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Plane Strain Plastic Deformation in Geometrically Nonlinear Arrays in Tresca-Saint-Venant Plasticity Condition

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Abstract: In this study is considered the creation of resolving combined differential equations of plane strain plastic deformation in Tresca-Saint-Venant plasticity condition for continuous bodies, the mechanics of which is described by geometrically nonlinear models in the sense of V. Novozhilov which is to say abandoning the consolidation principle. It is shown that the resolving combined differential equations of plane strain plastic deformation in Tresca-Saint-Venant plasticity condition is the system of hyperbolic type and does not permit to obtain H. Hencky integrals.

Key words: Plasticity, plane strain deformation, sliding lines, geometrical nonlinearity, Russia

INTRODUCTION

The problem of determination of sliding lines when considering the stress-strain state of objects being in the conditions of plastic deformation is fundamental. For continuous bodies which are described by geometrically linear models this problem is considered extensively enough (Cachanov, 1969; Malinin, 1975; Gueniev and Leytes, 1981). Solving a plane problem of the theory of plasticity the method of characteristics became a frequent practice. The idea of coincidence of combined differential equations characteristics with sliding lines forms the basis of this method. Differential equations of equilibrium (ignoring volumetric forces) in the system of coordinates coinciding at each point of continuous bodies with the lines tangent to sliding lines are reduced to H. Hencky algebraic relations. This fact led to the possibility of constructing the solution of the boundary value problem for plane strain plastic deformation, basing only on the properties of sliding lines.

The problem of constructing of plastic constitutive equations linking stresses and strains is not solved completely in the general case due to the complexity of the process of plastic deformation, though a lot of theories were proposed. However, all these theories are grounded on the hypothesis on the smallness of deformations and at the heart of this hypothesis lies the initial dimensions invariance principle (the principle of consolidation). Along with this deformations cannot be considered small under plastic warping and the construction of resolving equations should be accomplished after deformation, according to V. Novozhilov terminology.

MATERIALS AND METHODS

Sliding lines: Let us analyze the continuous body (half-space) in plane strain deformation state (Fig. 1). Following Novozhilov, we consider stress condition on the faces of elementary right-angled parallelepiped in the state after deformation:

$$\sigma_{\xi\xi} = \sigma_{\xi\xi}(\xi,\eta); \sigma_{\eta\eta} = \sigma_{\eta\eta}(\xi,\eta);$$
$$\sigma_{\xi\eta} = \sigma_{\xi\eta}(\xi,\eta); \sigma_{\eta\xi} = \sigma_{\eta\xi}(\xi,\eta);$$
$$\sigma_{\xi\eta} = \sigma_{\eta\xi}$$

Here, ξ, η are grid coordinates of the points of the body after deformation. We choose alignment of the parallelepiped so that after deformation its edges are parallel to the axes of rectangular coordinates X-Z. So, in the state before deformation this parallelepiped was oblique-angled. The location of the basic areas parallel to the Z axis in the after-deformation state is determined from the correlation (Bakushev, 2013):

$$\text{tg}\, 2\phi = \frac{2\sigma_{\xi\eta}}{\sigma_{\xi\xi} - \sigma_{\eta\eta}} \qquad (1)$$

Here, $\phi = \phi\,(\xi, \eta)$ is the angle between the normal line to one of the basic areas and axis (Fig. 2). For principal stress and the maximum shear stress functionating in the areas parallel to axis in the state after deformation we have correlations:

$$\sigma_{1,2} = \frac{\sigma_{\xi\xi} + \sigma_{\eta\eta}}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_{\xi\xi} - \sigma_{\eta\eta}\right)^2 + 4\sigma_{\xi\eta}^2} \qquad (2)$$

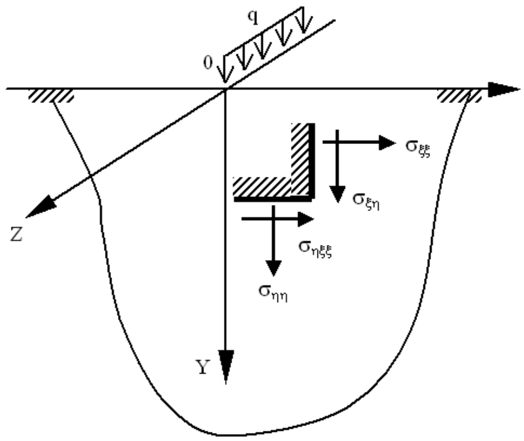

Fig. 1: Sliding lines

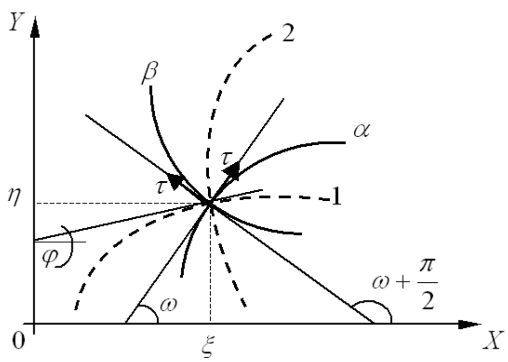

Fig. 2: Normal lines

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{\left(\sigma_{\xi\xi} - \sigma_{\eta\eta}\right)^2 + 4\sigma_{\xi\eta}^2} \tag{3}$$

Let us assume that in point M* of the deformed body the stress condition in principal axes is known: σ_1 and σ_2 then:

$$\begin{aligned} \sigma_{\xi\xi} &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\phi \\ \sigma_{\eta\eta} &= \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2}\cos 2\phi \\ \sigma_{\xi\eta} &= \frac{\sigma_1 - \sigma_2}{2}\sin 2\phi \end{aligned} \tag{4}$$

In symboling: The $\sigma = \sigma_1+\sigma_2/2$; $\tau = \sigma_1+\sigma_2/2$ and taking into consideration that normal lines to the basic areas divide the angles between the normal lines to the areas with the maximum shear stresses in halves, i.e., $\omega = \phi+\pi/4$, we rewrite Eq. 4 as:

$$\begin{aligned} \sigma_{\xi\xi} &= \sigma + \tau\sin 2\omega \\ \sigma_{\eta\eta} &= \sigma - \tau\sin 2\omega \\ \sigma_{\xi\eta} &= -\tau\cos 2\omega \end{aligned} \tag{5}$$

Here, $\tau = \tau\ (\xi, \eta)$ is the maximum shear stress functionating in the area which divides the angle between directions of principal stresses σ_1 and σ_2 in halves; $\sigma = \sigma\ (\xi, \eta)$ the normal stress on the area where the maximum shear stress functionates; $\omega = \omega\ (\xi, \eta)$ the angle between the direction of the maximum shear stress and X axis.

Lines α, и, β, the tangent lines to which coincide with the directions of the maximum shear stresses t in every point of the deformed body are called the maximum shear stress trajectories or the sliding lines. The principal normal stress trajectories 1 and 2 pierce the sliding lines α, и, β, at the angle of π/4 (Fig. 2). After deformation the differential equations of the sliding lines are:

$$\frac{d\eta}{d\xi} = \mathrm{tg}\,\omega, \quad \frac{d\eta}{d\xi} = -\mathrm{ctg}\,\omega \tag{6}$$

RESULTS AND DISCUSSION

Tresca-Saint-Venant plasticity condition: Taking Tresca-Saint-Venant plasticity condition for the scheme of the rigid plastic body:

$$\tau = \tau_{max} = \tau_s = \mathrm{Const} \tag{7}$$

We obtain the resolving combined differential equations of plane strain state under stresses «after» deformation in absence of volume forces:

$$\frac{\partial\sigma_{\xi\xi}}{\partial\xi} + \frac{\partial\sigma_{\eta\xi}}{\partial\eta} = 0, \quad \frac{\partial\sigma_{\xi\eta}}{\partial\xi} + \frac{\partial\sigma_{\eta\eta}}{\partial\eta} = 0 \tag{8}$$

$$\left(\sigma_{\xi\xi} - \sigma_{\eta\eta}\right)^2 + 4\sigma_{\xi\eta}^2 = 4\tau_s^2 \tag{9}$$

The substituting of correlations (Eq. 5) into plasticity condition (Eq. 9) shows that it is identically satisfied. Inserting the correlations (Eq. 5) into the equations of equilibrium (Eq. 8) we obtain the system of two nonlinear differential equations in partial derivatives of the first order for functions $\sigma = \sigma\ (\xi, \eta)$ and $\omega = \omega\ (\xi, \eta)$:

$$\left.\begin{aligned} \frac{\partial\sigma}{\partial\xi} + 2\tau_s\left(\cos 2\omega\frac{\partial\omega}{\partial\xi} + \sin 2\omega\frac{\partial\omega}{\partial\eta}\right) = 0 \\ \frac{\partial\sigma}{\partial\eta} + 2\tau_s\left(\sin 2\omega\frac{\partial\omega}{\partial\xi} - \cos 2\omega\frac{\partial\omega}{\partial\eta}\right) = 0 \end{aligned}\right\} \tag{10}$$

This system of equations is hyperbolical. Performance equations of the combined differential Eq. 10 are of the form:

$$\left(\frac{d\eta}{d\xi}\right)_1 = \text{tg}\,\omega; \left(\frac{d\eta}{d\xi}\right)_2 = -\text{ctg}\,\omega \tag{11}$$

The intercomparison of correlations (Eq. 6 and 11) shows that the characteristics of the system (Eq. 10) coincide with the sliding lines. The usage of differential Eq. 10 in calculations is inconvenient because the differentiation in them is realized with respect to the grid coordinates of the points of the body after its deformation, i.e. with respect to parameters enclosing unknown quantities displacements u = u (x, y), v = v (x, y). If we determine the location of the points of the deformed body not by the grid coordinates ξ, η but by curvilinear axials X, Y (which are the grid coordinates for the body in the state before deformation), then Eq. 8, on the basis by Bakushev (2013) are of the form:

$$\frac{\partial\sigma^*_{x\xi}}{\partial x} + \frac{\partial\sigma^*_{y\xi}}{\partial y} = 0, \frac{\partial\sigma^*_{x\eta}}{\partial x} + \frac{\partial\sigma^*_{y\eta}}{\partial y} = 0 \tag{12}$$

Here, $\sigma^*_{x\xi}$, $\sigma^*_{y\xi}$, $\sigma^*_{x\eta}$, $\sigma^*_{y\eta}$ are the stresses which are the functions of curvilinear axials X, Y functionating in the areas which were perpendicular to X, Y, Z axes before deformation. The connection between the stresses functionating in the areas which were perpendicular to X, Y, Z axes before deformation and the stresses functionating in the areas which became perpendicular to X, Y, Z axes after deformation is established by correlations (Bakushev, 2013):

$$\begin{aligned} \sigma^*_{x\xi} &= \alpha_{11}\sigma_{\xi\xi} + \alpha_{12}\sigma_{\eta\xi};\ \sigma^*_{y\xi} = \alpha_{21}\sigma_{\xi\xi} + \alpha_{22}\sigma_{\eta\xi} \\ \sigma^*_{x\eta} &= \alpha_{11}\sigma_{\xi\eta} + \alpha_{12}\sigma_{\eta\eta};\ \sigma^*_{y\eta} = \alpha_{21}\sigma_{\xi\eta} + \alpha_{22}\sigma_{\eta\eta} \end{aligned} \tag{13}$$

Where:

α_{11} = $1+\partial v/\partial y$

α_{22} = $1+\partial u/\partial x$

α_{12} = $1+\partial u/\partial y$

α_{21} = $1+\partial u/\partial x$

Inserting correlations (Eq. 13) into Eq. 12 and taking into consideration the formulae (Eq. 5 and 7), we obtain:

$$\begin{cases} \alpha_{11}\dfrac{\partial\sigma}{\partial x} + \alpha_{21}\dfrac{\partial\sigma}{\partial y} + 2\tau_s\left(\alpha_{11}\cos 2\omega + \alpha_{12}\sin 2\omega\right)\dfrac{\partial\omega}{\partial x} + \\ \quad + 2\tau_s\left(\alpha_{21}\cos 2\omega + \alpha_{22}\sin 2\omega\right)\dfrac{\partial\omega}{\partial y} = 0 \\ \alpha_{12}\dfrac{\partial\sigma}{\partial x} + \alpha_{22}\dfrac{\partial\sigma}{\partial y} + 2\tau_s\left(\alpha_{11}\sin 2\omega - \alpha_{12}\cos 2\omega\right)\dfrac{\partial\omega}{\partial x} + \\ \quad + 2\tau_s\left(\alpha_{21}\sin 2\omega - \alpha_{22}\cos 2\omega\right)\dfrac{\partial\omega}{\partial y} = 0 \end{cases} \tag{14}$$

Correlations (Eq. 14) represent the system of two differential equations in partial derivatives of the first order for functions **σ** = **σ** (X, Y) and **ω** = **ω** (x, y). The system (Eq. 14) is a geometrically nonlinear analog of the resolving system of plane strain plastic deformation. The differential equations of the sliding lines (Eq. 6) in the system of curvilinear axials (x, y, z) are of the form:

$$\begin{aligned} &\text{for } x = \text{Const},\ \frac{\partial\xi}{\partial y} = \text{ctg}\,\omega\frac{\partial\eta}{\partial y} \\ &\text{for } y = \text{Const},\ \frac{\partial\xi}{\partial x} = -\,\text{tg}\,\omega\frac{\partial\eta}{\partial x} \end{aligned} \tag{15}$$

In contrast to geometrically linear analog the system (Eq. 14) does not permit to obtain H. Hencky integrals in passing to the system of curvilinear axials in every point of the deformed body where the axes coincide with the direction of the tangent lines to sliding α, **и**, β. The system (Eq. 14) in this passing (Smirnov, 1963) is conversed to the form:

$$\begin{cases} \alpha_{11}\dfrac{\partial\sigma}{\partial\alpha} + \alpha_{21}\dfrac{\partial\sigma}{\partial\beta} + 2\tau_s\left(\alpha_{11}\dfrac{\partial\omega}{\partial\alpha} + \alpha_{21}\dfrac{\partial\omega}{\partial\beta}\right) = 0 \\ \alpha_{12}\dfrac{\partial\sigma}{\partial\alpha} + \alpha_{22}\dfrac{\partial\sigma}{\partial\beta} - 2\tau_s\left(\alpha_{12}\dfrac{\partial\omega}{\partial\alpha} + \alpha_{22}\dfrac{\partial\omega}{\partial\beta}\right) = 0 \end{cases} \tag{16}$$

It means that the construction of H. Hencky integrals becomes impossible.

CONCLUSION

The obtained systems of resolving equations of plane strain plastic deformation in Tresca-Saint-Venant plasticity condition can be used in solving the problems of plastic deformation of continuous bodies, the mechanics of which are described by geometrically nonlinear models.

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