} + b_{12}^{(m)} - b_{21}^{(m)} + b_{22}^{(m)}; \\
 \sigma_8^{(m)} &= b_{00}^{(m)} + b_{01}^{(m)} + b_{02}^{(m)}; \sigma_9^{(m)} = b_{00}^{(m)} + b_{01}^{(m)} + b_{10}^{(m)} + b_{02}^{(m)} + b_{20}^{(m)} + b_{11}^{(m)} + b_{12}^{(m)} + b_{21}^{(m)} + b_{22}^{(m)};
 \end{aligned} \tag{6}$$

After solving of this system, we shall obtain following expressions for coefficients $b_{ij}^{(m)}, i = \overline{0,2}; j = \overline{0,2}$:

$$\begin{aligned}
 b_{ij}^{(m)}, i = \overline{0,2}; j = \overline{0,2}: \\
 b_{00}^{(m)} &= \sigma_5^{(m)}; b_{01}^{(m)} = (\sigma_6^{(m)} - \sigma_4^{(m)}) / 2; b_{10}^{(m)} = (\sigma_8^{(m)} - \sigma_2^{(m)}) / 2; b_{11}^{(m)} = (\sigma_1^{(m)} - \sigma_3^{(m)} - \sigma_7^{(m)} + \sigma_9^{(m)}) / 4; \\
 b_{02}^{(m)} &= (\sigma_4^{(m)} + \sigma_6^{(m)}) / 2 - \sigma_5^{(m)}; b_{12}^{(m)} = (\sigma_2^{(m)} - \sigma_8^{(m)}) / 2 - (\sigma_1^{(m)} + \sigma_3^{(m)} - \sigma_7^{(m)} - \sigma_9^{(m)}) / 4; \\
 b_{20}^{(m)} &= (\sigma_2^{(m)} + \sigma_8^{(m)}) / 2 - \sigma_5^{(m)}; b_{21}^{(m)} = (\sigma_4^{(m)} - \sigma_6^{(m)}) / 2 - (\sigma_1^{(m)} - \sigma_3^{(m)} + \sigma_7^{(m)} - \sigma_9^{(m)}) / 4; \\
 b_{22}^{(m)} &= (\sigma_1^{(m)} + \sigma_3^{(m)} + \sigma_7^{(m)} + \sigma_9^{(m)}) / 4 - (\sigma_2^{(m)} + \sigma_4^{(m)} + \sigma_6^{(m)} + \sigma_8^{(m)}) / 2 + \sigma_5^{(m)}
 \end{aligned} \tag{7}$$

For values $\sigma_1^{(m)}, t = \overline{1,9}$ of Table 1 in accordance with Eq. 7 the values $b_{ij}^{(m)}$ are obtained:

$$\begin{aligned}
 b_{00}^{(m)} &= 70; b_{01}^{(m)} = 6; b_{10}^{(m)} = 7.5; b_{02}^{(m)} = -11; b_{20}^{(m)} = -2.5 \\
 b_{11}^{(m)} &= -1.25; b_{12}^{(m)} = -1.25; b_{21}^{(m)} = -0.75; b_{22}^{(m)} = 1.25
 \end{aligned}$$

In that way, for body point “m” Eq. 4 at nondimensional parameters (\bar{p}, \bar{t}) looks like:

$$\begin{aligned}
 \sigma^{(m)} &= 70 + 6\bar{p} + 7.5\bar{t} - 11\bar{p}^2 - 2.5\bar{t}^2 - \\
 &1.25\bar{p}\bar{t} - 1.25\bar{p}^2 - 0.75\bar{t}^2 + 1.25\bar{p}\bar{t}^2
 \end{aligned} \tag{8}$$

And for determination of function $\sigma^{(m)} = \sigma^{(m)}(p, t)$, it is necessary to transform expression Eq. 8 using dependences (Eq. 5). Response function (Eq. 4) at varying of pressure and temperature in intervals $p_{\min} = 1.8$, $MPa \leq p \leq p_{\max} = 5.0$ MPa; $t_{\min} = -28^\circ C \leq t \leq t_{\max} = 40^\circ C$ is shown on Fig. 5.

Describing the functions of density of distribution of limit stresses: Now, we shall return to task presented by Fig. 1. Obtained Eq. 8 taking into account dependences Eq. 5 allows to generate for each point “m” the sample of stresses $\sigma_n^{(m)}, n = \overline{1, N}$ by nonparametric generators of pressure $(p_n, n = \overline{1, N})$ and temperature $(t_n, n = \overline{1, N})$ and then to describe by methods (Botev *et al.*, 2010; Syzrantsev and Cheranya, 2014, Syzrantsev *et al.*, 2015) the functions of density of distribution of stresses:

$$f_{N\sigma}(\sigma^{(m)}) = \sum_{k=1}^{N_\sigma} \lambda_{\sigma k} \times \varphi_k \left[\frac{(\sigma^{(m)} - A_\sigma)}{(B_\sigma - A_\sigma)} \right] \tag{9}$$

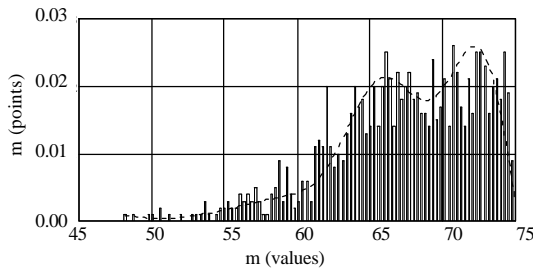


Fig. 6: Functions of density of distribution of stresses in points “m” researched body of wedge valve

Where:

$$A_{\sigma} = \min_n \{\sigma_n^{(m)}\}$$

$$B_{\sigma} = \max_n \{\sigma_n^{(m)}\}$$

$$\varphi_k(\alpha) = \cos [(2 \times k - 1) \times \pi / 2 \times \alpha]$$

As an example Fig. 6 illustrates the functions of density of distribution of stresses and its approximation in kind (Eq. 9) for one of points “m” researched body of wedge valve. Coefficients of approximation function are determined with the help of method of minimization of empirical risk (Syzrantseva, 2009a, b). Then by nonparametric generator S_k of limiting stresses (as endurance limit σ_{-1}), we shall obtain the sample with required length and describe the functions of density of distribution of limit stresses:

$$f_{N_s}(\sigma_{-1}) = \sum_{r=1}^{N_s} \lambda_{sr} \times \varphi_r \left[\frac{(\sigma_{-1} - A_s)}{(B_s - A_s)} \right] \quad (10)$$

$$A_s = \min_r \{\sigma_{-1r}\}, B_s = \max_r \{\sigma_{-1r}\}$$

Calculating the probability of no-failure operation: After that, we shall calculate for each point “m” of researched body the probability of no-failure operation:

$$R^{(m)} = \int_0^{\infty} \left\{ \int_0^{\infty} \left[\sum_{r=1}^{N_s} \lambda_{sr} \times \varphi_r \left(\frac{\sigma_{-1} + \sigma - A_s}{B_s - A_s} \right) \right] \times \left[\sum_{k=1}^{N_{\sigma}} \lambda_{\sigma k} \times \varphi_{\sigma k} \left(\frac{\sigma - A_{\sigma}}{B_{\sigma} - A_{\sigma}} \right) \right] d\sigma \right\} d\sigma_{-1} \quad (11)$$

Total durability of body R is defined that point “m”, which is characterized by minimal calculated in accordance with Eq. 10 value $R^{(m)}$:

$$R = \min_m (R^{(m)})$$

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