Journal of Engineering and Applied Sciences 11 (2): 225-231, 2016
ISSN: 1816-949X

A Study of the Effect of Distance of the Cross Beams on Retrofitting the Girders

Nobakht Bakhtiari
Department of Civil Engineering, Islamic Azad University, Maragheh Branch, Maragheh, Iran

Abstract: Plate girders are used frequently in major industrial structures. In the systems supported by these types of beams, reduced deflection and decreased maximum bending moment can be considered as a retrofitting method which can be done using cross beams or braced beams as the main elements perpendiculars to the girders. In the present study, the effect of cross beams and their distance on retrofitting the girders has been studied. Differential equation governing the behavior of the new structures is a fourth grade one is determined. Later, its solution is continued using mathematical transformations and the fundamental relationships are reached at. The produced relationships have led to solution of a practical issue. The results offered have been compared with those obtained from modelling in SAP 2000, showing precise consistency. It is concluded that different values of stress, bending moments and forces in the systems may be determined by changing the stiffness and distance of the cross beams.

Key words: Girders, retrofitting, stiffness, cross beam, effect

INTRODUCTION

Using plate girders in building construction of about 100 m may be economical but, in buildings of higher spans, it requires trusses. Web thin flexural members are influenced by the flexural behavior of stiffness and are placed in the cross beam having the ability to neutralize the destroying forces. In two-dimensional structures, the beam for plate industrial floor systems are used. We can add elements and new components perpendicular to the bearing element in a two-dimensional structure so that the entire stiffness of structure to increase. These members can be placed both horizontally or vertically between the existing structures and turn their behavior into a spatial or three dimensional one. By creating three-dimensional structures, the upper connections will be available in two directions perpendicular to communicate and connect parts.

LITERATURE REVIEW

Since, 1960 extensive researches have been conducted in USA and Japan on the slender girders. The research in this area led to determination of the effect of the cross beams in the lateral torsional buckling (Masoumy, 1980). Sadovsky *et al.* (2005) focuses on a method to determine the basic principle of stable energy to measure the degree of distortion and deformation effects of errors. Hu and Cui (2003) compared the ultimate strength of stiffened and non-stiffened roof systems by simple analytical formulas designed to measure final strength of beams. Paik and Lee (2003) designed a method for treatment of stiffened plastic lateral plates under different loads until reaching their final strength (Bruneau and Lee, 2005). Zhang (2011) worked on a method for designing oblique intersection beams in the roof structures. The investigations in this area have continued, leading to optimization of this system (Gennadi and Sergei, 2013).

The effects of the beam span length, depth, number and spacing of plate girders, flange width and curvature of the stress ratio distortion on bending stress was evaluated by Davidson *et al.* (1996). Sharafbayani and Linzel showed that using a diagonal cross beams can optimize inhibitory length of plate girders and thus eliminate providing additional cross beams. These researchers extend their proposed results to multi-span bridge and the findings show adequate control of vertical and lateral deformation in reducing the number of cross-beam.

STATEMENT OF THE PROBLEMS AND RESEARCH OBJECTIVES

A glance at the literature review reveals that sufficient research on determining the analytical effects of cross forces are not available. Taking that into consideration, it is shown in the present study that by adding cross beams, the general behavior of roof coverage system shifts from buckling mode towards inelastic local buckling in the compression flange of the plate girders and as the result, the shear buckling strength along with the elastic stiffness of the structure increase. On the other hand, securing the stability of the plate girders embedded in the bracing points and the general stability of the system against the inflicted loads are of utmost importance. It needs mentioning that the plate girders only undergo vertical dispositioning under the gravity loads if enough lateral bracing does exist. The main function of the cross beams, in addition to that of creating lateral bracing is to prevent lateral-torsion buckling. Furthermore in the provided method, the cross beams are let into the calculations as the secondary load bearing component.

Behavioral changes of a two-dimensional structure to make a three-dimensional one leads to increased stiffness in the whole space and thereby, decreased deflection of structure. This issue constitutes one of the main points in investigating the linear elastic behavior of the structures and particularly, the steel elements in the structural retrofitting projects. In the present study, a particular analytical method is provided and compared with computerized finite element method.

THE MAIN DIFFERENTIAL EQUATION

Let us consider cross beams (bracing beams) vertically on the girders having equal distance from each other as shown in Fig. 1a. We assume profile their stiffness and load equal. We suppose the specificities of the main girders, their stiffness and loading are equal. Reaction forces exerted by the main beams onto the cross beams are shown in Fig. 1b. For the cross beam in the optional point of intersection, the following differential equation for deformation (Chen and Lui, 1987) is provided as following:

$$Z_i = +\beta \frac{QL_1^3}{EI_i} - \alpha \frac{R_j L_1^3}{EI_i} \tag{1}$$

Where:

- Z_i = Maximum deflection of the girder or truss
- Q = Loads of the main beam
- R_j = Reaction of cross beam in response of the main beam
- E = Modulus elasticity of materials
- I_i = Moment of inertia for main beam
- α and β = Coefficients related to the specification of the load

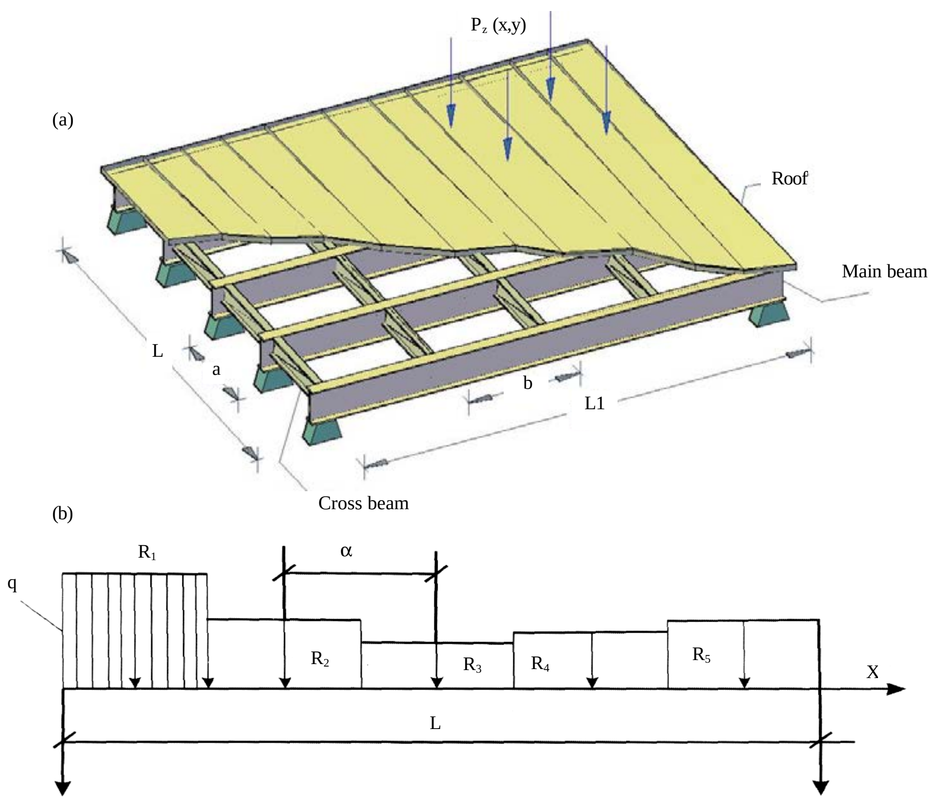


Fig. 1: Girders in roof systems, cross beam reactions on the main beam: a) Original envelope girders; b) Reaction forces exerted by the main beams onto the cross beams

Such coefficients can be pre-set according to the type of load and the rest points. If the primary beam number is no.<5, it will still be possible to substitute single reactions with massive loads, i.e., $q = R_j/a$ q: a massive load of intensity (Hu and Cui, 2003).

Thus, the existing beams (main beams) are considered in the direction of i shot at the intersection of the main beam transverse nodes j and optionally, a set of a×b are obtained from Eq. 2:

$$Z_i = \beta_i Q \frac{L_1^3}{EI_i} - \frac{L_1^3}{EI_i} \sum_{j=1}^{j=m} \alpha_j R_j \tag{2}$$

The β_i is the deflection influence coefficient of the main beam which is created due to the influence of the load on the same in the intersection of the ith main beam with jth cross beams.

Equation 2, m is the number of beams added to the cross section of roof system. By placing and simplifying the procedure, we will have:

$$\begin{aligned} q &= \frac{d^2M}{dx^2} = EI_i \frac{d^4z}{dx^4},\ R_j = aq = aEI_i\left(\frac{d^4z}{dx^4}\right), \\ Z_i &= \beta_i Q \frac{L_1^3}{EI_i} - \frac{aL_1^3}{EI_i} \sum_{j=1}^{m} \alpha_j EI_i \frac{d^4z}{dx^4} \end{aligned} \tag{3}$$

MATHEMATICAL CONVERSIONS

Understanding Eq. 3 is rather difficult and complicated and mathematical transformations will be needed to solve it. If we assume that in both sides the numbers of beams (or trusses) are high and their stiffness are equal then solving the problem would be easier and solving differential equation of degree 4 will continue as follows. According to Fig. 1, the amount of load will be as p.a.b for each node in the intersection of the beams. In general cases of $P_z(x, y)$, node load at the intersection of the incoming beam is split into two parts. The R value for the main beams is parallel to the x-axis and y-axis is parallel to the cross beam of R and ω as deflection in any part of the roof:

$$\begin{aligned} R + \bar{R} &= P_z(x,y)ab \\ R = aEI_? \frac{\partial^4\omega}{\partial x^4},&\ \bar{R} = bEI_y \frac{\partial^4\omega}{\partial y^4} \end{aligned} \tag{4}$$

By substituting R and $\bar{R}$ in Eq. 4, we will have:

$$aEI_x \frac{\partial^4\omega}{\partial x^4} + bEI_y \frac{\partial^4\omega}{\partial y^4} = P_z(x,y)ab \tag{5}$$

Considering the size of the system created in two directions x and y and simulating a structural steady a consistent performance space in three-dimensional format, we can skip the grid ceiling system and an integrated suite can be written as (Makowski, 1981; Alinia and Kashizadeh, 2006):

$$\omega = f \sin\frac{\pi x}{L} \sin\frac{\pi y}{L_1} \tag{6}$$

Where:

ω	=	Deflection in any part of the roof (ω is supposed as equivalent to z)
f	=	Deflection in the middle of the roof
L_1 and L	=	Length of main and cross beams

Ordinary differential equations can be obtained by replacing relation Eq. 5 in Eq. 6 whose results will be equal to the solution of the differential Eq. 3. But, before continue to further solving, the following mathematical transformations are required to apply. In general cases, Eq. 6 can be written as: z

$$\omega = \Sigma X_n(x) Y_n(y) \tag{7}$$

Derivatives of X_n (x) function will change with respect to the constant factor, in other words, their properties change iteratively and can be written as:

$$\frac{d^4X_n(x)}{dx^4} = \eta_n^4 X_n(x) \tag{8}$$

Using orthogonal properties of these relationships, it is possible to convert the right side of Eq. 5 to form a series of related mathematical functions of X_n (x) and then, write it as follow:

$$P_z(x,y)ab = b\Sigma P_n(y) X_n(x) \tag{9}$$

P_n (y) is the arguments function of y. To determine the expression P_n (y) , we can multiple X_n (x) to both sides in Eq. 9 and integrate it as follow:

$$\int_0^L P_z(x,y)abX_m(x)dx = b\sum P_n(y)\int_0^L X_m(x)X_n(x)dx$$

Here, for X_n (x) orthogonal properties are considered and it can written:

$$P_n(y) = a\frac{\int_0^L P_z(x,y)X_n(x)dx}{\int_0^L X_n^2(x)dx} \tag{10}$$

By substituting Eq. 7 in Eq. 5 and considering the Eq. 8, the below equation will be achieved:

$$\sum X_n(x)\left[bEI_y Y_n^{IV}(y)+\eta_n^4 aEI_x Y_n(y)-P_n(y)b\right]=0$$
$$EI_y Y_n^{IV}(y)=P_n(y)-k_n Y_n(y) \quad (11)$$

$$k_n=EI_x\eta_n^4\frac{a}{b} \quad (12)$$

Equation 11 is the relation of the beam on the elastic foundation where in the Y_n parameter can be determined according to the condition of the anchor. Thus, instead of applying complicated Eq. 4, we can use ordinary differential Eq. 8 and 11. After calculating the values of $X_n(x)$ and $Y_n(y)$ by placing them in Eq. 7, the values of ω will be calculated easily. The obtained values ω will be derived two or three times with respect to x and y and the bending moment and shear force will be calculated in both directions then the amount to which the stress decreases with respect to two-dimensional roof will be determined. Choosing proper distance and stiffness for cross beams will adjust the stress imposed on the existing main beams.

APPLICATION OF RELATIONS

Characteristic relation of Eq. 8 will be as:

$$\lambda_4-\eta_n^4=0$$

The roots of the equation are: λ-1 = η-n; $\lambda_2=-\eta_n$; $\lambda_3=i\eta_n$; $\lambda_4=-i\eta_n$. Particular solutions are as follow: $e^{\eta_n^x};e^{-\eta_n^x};e^{i\eta_n^x};e^{-i\eta_n^x}$. The combination of particular solutions with general ones in the so called equation will be as follow:

$$X_n(x)=A\,ch\eta_n^x+B\,sh\eta_n^x+C\cos\eta_n^x+D\sin\eta_n^x \quad (13)$$

By integrating from Eq. 11 and applying the boundary conditions, we can determine the coefficients of Eq. 13 and then by applying mathematical transformations and integration, we have:

$$\frac{d^4Y_n(y)}{dy^4}+4\beta_n^4Y_n(y)=\frac{P_n(y)}{EI_y} \quad (14)$$

$$\beta_n^4=\frac{k_n}{4EI_y};\ \beta_n=4\sqrt{\frac{a}{4b},\frac{EI_x}{EI_y}}\frac{n\pi}{L} \quad (15)$$

To solve Eq. 14, we use hyperbolic functions and optional requirements for simplified integration which are as follow:

$$\bar{A}_{on}=-\frac{aq}{n\pi\beta_n^4EI_y}$$
$$\bar{B}_{on}=-\frac{aq}{n\pi\beta_n^4EI_y}\times\frac{\sin 2\beta_n l_1-2\,chs\,\beta_n l_1}{\cos 2\beta_n l_1-ch2\beta_n l_1} \quad (16)$$
$$\bar{C}_{on}=-\frac{aq}{n\pi\beta_n^4EI_y}\times\frac{sh\,2\beta_n l_1-2shc\,\beta_n l_1}{\cos 2\beta_n l_1-ch2\beta_n l_1}$$

Finally, the general solution of the deflection equation will be as follow:

$$\omega=\sum_{n=1}\sin\frac{n\pi}{L}\times\begin{pmatrix}-\dfrac{aq}{n\pi^4\beta_n^4EI_y}+\bar{A}_{on}chc\beta_n y+\\ \bar{B}_{on}chs\beta_n y+\bar{C}_{on}\,chc\beta_n y\end{pmatrix} \quad (17)$$

Here:

$$chc\beta_n y=ch\beta_n y\times\cos\beta_n y$$
$$chs\beta_n y=ch\beta_n y\times\sin\beta_n y$$
$$shc\beta_n y=sh\beta_n y\times\cos\beta_n y$$
$$shs\beta_n y=sh\beta_n y\times\sin\beta_n y$$

Using the relations M_x = -EI_x ω_x”, Q_x = -EI_x ω_x” (and M_y = -EI_y ω_y”, Q_y = -EIy ω_y”) we can determine shear the applied moments to the values of the reaction of rely and also calculate the influence of the main and cross beams:

$$M_x=-\frac{n^2\pi^2EI_x}{L^2}\sum_{n=1}^{n=\infty}\sin\frac{n\pi}{L}\begin{pmatrix}\dfrac{aq}{n\pi^4\beta_n^4EI_y}+\bar{A}_{on}chc\beta_n y+\\ \bar{B}_{on}ch+\bar{C}_{on}shc\beta_n y\end{pmatrix}$$
$$Q_x=-\frac{n^3\pi^3EI_x}{L^3}\sum_{n=1}^{n=\infty}\cos\frac{n\pi}{L}\begin{pmatrix}\dfrac{aq}{n\pi^4\beta_n^4}+\bar{A}_{on}chc\beta_n y+\\ \bar{B}_{on}chs\beta_n y+\bar{C}_{on}shc\beta_n y\end{pmatrix} \quad (18)$$

SOLVING THE PROBLEM

For plan and plate girder sections shown in Fig. 2, beams with a length of 30 m direction (cross beams for retrofitting) and 24 m in direction y are considered aiming to regulate the stress. The distance of the beams is considered 3 m in longitudinal direction and 5 m in transverse direction. Flexural stiffness is assumed to be equal in two directions. The load imposed by the roof covering equals to: q = 5 KN m^{-2}. Deflection and anchor of the roof cover are studied and compared in both the situations of using and not using the cross beams.

For various nodes, the amount of deflection is calculated according to the functions provided in the present study and the results have been presented in Table 1. Furthermore, problem modeling has been done by SAP 2000 and the results of deflection analysis have been presented using two methods for comparison in Table 1.

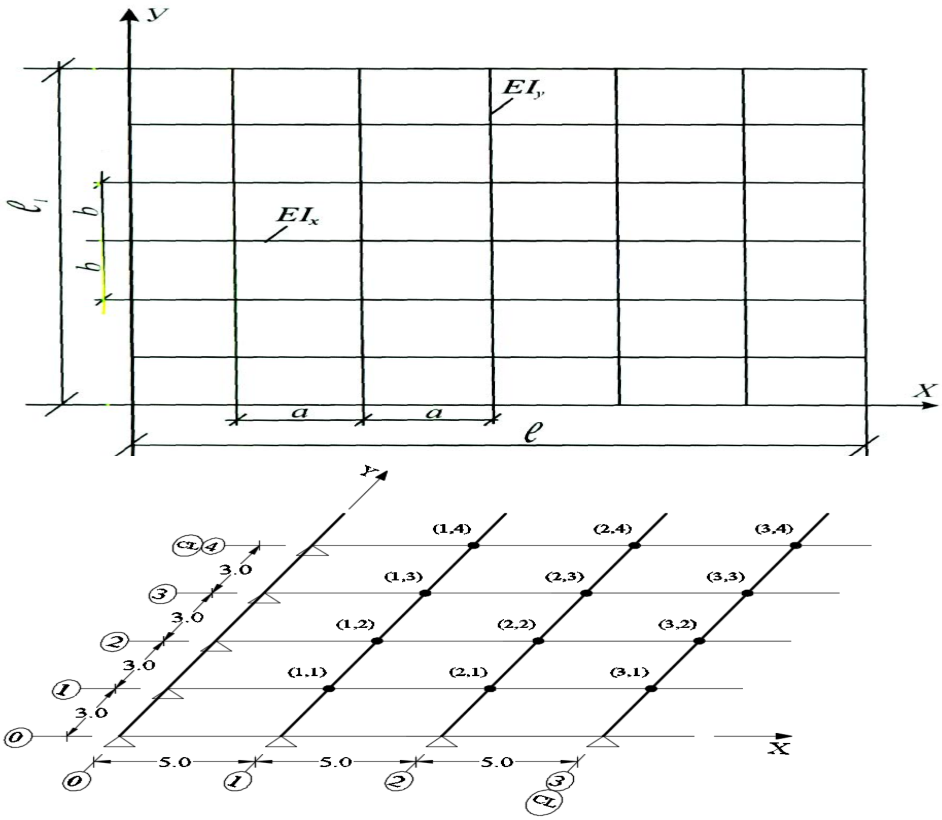


Fig. 2: Main grid system with accessory added beams and coordinates of nodes

Table 1: Comparing the proposed method with the results from SAP2000

Node	ω			M_x			M_y			Calculation method
	1	2	3	1	2	3	1	2	3	
1	1.25	2.10	2.40	224	292	314	345	545	612	Analytical
	1.19	2.00	2.28	218	290	308	336	537	610	SAP 2000
2	2.20	3.85	4.41	391	543	585	537	903	1024	Analytical
	2.16	3.65	4.17	390	537	578	531	897	1018	SAP 2000
3	2.94	4.99	5.11	489	708	766	634	1045	1265	Analytical
	2.78	4.71	5.39	483	701	762	625	1038	1259	SAP 2000
4	3.13	5.32	5.85	579	754	824	666	1160	1335	Analytical
	3.00	5.08	5.82	574	752	821	661	1156	1331	SAP 2000

Figure 3 shows the maximum deflection of ω and maximum moment of M_x at nodes (4, 3) in the middle of the retrofit roof system in simply-supported beams ($\omega_{4,3} = \omega_{max} = 5.85$ cm and $M_{x,max} = M_{4,3} = 824$ KN m):

$$\omega_{max} = \frac{5ql^4}{384El_y} = 11.42 \text{ cm}$$

And:

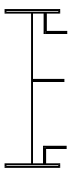
$$M_{max} = \frac{ql^2}{8} = 1675.8 \text{ KNm}$$

The results computed by SAP 2000 for deflections are shown in Fig. 4 (U3 = 5.81 cm). In Table 1, the results of analysis achieved by computer program SAP2000 are compared with the results of the method presented in this study. The results show an excellent compatibility of two methods which shows the maximum deflection of ω and maximum moment of M_x at nodes (4,3) or in the middle of the roof system the digits show a 50% decline in strengthen state in comparison to non-strengthen one.

In Fig. 5, the amount of deflection is calculated by Excel is in accordance with other constant parameters shown by the results of the method presented in this study. Furthermore, it is shown that by reducing the distance between cross beams, their power in reducing deflection becomes higher.

Equation 15	$\beta_1 = 0.0842; \beta_2 = 0.2526; \beta_3 = 0.421; \beta_1^2 = 0.00709; \beta_2^2 = 0.0638; \beta_3^2 = 0.1772;$ $\beta_1^4 = 5\times10^{-1}; \beta_2^4 = 4.07\times10^{-2}; \beta_3^4 = 3.1414\times10^{-2}$		
Equation 16	$\bar{A}_{01} = -\frac{159155}{EI_y}$	$\bar{A}_{02} = -\frac{652}{EI_y}$	$\bar{A}_{03} = -\frac{51}{EI_y}$
	$\bar{B}_{01} = -\frac{42097}{EI_y}$	$\bar{B}_{02} = -\frac{70}{EI_y}$	$\bar{C}_{03} = -\frac{51}{EI_y}$
	$\bar{C}_{01} = \frac{173298}{EI_y}$	$\bar{C}_{02} = \frac{649}{EI_y}$	$\bar{C}_{03} = \frac{b1}{EI_y}$

Sections properties:



$I_x = 653100\ cm^4$

$E = 2.1\times10^4\ KN\ cm^{-2}$

Equation 17

$\omega_{1,1} = \omega_{1,5} = 1.25$ cm

	1	2	3	4	5
1	1.25	2.10	2.40	2.10	1.25
2	2.20	3.85	4.41	3.85	2.20
3	2.94	4.99	5.11	4.99	2.96
4	3.13	5.32	5.85	5.32	3.13
5	2.94	4.99	5.11	4.99	2.20
6	2.20	3.85	4.41	3.85	2.20
7	1.25	2.10	2.40	2.10	1.25

Equation 18

$M_{x(1,1)} = M_{x(1,5)} = 224$ KN m

	1	2	3	4	5
1	224	292	314	292	224
2	391	543	282	543	391
3	489	707.5	766	707.5	482
4	579	754	824	754	579
5	486	707.5	766	707.5	482
6	391	543	585	543	391
7	224	292	314	292	284

Fig. 3: Calculating the deflection and moment at the junction of the main and secondary beams

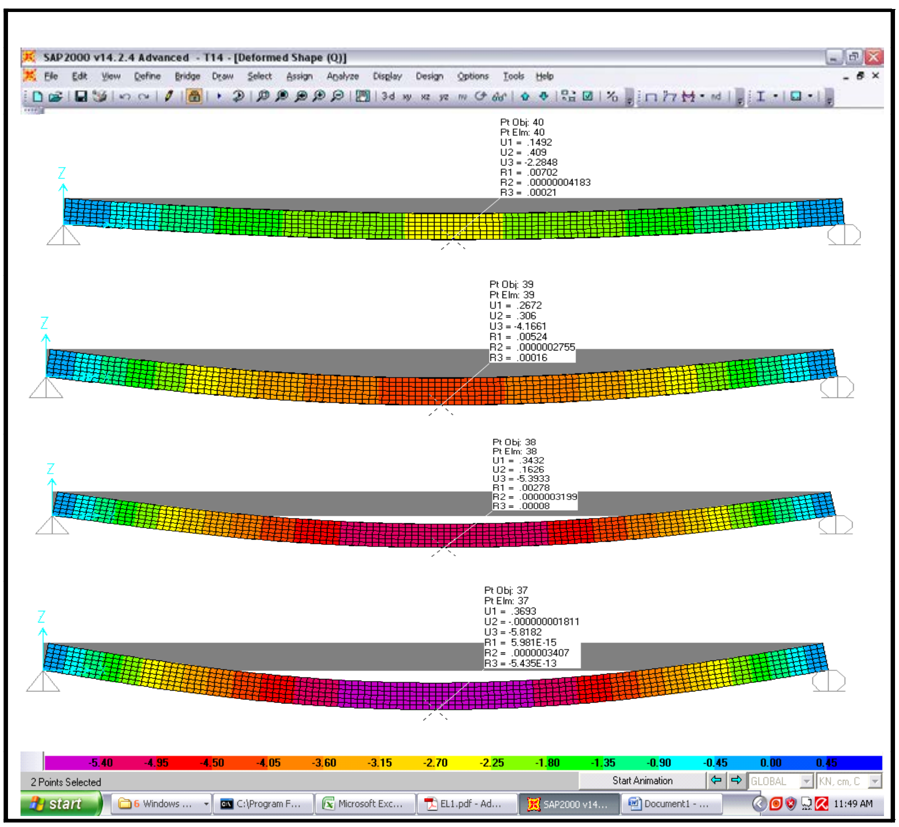


Fig. 4: Computed results by sap 2000 for deflections

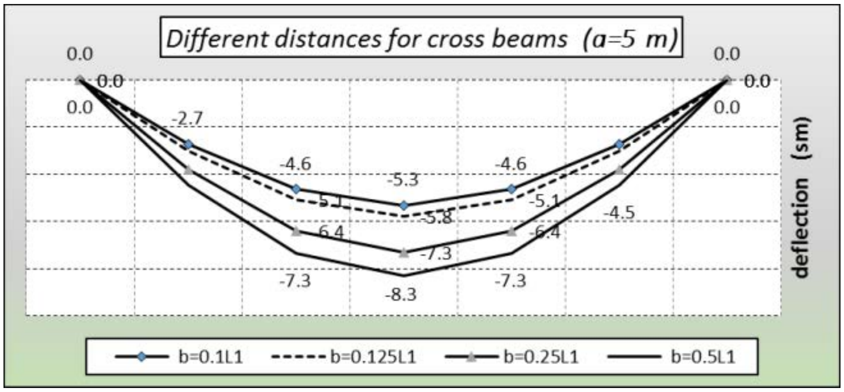


Fig. 5: The effect of reducing the distance between cross beams and deflection

CONCLUSION

The amount of the decreased stress and bending moments can be calculated by changing several related parameters aiming to reach the optional adjustment of stress. Other factors such as the support condition, type of loading and the cross beam and main beam connection point are supposed to be among the main factors in reaching the optimal stress levels and retrofitting roof system. Other conclusions are as follow:

- By means of determining the differential equation governing the behavior of the main and cross beams at the intersection, adding to the roof load bearing system and converting it to an ordinary differential equations and then providing analytical solution, accurate results were obtained. These results were compared with the results of detailed analysis with computer approving that the final results of both methods closely match each other
- The amount of applied work can be reduces by keeping constant the load pressure and executing the appropriate technology
- The input load on the roof structure can be increased by converting the behavior of structure from two-dimensional to three-dimensional one and keeping the bending moment and shear force as constant amounts
- In roof coverings, the physical and geometrical characteristics of cross beams have direct effect on adjusting tensions

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