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Effect of Newtonian Heating on the Mixed Convection Boundary Layer Flow of Eyring-Powell Fluid Across a Nonlinearly Stretching Sheet

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Abstract: The problem on mixed convection boundary layer flow of Eyring-Powell fluid across a nonlinear stretching sheet together with the effect of Newtonian heating is examined. The governing equations are transformed into a system of ordinary differential equations using a similarity transformation and then solved by employing the Range-Kutta method. The effects of the Prandtl number, p_r fluid ε parameter conjugate parameter for Newtonian heating γ and mixed convection parameter λ on the velocity and temperature profiles are computed and discussed. Increasing on the values of parameters ε, γ and λ resulted in an increase in the velocity while increasing on the values of caused in decreasing on the values of fluid velocity. In addition, temperature of fluid decreased with an increasing in the value of parameters Pr, ε and λ but increases with increasing γ.

Key words: Eyring-Powell fluid, mixed convection, heat transfer, Newtonian heating, parameters

INTRODUCTION

In recent years, considerable attention has been given to the problem on non-Newtonian fluids. Such fluids are quite common in the process of manufacturing optical fibres, coated sheets, plastic polymers, foods and drilling muds. The connection between a flow field and shear stress in these fluids is very tedious and, therefore, it offers interesting challenges to researchers. In spite of all these challenges, the researchers in this field are making valuable contributions to the investigations on non-Newtonian fluids (Fetecau and Fetecau, 2006; Fetecau *et al*., 2006; Tan and Masuoka, 2005; Tan and Xu, 2002; Ariel, 1992; Sahoo, 2010; Liao, 2003; Nazar *et al*., 2004; Hayat *et al*., 2010; Cortell, 2007; Hayat *et al*., 2009). In a recent published paper, Jalil *et al*. (2013) studied the problem of non-Newtonian Eyring-Powell fluid over a continuously moving permeable surface in a parallel with free stream. The researchers were motivated to study this problem by the fact that the Eyring-Powell fluid type has a clear characteristic as compare with other non-Newtonian fluids and it is derived from the kinetic theory of gases instead of from empirical relations. Most significantly, it has behaves like a viscous fluid at increase shear rates. Hayat *et al*. (2012) studied an analytical solution for effects of convective boundary conditions over a moving surface in the presence of constant free stream with the flow of a Eyring-Powell fluid.

The stretching sheet is the geometry of fluid flow and has a large number of applications in numerous engineering processes. These applications involve the cooling process of a metallic plate in a cooling bath, polymer industries, glass, aerodynamic extrusion of plastic sheets and the boundary layer along a liquid film condensation process. The stretching flow problem has been studied by Sakiadis (1961). A vast body of knowledge that includes numerical and analytical investigation of various aspects of stretching flow is now available (Cortell, 2006; Sadeghy *et al*., 2005; Liu, 2005; Xu and Liao, 2005). Such studies depict a stretching condition such that stretching sheet with speed of the fluid that is proportional to the distance from the origin. While, a flow over an exponentially stretching sheet has limited attention. Magyari and Keller (1999) described the mass and heat transfer occurring in the boundary layers of an exponentially stretching continuous surface.

Heat transfer characteristics rely on thermal boundary conditions. Heating processes have common four types, namely, surface-heat-flux, wall temperature, Newtonian heating and convective boundary condition. The value of temperature depends on the fundamental properties of the system due to the interaction of solid and fluid. Merkin (1994) investigated on the problem of the effect on Newtonian heating on free convection flow across vertical surface. Convective boundary layer flows

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in fluid-saturated porous media have been derived from Newtonian heating have also received some interest (Lesnic *et al*., 1999, 2004).

A combination of free and forced convection (mixed convection) flow is important under the condition of a large buoyancy forces which attributable to the temperature difference between the free stream and the surface. This situation significantly influences the thermal condition and the flow fields. The study of free (natural) convection, forced convection or the combination of both convections has motivated numerous investigations (Saeid, 2006; Roy and Anilkumar, 2006; Kasim *et al*., 2013; Ali *et al*., 2014; Elatar and Siddiqui, 2015). Ramachandran *et al*. (1988) studied laminar mixed convection in two-dimensional stagnation flows around heated surfaces at rest. Their results shows the dual solutions exist for a certain range of the buoyancy parameter and a reverse flow is developed in buoyancy opposing flow region. Motivated to the significant result to literature, the current examine the flow and heat transfer across a surface study primarily aims to analyse an Eyring-Powell fluid and sheet under Newtonian heating conditions. The solution for the velocity and temperature fields are obtained by employing Runge-Kutta method and the results are presented graphically for various parameter values.

MATERIALS AND METHODS

Mathematical model: An eyring-powell fluid's mixed convection boundary layer flow was examined over a stretching sheet at a constant velocity U_W in the direction of a uniform free stream velocity u_∞. The boundary layer problems that emerged are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\nu+\frac{1}{\rho\tilde{\beta}C^*}\right)\frac{\partial^2 u}{\partial y^2}-\frac{1}{2\rho\tilde{\beta}C^{*3}}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad (3$$

$$U=U_W,\ v=0\frac{\partial T}{\partial Y}=-h_s T\ \ ay\,Y=0$$
$$U\to U_\infty,\ T\to T_\infty\ \ as\,Y\to\infty \quad (4)$$

Where:

U and V	=	The velocity components in the X and Y directions, respectively
$v=\mu/\rho$	=	The kinematic viscosity
ρ	=	The fluid density
T	=	The temperature field
h_s	=	Parameter of heat transfer for the Newtonian heating
K	=	The thermal conductivity
g	=	The gravitational acceleration
β_T	=	The thermal expansion coefficient

A cold fluid stream at temperature T_∞ flowed across the stretching sheet at a uniform velocity U_∞ with T_f as the heat the surface of the plate and h_t as the heat transfer coefficient. C_p the specific heat at a constant pressure whereas $\tilde{\beta}$ and are C^* the features of the Eyring-Powell fluid model. The following transformations has been proposed:

$$u=cx^{-n}f'(\eta),\ v=-\frac{1}{2}\sqrt{c\nu(n+1)}x^{-\left(\frac{n+1}{2}\right)}[f(\eta)-\eta f'(\eta)],$$
$$\eta=y\sqrt{\frac{c(n+1)}{\nu}}x^{-\left(\frac{n+1}{2}\right)},\ (NH)\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}\ (CBC) \quad (5)$$

where, $cx^{-n}=u_w+u_\infty$. After applying Eq. 5 (Eq. 1-3) reduced to:

$$\left(1+\varepsilon\right)f'''+\frac{1}{2}ff''-\varepsilon\delta\left(f''\right)^2f'''+\lambda\theta=0 \quad (6)$$

$$\theta''+\frac{1}{2}\Pr f\theta'=0 \quad (7)$$

with respected to boundary condition:

$$f'(\eta)=\alpha,\ f(\eta)=0,\ \theta'(\eta)=-\gamma\left(1+\theta(\eta)\right)\ (NH)\ at$$
$$\eta=0,\ f'(\eta)=1-\alpha,\ \theta(\eta)=0\ as\ \eta\to\infty \quad (8)$$

For the case of convective boundary condition, one of the boundary condition at is:

$$\theta'(\eta)=-\gamma\left(1-\theta(\eta)\right)\ (CBC)$$

Where:

ε and δ	=	The fluid parameters
pr	=	The Prandtl number (w)
λ	=	The mixed parameter
α	=	A constant parameter
γ	=	Conjugate parameter for Newtonian heating
c	=	The stretching rate along with its subscripts

Notably, when, γ = 0 an insulated wall is present whereas when $\gamma\to\infty$, the wall temperature remains constant. These parameters are defined as follows:

$$\alpha = \frac{u_w}{cx^{-n}},\ Pr = \frac{\mu C_p}{k},\ \delta = \frac{c^3(n+1)x^{-3n}}{2vxC^{*2}} \qquad (9)$$

$$\lambda = \frac{g\beta_T T_\infty}{c^2(n+1)x^{-(n+1)}x^{-n}}$$

$$\gamma = h_s\sqrt{\frac{v}{c(n+1)}}\times^{(\frac{n+1}{2})}\ (NH),\ \varepsilon = \frac{1}{\rho v\tilde{\beta}C^*} \qquad (10)$$

$$\gamma = \frac{h_f}{k}\sqrt{\frac{v}{c(n+1)}}\times^{(\frac{n+1}{2})}\ (CBC)$$

Particularly, $\alpha = 0$ is similar to the flow over an inert surface generated by the free stream velocity, whereas$\alpha = 1$ is similar to the behaviour of a mobile sheet in fluid. When, $0<\alpha<1$ the plate and fluid exhibit the same direction of movement. However, in this present work, the only value$\alpha \leq 1$ is considered.

RESULTS AND DISCUSSION

The numerical scheme was utilised on various factors like fluid parameter ε conjugate parameter for Newtonian heating γ Prandtl number Pr and mixed convection parameter λ. The numerical values were plotted to illustrate the results of the problem. Table 1 shows a comparison between the previous published results with the present solutions The (CBC) case from Aziz (2009) provided the results of and θ (0), -θ (0). It was very encouraging to see the present results and those of Aziz (2009) being in agreement with each other. Table 2 shows the values of θ (0) and $-\theta'$ (0) for the different values of γ and Pr at $\alpha = 1$, $\delta = 0.01$. $\varepsilon = 0.01$ and $\lambda = 5$ (for the NH case). It is noticed that the trend showed by the case of NH is similar to the trend that represent by CBC case.

Figure 1 indicates the distribution of velocity of fluid for different value of Pr. An increase in values of caused the velocity of fluid decreasing. The figure clearly indicates, at far from the plate, the velocity of fluid is decreased until asymptotically zero which is correctly fulfil the boundary condition. Figure 2 illustrates the distribution on temperature of fluid for different Pr values. The temperature of fluid θ, decreased at far from the plate (larger η) for all considered values of Pr. With an increase in Pr values, the temperature significantly decreases. The effects of the fluid parameter on the values of velocity and temperature were shown in Fig. 3 and 4. As the fluid parameter ε, increased, i t was found that the velocity of fluid, $f'(\eta)$ also increased (Fig. 3). However, the temperature of fluid θ (η) displays a decreasing trend as the fluid parameter increased (Fig. 4). These results concurred with the physical reality where thermal boundary layer thickness decreases due to increasing on the values of ε.

Figure 5 and 6 show the effects of the conjugate parameter for Newtonian heating γ, on the distribution for velocity and temperature. It was observed that the fluid velocity and temperature increased as the values of γ bigger. With an increase in the values of mixed convection parameter, the velocity of fld is increased (Fig. 7) but the temperature decreasedui (Fig. 8).

Table 1: Comparison between the previously published results for boundary condition and the present solution (CBC)

		θ (0)		_θ (0)	
Pr	γ	Aziz (2009)	Present	Aziz (2009)	Present
0.72	0.05	0.1447	0.1447	0.0428	0.0428
	0.10	0.2528	0.2528	0.0747	0.0747
	0.20	0.4035	0.4035	0.1193	0.1193
	0.40	0.5750	0.5750	0.1700	0.1700
	0.60	0.6699	0.6699	0.1981	0.1981
	0.80	0.7302	0.7302	0.2159	0.2159
	1.00	0.7718	0.7718	0.2282	0.2282
	5	0.9441	0.9441	0.2791	0.2791
	10	0.9713	0.9713	0.2871	0.2871
	20	0.9854	0.9854	0.2913	0.2913
10	0.05	0.0643	0.0643	0.0468	0.0468
	0.10	0.1208	0.1208	0.0879	0.0879
	0.20	0.2155	0.2155	0.1569	0.1569
	0.40	0.3546	0.3546	0.2582	0.2582
	0.60	0.4518	0.4518	0.3289	0.3289
	0.80	0.5235	0.5235	0.3812	0.3812
	1.00	0.5787	0.5787	0.4213	0.4213
	5	0.8729	0.8729	0.6356	0.6356
	10	0.9321	0.9321	0.6787	0.6787
	20	0.9649	0.9649	0.7026	0.7026

Table 2: Values of θ (0), -θ (0) for different values of Pr and λ when 1, α = 1, δ = 0.01, ε = 0.01 and λ = 5 (NH)

Pr	γ	θ (0)	-θ (0)
0.72	0.05	0.12954	0.056477
	0.10	0.26487	0.126490
	0.20	0.58778	0.317560
10	0.05	0.03054	0.051530
	0.10	0.06273	0.106270
	0.20	0.13259	0.226520
	0.40	0.29857	0.519430

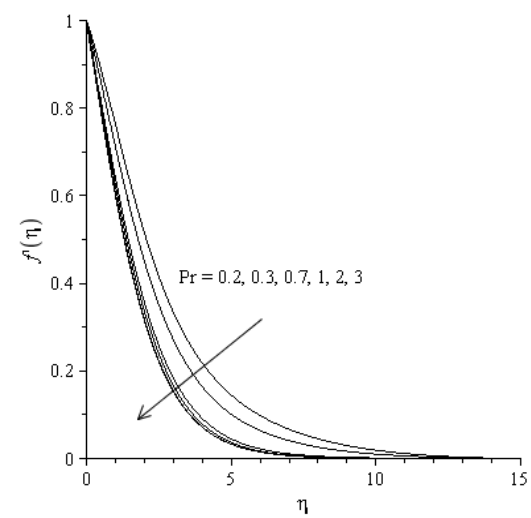

Fig. 1: Effect of Pr on f' (η) when δ = 0.05, α = 1, γ = 0.005 and ε = 0.05

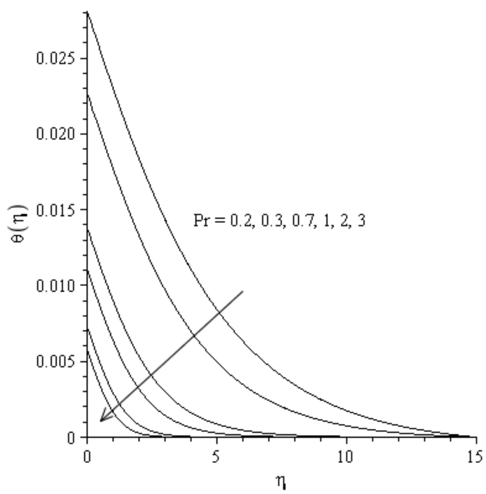

Fig. 2: Effect Pr on θ (η) of on when δ = 0.05, α = 1, γ = 0.005, λ = 5 and ε = 0.05

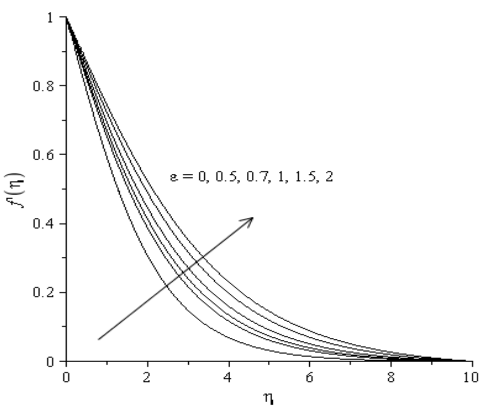

Fig. 3: Effect of ε on f' (η) when δ = 2, α = 1, γ = 0.005, λ = 0.5 and Pr = 0.7

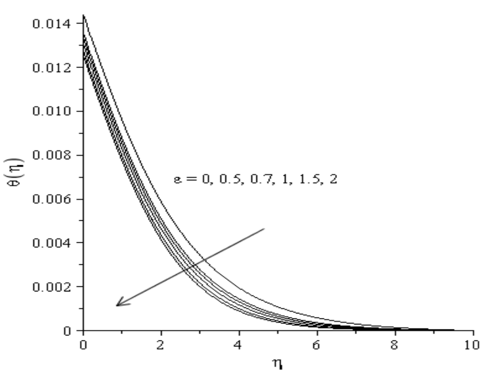

Fig. 4: Effect of ε on θ (η) when δ = 2, α = 1, γ = 0.005, λ = 0.5 and Pr = 0.7

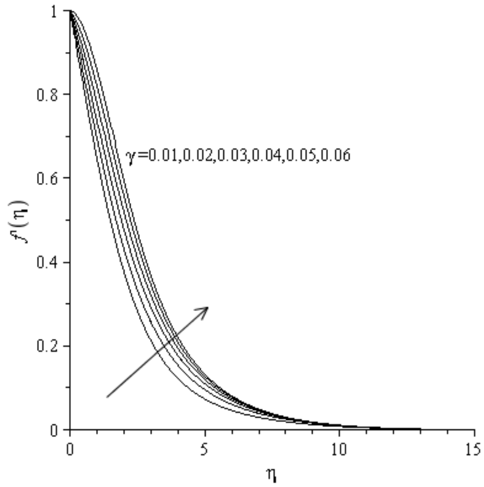

Fig. 5: Effect of γ on f' (η) whenδ = 0.01, α = 1, ε = 0.01, λ = 1 and Pr = 0.3

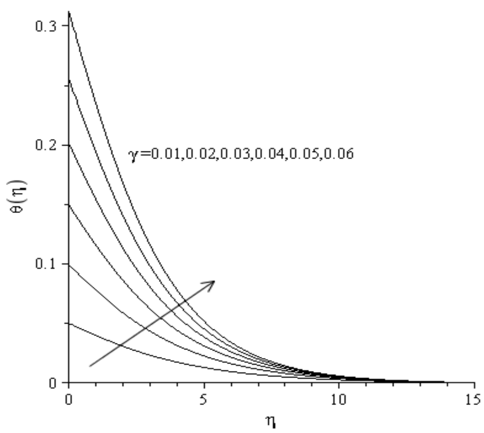

Fig. 6: Effect of γ on θ (η) when δ = 0.01, α = 1, ε = 0.01, λ = 1 and Pr 0.3

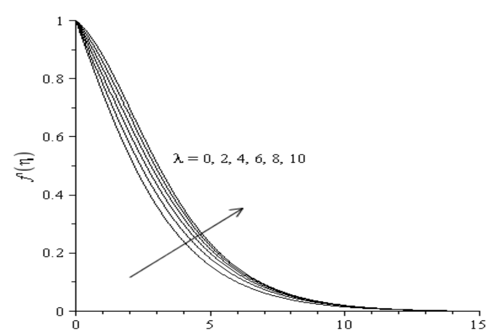

Fig. 7: Effect of λ f' (η) whenδ = 0.1, α = 1, ε = 2, γ = 0.009 and Pr = 0.3

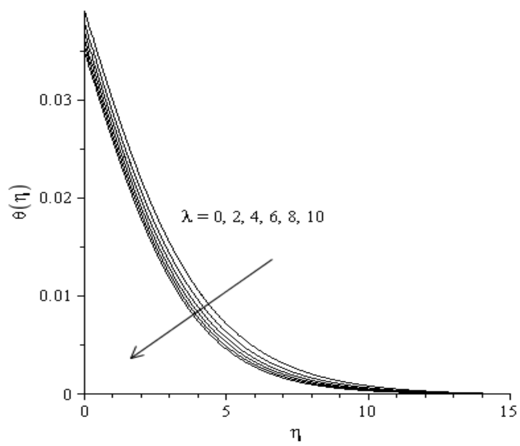

Fig. 8: Effect of λ on when θ (η) when δ = 0.1, α = 1, ε = 2, γ = 0.009 and Pr = 0.3

CONCLUSION

This study analysed the influence of Newtonian heating on the mixed convection boundary layer flow of an Eyring-Powell fluid across a nonlinearly stretching sheet. The Runge-Kutta method was applied to determine solutions for governing equation of the proposed model after applying the appropriate similarity transformation. A summary of the main results are listed as follows:

- Increasing in the Prandtl number Pr, reduced the velocity and the temperature of fluid
- An increase in the values of velocity and a decrease in the values of temperature occurred when the values of fluid parameter, ε, increased
- Velocity and the temperature increased in the increment on the values of γ
- An increase in the values of mixed convection parameter, λ, caused an increment in velocity but a declining in temperature of fluid

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