

## The Characteristics of Unit Hydrographs with Different Scale of Antecedent Soil Moisture Condition

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**Abstract:** Unit Hydrograph (UH) is a popular method for predicting flood caused by rainfall in a catchment. This method has little flexibility to different storm, consequently the unit hydrographs produced differ considerably from storm to storm. This is partially due to the unit hydrograph method ignores the influence of Antecedent Soil Moisture Condition (AMC) on runoff generating processes. This paper presents the research result on the characteristics of unit hydrograph with different scale of AMC. In this research the AMC was approached by parameter of Antecedent soil Moisture Deficit (AMD) which has opposite meaning with the AMC. The AMD was represented by the dependent variable of  $S_c$  (Storage capacity) that is the space volume remaining the soil moisture evaporated. The research was conducted using hydrologic data (rainfall, discharge and evaporation) assembled from four catchments. The characteristics of unit hydrograph were presented by peak discharge ( $q_p$ ) and time to peak ( $t_p$ ) of unit hydrograph. The research experiment were performed using three parallel tanks and one serial tank model of Yue and Hashino. The research result are concluded; the unit hydrographs derived with base-flow separation using tank model (HSSim) show more consistent than the unit hydrographs calculated by conventional method HScol; AMC influences the effective rainfall as the AMC becomes bigger than so do the effective rainfall as the AMC trends to smaller than the peak discharge ( $q_p$ ) becomes smaller; the influence of AMC to time to peak ( $t_p$ ) cannot detected by this model.

**Key words:** AMC, AMD, unit hydrograph, tank model, peak discharge, time to peak

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### INTRODUCTION

Unit Hydrograph (UH) was presented for the first time by Sherman in 1932 (Shaw *et al.*, 2010). This method has been applied for flood estimation in the wide world. The unit hydrograph theory is derived based on assumption of uniformity and adopting principle of linearity and time-invariant. This theory consist of limitation namely it always produces different unit hydrographs from storm to storm. This partially because the theory ignores the influence of Antecedent soil Moisture Condition (AMC) on runoff generating processes (Yue and Hashino, 2000) whereas the AMC has significant role on the process of runoff generating (Beven, 2011). The high AMC produces bigger runoff than the small AMC.

Study the influence of AMC to runoff has been conducted that studied the influence of variability of AMC to runoff. Zhang *et al.* (2011) investigated the effect of AMC on runoff modeling. Unfortunately, most of research did not relate to the unit hydrograph. This study presents the research result of the effect of AMC on the characteristics on unit hydrograph.

### MATERIALS AND METHODS

**Hydrologic data:** The hydrologic data (rainfall, water level depth and evaporation) required were collected from four upper catchments where three catchments are located in the hill slope of Mount Merapi at DIY Province and a catchment located in the hill slope of Mount Rinjani at Lombok Island. The three catchments located at DIY Province are shown on Fig. 1 consists of:

- Catchment of code at pogung
- Catchment of Code at Kaloran
- Catchment of Gajahwong at Papringan

The Palung catchment is shown on Fig. 2. The location of the hydrological stations are shown on Fig. 1 and 2. There are at least two Automatic Rainfall Recorders (ARR) in a catchment. The areal rainfall is calculated using Thiessen Polygon. The water level depth is transferred to discharge flow using rating curve at the Automatic Water Level Recorder (AWLR) station.

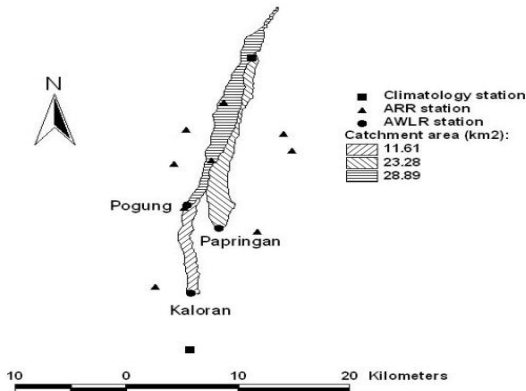


Fig. 1: Hydrologic station network of code catchment and gajahwong catchment at DIY Province

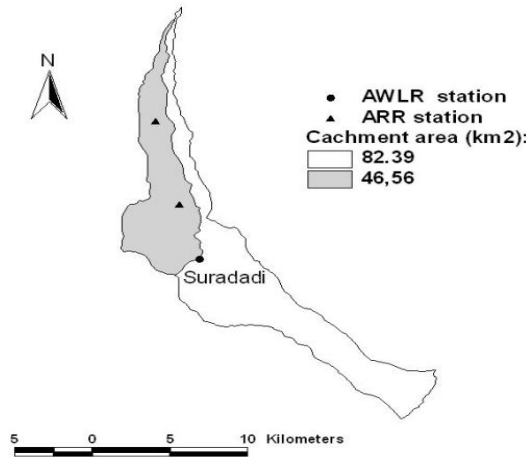


Fig. 2: Hydrologic station at Palung Catchment at NTB Province

**Antecedent soil Moisture Condition (AMC):** Antecedent soil Moisture Condition (AMC) is define as the value of soil moisture at time before rainfall event. In this research the AMC is approximated using variable  $S_c$  (Storage capacity). Variable  $S_c$  is defined as an initial loss of rainfall and as a comprehensive index representing the influence of the antecedent soil moisture conditions on runoff generating processes (Yue and Hashino, 2000).

The minimum value of  $S_c$  is zero in which all pore volume is full filled of water. In this stage, the AMC is maximal; the soil is in saturated condition. As the AMC is minimum or zero then the  $S_c$  is maximal; the soil is completely dry. Figure 3 shows structure of soil and the physical meaning of variable  $S_c$ . The value of  $S_c$  is estimated using the daily water balance equation as follows (Yue and Hashino, 2000):

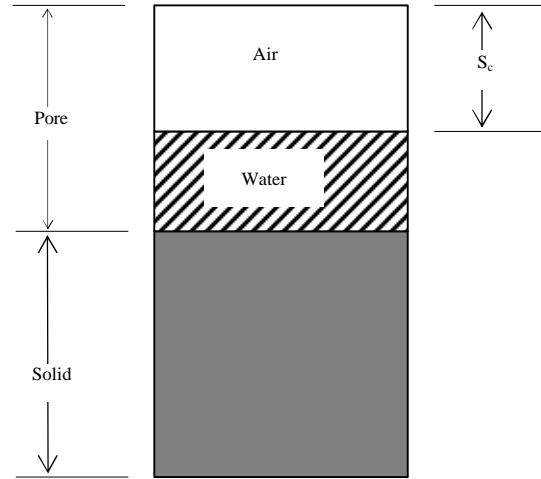


Fig. 3: Soil structure and variable Storage capacity ( $S_c$ )

$$S_c = \sum_{i=1}^m [E_{day}(i) - P_{day}] \quad (1)$$

Where:

$P_{day}$  = The daily rainfall amount that is not enough to produce runoff

$E_{day}$  = The daily evapotranspiration amount (mm)

$m$  = The duration from the last strong rainfall to the present rainfall (days)

The  $S_c$  calculation starts just after the last strong rainfall and at that time the soil moisture content is equal to the areal mean saturated soil moisture content in this condition the value of  $S_c$  is null. The  $E_{day}$  is directly obtained from observed station at Plunyon and Barongan. The evaporation data is observed using an evaporation pan with adjustments using Eq. 2 to account for the condition of the vegetation and soil (Chay, 1995):

$$PET = C_e E_p \quad (2)$$

Where:

PET = Potential evapotranspiration ( $\text{mm day}^{-1}$ )

$C_e$  = Coefficient of pan (0.5-0.8)

$E_p$  = Pan evaporation ( $\text{mm day}^{-1}$ )

**Tank model of Yue and Hashino:** Yue and Hashino developed unit hydrograph model to quick and slow runoff using model of three serial tanks and one parallel tank (Fig. 4) (Yue and Hashino, 2000). This model still adopts the assumption of space uniformity and linearity principle. They represented a catchment as a number of connected tanks with exponential function to describe the movement of water into and out of them. In this model, the

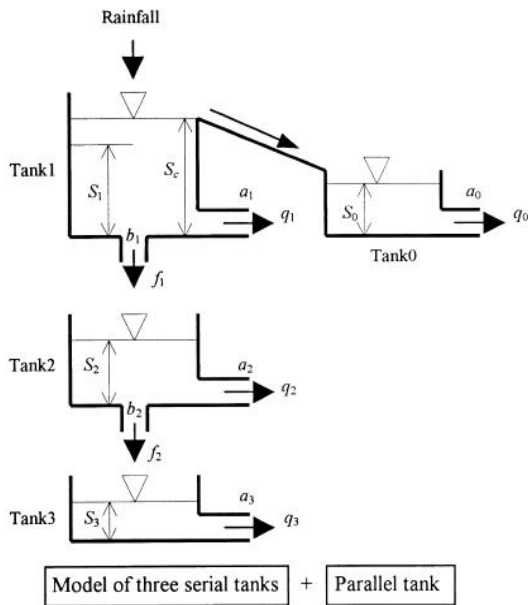


Fig. 4: Structure of tank model

influence of antecedent soil moisture conditions on runoff generating is considered by the variable of  $S_c$  (Fig. 4).

Runoff components flow out through holes with dimension of  $a_0, a_1, a_2, a_3$ . The infiltrations flow out from Tank 1 and 2 from holes with size of  $b_1$  and  $b_2$ , respectively. The runoff components are:  $q_0$  is surface flow (quick flow) where flow out from tank0;  $q_1$ - $q_3$  are subsurface flow represent rapid subsurface flow; delayed flow and groundwater flow; occurring from Tank1-Tank3, respectively. Flow component of  $f_1$  and  $f_2$  are the infiltrations from Tank 1-2 and from Tank 2-3, respectively.

In the model, rainfall  $r$  first fills Tank1. Rapid subsurface runoff  $q_1$  and infiltration  $f_1$  occur when  $S_1 > 0$ . Delayed subsurface flow  $q_2$  and infiltration  $f_2$  occur when  $S_2 > 0$ . Groundwater flow  $q_3$  occurs when  $S_3 > 0$ . Surface runoff  $q_0$  occurs when Tank1 is full and water overflows from Tank1-2.

The unit response for the flow components ( $q_1$ - $q_3$ ) and infiltrations  $f_1$  and  $f_2$  are represented as exponential function and can be calculated by the following equations:

$$q_i(t) = a_i S_i(t) \quad (i = 1, 2, 3) \text{ (mm/h)} \quad (3)$$

$$f_i(t) = b_i S_i(t) \quad (i = 1, 2) \text{ (mm/h)} \quad (4)$$

$$c_i = a_i + b_i \quad (5)$$

$$c_2 = a_2 + b_2 \quad (6)$$

$$c_3 = a_3 \quad (7)$$

Tank 1: For ( $0 < t \leq \Delta t$ ):

$$S_1(t) = \frac{r_u}{\Delta t} \frac{(1 - e^{-c_1 t})}{c_1} \quad (8)$$

$$f_1(t) = \frac{r_u b_1}{\Delta t c_1} (1 - e^{-c_1 t}) \quad (9)$$

For ( $t > \Delta t$ ):

$$S_1(t) = \frac{r_u}{\Delta t} \frac{(e^{-c_1 \Delta t} - 1)}{c_1} \quad (10)$$

$$f_1(t) = \frac{r_u b_1}{\Delta t c_1} (e^{-c_1 \Delta t} - 1) \quad (11)$$

Tank 2: For ( $0 < t \leq t$ ):

$$S_2(t) = \frac{r_u b_1}{\Delta t c_1 c_2} \left[ 1 + \frac{c_2}{c_1 - c_2} e^{-c_1 t} - \frac{c_1}{c_1 - c_2} e^{-c_2 t} \right] \quad (12)$$

$$f_2(t) = \frac{r_u b_1 b_2}{\Delta t c_1 c_2} \left[ 1 + \frac{c_2}{c_1 - c_2} e^{-c_1 t} - \frac{c_1}{c_1 - c_2} e^{-c_2 t} \right] \quad (13)$$

For ( $t > \Delta t$ ):

$$S_2(t) = \frac{r_u b_1}{\Delta t c_1 c_2} \left[ \frac{-c_2 (e^{c_1 \Delta t} - 1)}{c_1 - c_2} e^{-c_1 t} + \frac{c_1 (e^{c_1 \Delta t} - 1)}{c_1 - c_2} e^{-c_2 t} \right] \quad (14)$$

$$f_2(t) = \frac{r_u b_1 b_2}{\Delta t c_1 c_2} \left[ \frac{-c_2 (e^{c_1 \Delta t} - 1)}{c_1 - c_2} e^{-c_1 t} + \frac{c_1 (e^{c_2 \Delta t} - 1)}{c_1 - c_2} e^{-c_2 t} \right] \quad (15)$$

Tank 3: For ( $0 < t \leq \Delta t$ ):

$$S_3(t) = \frac{r_u b_1 b_2}{\Delta t c_1 c_2 a_3} \left[ 1 - \frac{c_2 a_3 e^{-c_1 t}}{(c_1 - c_2)(c_1 - a_3)} + \frac{c_1 a_3 e^{-c_2 t}}{(c_1 - c_2)(c_2 - a_3)} - \frac{c_1 c_2 e^{-a_3 t}}{(c_1 - a_3)(c_2 - a_3)} \right] \quad (16)$$

$$S_3(t) = \frac{r_u b_1 b_2}{\Delta t c_1 c_2 a_3} \left[ \frac{c_2 a_3 (e^{-c_1 \Delta t} - 1) e^{-c_1 t}}{(c_1 - c_2)(c_1 - a_3)} - \frac{c_1 a_3 (e^{-c_2 \Delta t} - 1) e^{-c_2 t}}{(c_1 - c_2)(c_2 - a_3)} + \frac{c_1 c_2 (e^{-a_3 \Delta t} - 1) e^{-a_3 t}}{(c_1 - a_3)(c_2 - a_3)} \right] \quad (17)$$

The unit pulse response function for the slow runoff is the sum of rapid subsurface ( $q_1$ ), delayed subsurface ( $q_2$ ) and groundwater flow ( $q_3$ ):

$$u_{1,3}(t) = \sum_{i=1}^3 q_i(t) = \sum_{i=1}^3 \alpha_i S_i(t)$$

For ( $0 < t \leq \Delta t$ ):

$$u_{(1,3)}(t) = \frac{r_u}{\Delta t} (1 + D_{11}e^{(c_1 t)} + D_{12}e^{(c_2 t)} + D_{13}e^{(c_3 t)})$$

For ( $t > \Delta t$ ):

$$u_{1,3}(t) = \frac{r_u}{\Delta t} (D_{21}e^{(c_1 t)} + D_{22}e^{(c_2 t)} + D_{23}e^{(c_3 t)})$$

Where:

$$D_{11} = \frac{b_1}{c_1} \left( \frac{(c_1 a_2 - c_2 a_3) - a_1}{(c_1 - c_2)(c_1 - a_3)} - \frac{a_1}{b_1} \right)$$

$$D_{12} = \frac{b_1(a_3 - a_2)}{(c_1 - c_2)(c_2 - a_3)}$$

$$D_{13} = \frac{-b_1 b_2}{(c_1 - a_3)(c_2 - a_3)}$$

$$D_{21} = -D_{11}(e^{c_1 \Delta t} - 1)$$

$$D_{22} = -D_{12}(e^{c_2 \Delta t} - 1)$$

$$D_{23} = -D_{13}(e^{c_3 \Delta t} - 1)$$

For unit pulse input the unit pulse response function of quick runoff is calculated using (Eq. 10):

$$u_0(t) = \frac{1 - e^{-\alpha_0 t}}{\Delta t} \quad (0 < t \leq \Delta t)$$

$$u_0(t) = \frac{1}{\Delta t} (e^{-\alpha_0 \Delta t} - 1) \quad (t > \Delta t)$$

$$u(t) = u_{1,3}(t) \quad (S_1 \leq S_c)$$

$$u(t) = u_{1,3}(t) + u_0(t) \quad (S_1 > S_c)$$

Thus, the unit pulse response function  $u(t)$  for the total runoff is the summation of  $u_{1,3}(t)$  and  $u_0(t)$ . The effects of  $S_c$  to the characteristics of UH were explored

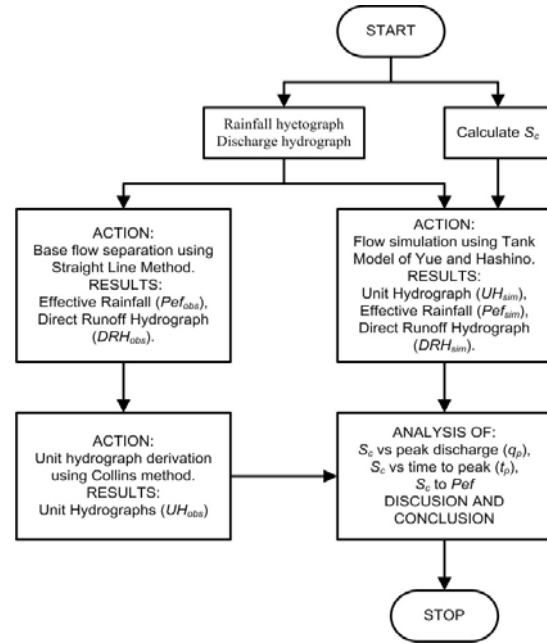


Fig. 5: Experimental work scheme

Table 1: Number of events used in the research

Catchment's name	Number of events
Palung at Suradadi	4
Code at Pogung	8
Code at Kaloran	7
Gajahwong at Papringan	6

using the relation between variable  $S_c$  to peak discharge ( $q_p$ ) and  $S_c$  to time to peak ( $t_p$ ) of unit hydrograph. The peak discharge and time to peak were derived based on experimental work scheme as shown in Fig. 5 for the four catchments. The variable of  $S_c$  was determined using (Eq. 1 and 2).

The experiments were conducted based on two methods on generating UH namely: Collins method ( $UH_{col}$ ) and flow simulation using Tank Model Yue and Hashino ( $UH_{sim}$ ). Table 1 shows the event of rainfall-runoff used.

## RESULTS AND DISCUSSION

**Unit Hydrographs (UH) obtained:** The Unit Hydrographs (UH) obtained from the simulations ( $UH_{col}$  and  $UH_{sim}$ ) are presented in the following figures. Figure 6 shows UH obtained on Palung Catchment at Suradadi while Figure 7 shows UH obtained on Code Catchment at Pogung. Then, Fig. 8 shows UH obtained on Code Catchment at Kaloran and Fig. 9 shows UH obtained on Gajahwong Catchment at Papringan.

According to the time invariance principle, unit hydrographs in a catchment is unique and no differences

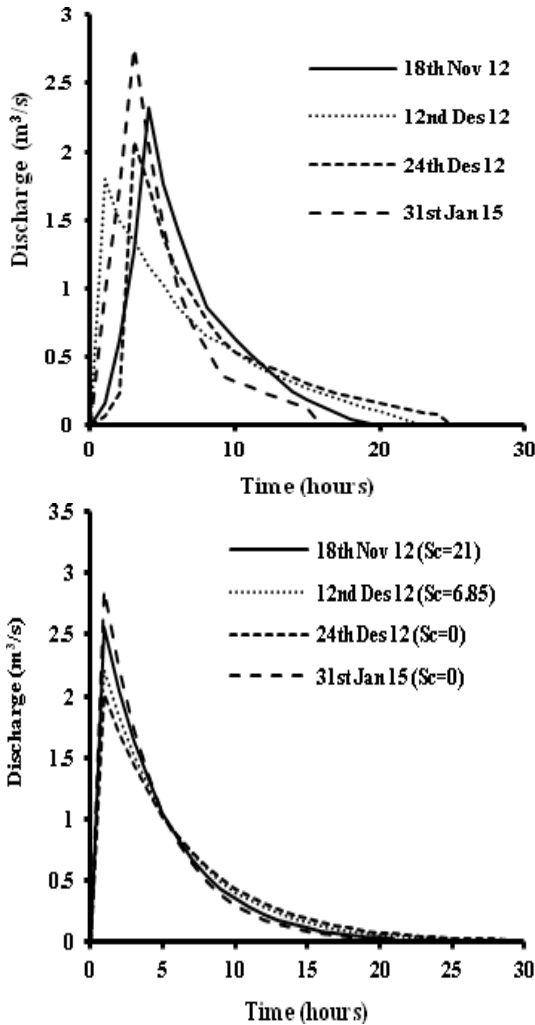


Fig. 6: UH derived from Palung Catchment at Suradadi: a) UH derived using Collins Method ( $UH_{col}$ ) and UH simulated using Tank Model ( $UH_{sim}$ )

among them (Chow *et al.*, 1988). The peak discharge ( $q_p$ ) and time to peak ( $t_p$ ) should be the same for all of the UH, but in fact those UH are varied as shown on Fig. 6-9a. It seems that  $UH_{col}$  has more variation than the  $UH_{sim}$ . The variation of  $UH_{col}$  is greater than  $UH_{sim}$  because the  $UH_{sim}$  considering the AMC in the runoff generating process.

Sasmito showed that UH which are derived by considering AMC are more consistent than UH which is ignoring the AMC. The study derived two kinds of UH as shown on Fig. 6a and 10. Both UHs were derived using Collins method but differ on base-flow separation, first UH using straight line method ( $UH_{sl}$ ) and the other UH ( $UH_{omt}$ ) using subsurface flow component that simulated by Tank Model (Li *et al.*, 2012).

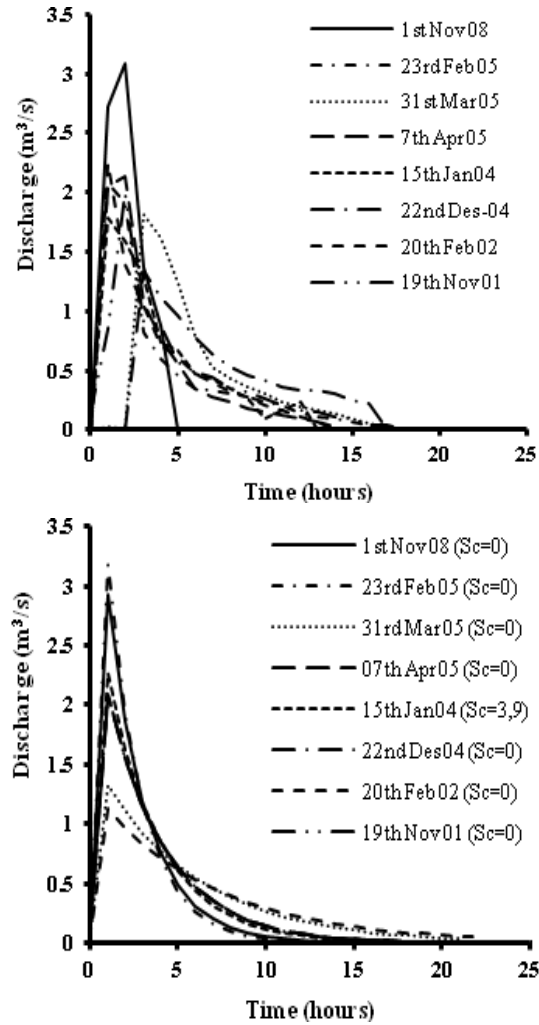


Fig. 7: UH derived from Code Catchment at Pogung: a) UH derived using Collins Method ( $UH_{col}$ ) and UH derived using Collins Method ( $UH_{col}$ )

The variation of UH were tested using  $RAE_p$  and  $RMAE$ .  $RAE_p$  (relative average of peak flow error) and  $RMAE$  (relative mean absolute error) are calculated using Eq. 12 and 13:

$$RAE_p (\%) = \frac{|Q_{pref} - Q_{psim}|}{Q_{pref}} \times 100$$

$$RMAE (\%) = \frac{1}{n} \sum_{t=1}^n \frac{|Q_{ref}(t) - Q_{sim}(t)|}{Q_{ref}(t)} \times 100$$

Where:

- $Q_{pref}$  = Peak flow reference
- $Q_{psim}$  = Peak flow simulated
- $Q_{ref}(t)$  = Flow discharge reference at time (t)
- $Q_{sim}$  = Flow discharge simulated at time (t)

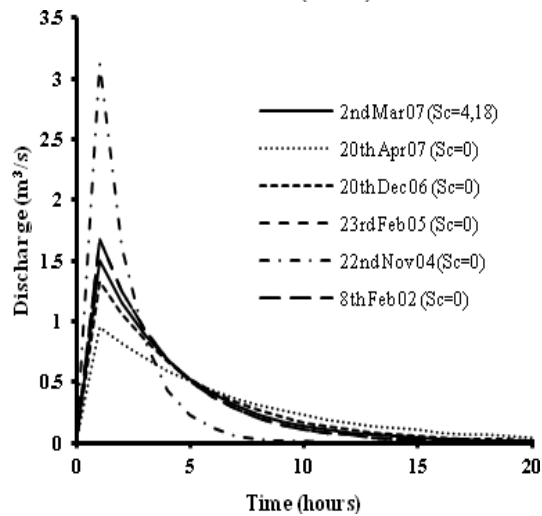
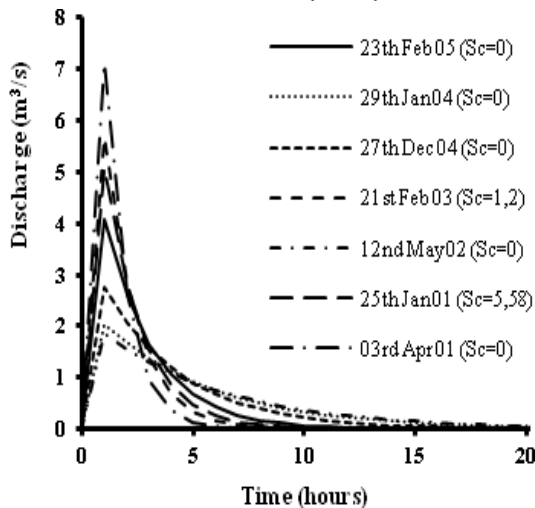
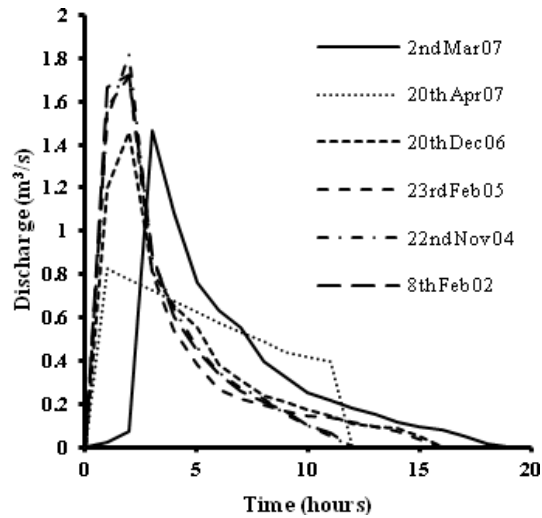
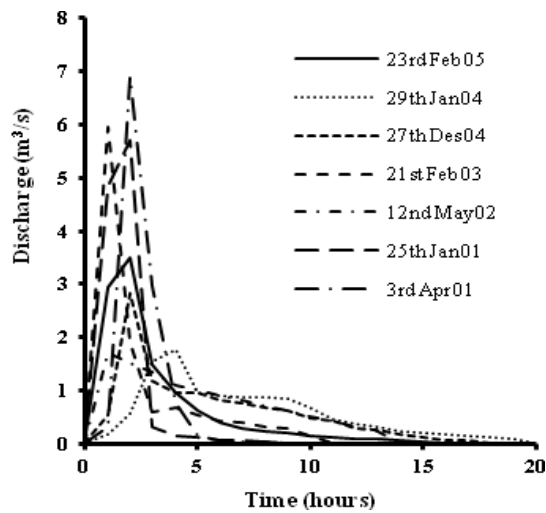


Fig. 8: UH derived from Code Catchment at Kalaran: a) UH derived using Collins Method ( $UH_{col}$ ) and UH simulated using Tank Model ( $UH_{sim}$ )

Fig. 9: UH derived from Gajahwong Catchment at Papringan: a) UH derived using Collins Method ( $UH_{col}$ ) and UH simulated using Tank Model ( $UH_{sim}$ )

The test result showed that  $RMAE$  and  $RAE_p$  of  $UH_{omt}$  is less than the  $RMAE$  and  $RAE_p$  of  $UH_{col}$  as presented on Table 2. It is indicated that  $UH_{omt}$  more consistent than  $UH_{col}$ .

Parameters	$UH_{col}$ (%)	$UH_{omt}$ (%)
$RAE_p$	19.470	17.32
$RMAE$	113.82	67.28

**The influence of  $S_c$  to effective rainfall (Pef):** Physically meaning of  $S_c$  (storage capacity) is the portion of pore volume which is filled with air as shown on Fig. 3. The  $S_c$  are used to represent the AMC but in opposite meaning. As the  $S_c$  higher means the AMC is lower but if  $S_c$  is lower than the AMC is higher.

subsurface storage for infiltrated rainfall water in the soil. If the space has been satisfied filled then the remaining rainfall water become effective rainfall (Pef). As the rainfall continue then the Pef magnitude getting bigger. This phenomena is illustrated on Fig. 11 when  $S_c = 0$  there are variation values of Pef/Ptot. Ptot is total rainfall.

Figure 11 illustrates the relationship between  $S_c$  and the effective rainfall (Pef). According to Fig. 11 as  $S_c$  greater the effective rainfall trends to smaller since more rainfall water infiltrated. The  $S_c$  provides space as

The effects of variable  $S_c$  on the characteristics of UH are analyzed using relationship between the variable  $S_c$  and the parameters of peak discharge ( $q_p$ ) and time to peak ( $t_p$ ) of UH and are presented in the following section.

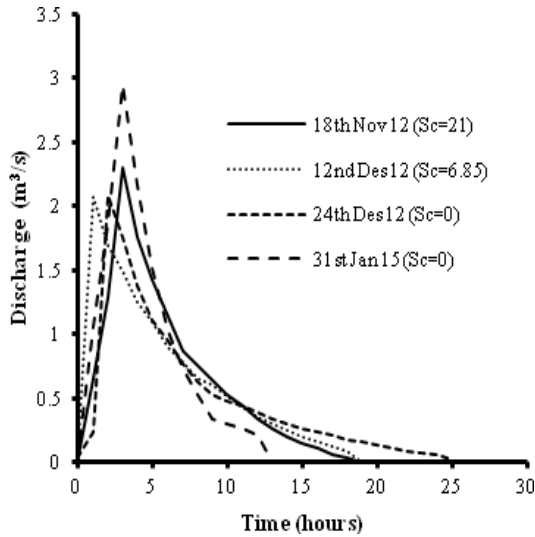


Fig. 10: UH derived with baseflow separation using tank model

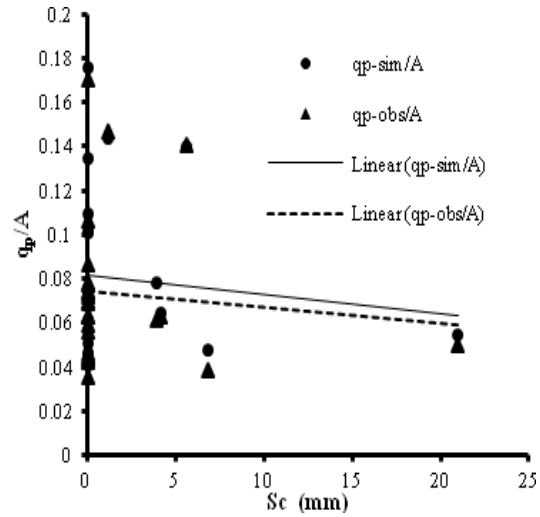


Fig. 12: The relationship between  $S_c$  and specific discharge ( $q_p/A$ )

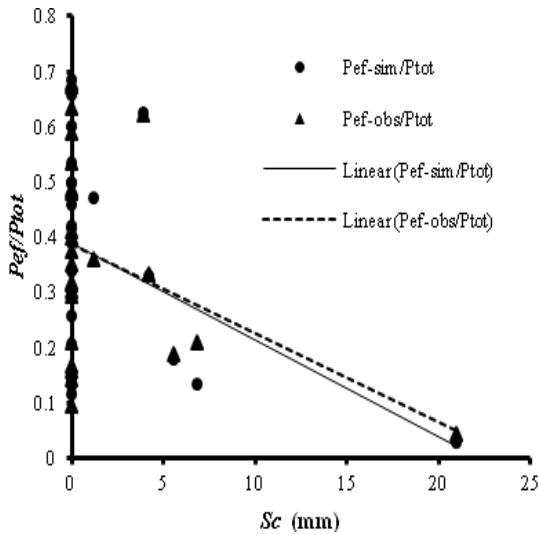


Fig. 11: Relationship between  $S_c$  and  $P_{ef}/P_{tot}$

**The influence of  $S_c$  to peak discharge ( $q_p$ ) of UH:** The influence of  $S_c$  to peak discharge of UH is shown on Fig. 12 where the peak discharge is transferred to specific discharge by dividing  $q_p$  by the catchment area ( $A$ ) in order to generalize the result in one graph.

According to Fig. 12, the peak discharge trends to lower as the  $S_c$  greater, this agreed with decreasing of  $P_{eff}$  when  $S_c$  increasing, it indicating that the peak discharge also decreasing. At  $S_c = 0$  the value of  $q_p/A$  varying because the value of  $q_p$  is influenced by  $P_{ef}$ . As  $P_{ef}$  increasing then  $q_p$  increasing also. So, at  $S_c = 0$  the range value of  $q_p/A$  is bigger.

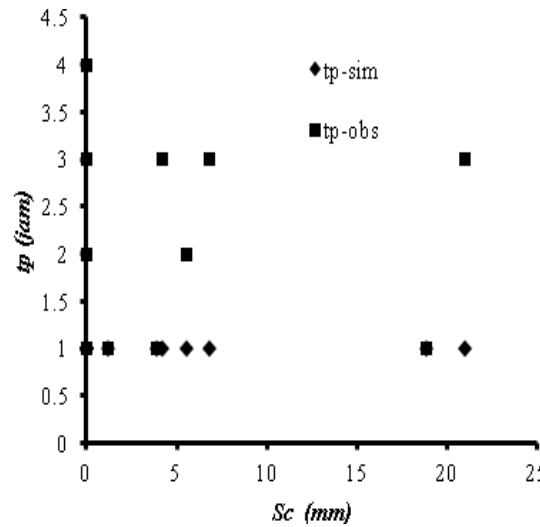


Fig. 13: The relationship between  $S_c$  and time to peak  $t_p$

**The influence of  $S_c$  to time of peak ( $t_p$ ) of UH:** The influence of  $S_c$  to time to peak discharge of UH is shown on Fig. 13. The time to peak obtained from simulation ( $t_{p-sim}$ ) is constantly 1 h but the time to peak produced from Collins method ( $t_{p-obs}$ ) are varied.

The tank model formulates the unit response function based on exponential equation which always produces maximum ordinate hydrograph curve at one hours time, thus the time to peak of simulated UH are always at one hours time. Therefore, the influence of  $S_c$  to time to peak cannot be detected if the simulation using tank model. Basically, variable  $t_p$  is depended to the catchment

area and rainfall duration. In big catchment with long duration of rainfall need long time to reach peak discharge.

### CONCLUSION

The research proof that the Unit Hydrograph (UH) that base flow separation using tank model more consistent than UH that baseflow separation using straight line method. The AMC influence the effective rainfall, if AMC increases then the effective rainfall also increases. The AMC influence the peak discharge of UH, if the AMC decreases then the peak discharge also decreases. The influence of AMC to time to peak of UH cannot be detected by tank model.

This research has shown the characteristics of UH on different scale of AMC. The future research will be focused on the formulation of the influence of AMC to the characteristics of the UH.

### NOMENCLATURE

UH	=	Unit Hydrograph
$q_p$	=	Peak discharge
AMC	=	Antecedent soil Moisture Condition
$t_p$	=	Time to peak
$S_e$	=	Storage capacity
$q_p$	=	Peak discharge
AMD	=	Antecedent soil Moisture Deficit
$P_{ef}$	=	Effective rainfall
ARR	=	Automatic Rainfall Recorders
RMAE	=	Relative Mean Absolute Error

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