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Finite Element Dynamic Analysis of a High-Speed Train on Ballasted Track

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Abstract: The impact of dynamic loading on ballasted railway track has become an important issue due to increased demand for high-speed freight and public transport and because it leads to progressive deterioration and settlement of railway. In this study, the LS-DYNA computer code is used to conduct finite element calculations for the dynamic analysis of a high-speed train running on ballasted railway track. The rails are modelled as beams of a plastic kinematic material and the ballast and sub-ballast structure are modelled using rectangular prisms of an elastic material. The elastic beams representing the sleepers are placed 0.6 m apart.The load of a high-speed train is applied using a simplified finite element model. The commands RAIL_TRACK and RAIL_TRAIN LS_DYNA are applied to simulate the train's interaction with the track. The CONTACT_SINGEL_SURFACE and CONTACT_SURAFCE_SURFCE commands which have not been applied in previous research are used to model the contact between connected parts. The results indicate that the LS-DYNA program can be used to model a high-speed train running on ballasted railway track and to calculate stress-strain, deformation and contact forces as well as sliding energy which are crucial for in railway track maintenance and in designing new ballasted railway track.

Key words: Dynamic analysis, ballasted railway, high-speed train, finite element, track

INTRODUCTION

Improvements to running speeds on existing railway lines can economically improve rail transport, an increasingly important piece of the infrastructure given its efficiency and low environmental impact. A modern rail network requires faster, more frequent and heavier trains. Such improvements can be challenging to effect and depend on the existing track's geotechnical dynamics. The quality of the subgrade and trackbed (and the dynamic loads on them) greatly influence the maximum safe running speed. Therefore, a deep understanding of the track dynamics and the trackbed properties is necessary.

High-speed tracks can allow running speeds over 300 km h^{-1}, much faster than possible on conventional rail. The weight per axle of trains has also increased. Greater loads at higher speeds lead to higher stresses in the track, which can increase track deterioration, leading to passenger discomfort and safety problems. As a result, high-speed track is more costly to maintain than conventional track.

Accumulated experience from high-speed railways has shown that even if the trains have a relatively low axle load, their dynamic loading degrades the ballast and its subgrade, causing severe track settlement problems along the line. Such settlement will not stop until the tack is reinforced; left unmaintained, permanent deformations in the ballast, sub-ballast and subgrade will follow with a severity depending on their quality and behaviour. As soon as the structure of ballasted track starts to degenerate, the variations of the dynamic train-track interaction forces increase, further promoting track deterioration. Commonly the ballast bed becomes unevenly distributed, resulting in loss of the full contact between the sleepers and the ballast bed and leaving some sleepers suspended from the rails rather than resting on the ballast.

The finite element method is a powerful and well-used general method of structural analysis. It can be applied to various physical complex systems; the analysed problem can have arbitrary geometry, loads and support conditions (Shao, 2003). Also, the meshing process can mix elements of different types, shapes and physical properties. This numerical method is useful when the problem is too complicated to be solved satisfactorily by classical analytical methods. Many finite element software programs are available for structural analysis: the LS-DYNA program is one of the most powerful and versatile. It is effective and efficient, especially for transient and impact load analysis and combines the advantages of other finite element software. It can handle a wide range of element types and material models in every area of engineering (Hallquist, 2006).

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Numerical modelling is crucial to the analysis of the dynamic responses of the combined system of a high-speed railway's ballast, sub-ballast and subgrade-collectively called the track substructure. A typical profile of track resting on its substructure can be modelled, analysed and simulatedusing LS-DYNA. Computational analytical parameters determined by selecting a relevant train load must be calculated at different time steps. The lateral distributions of vertical displacement, stress, strain and vertical acceleration through the substructure must be analysed to model its dynamic response to the load.

Ballast is an essential component of the track structure due to its mechanical properties: its structure can deform and readily return to the undeformed state without suffering damage. It is also easy to construct and maintain. Track engineers, both practical and theoretical, tend to focus mainly on the track superstructure (i.e., the vehicle, rails, sleepers and fasteners) and have devoted less attention to the substructure (Zhai *et al*., 2004).

Operational expenditure on preventive maintenance of railway substructure has increased because of the difficulties arising from ballast fouling. Keeping the ballast as clean as possible is an essential part of maintenance to ensure its longevity. The processes of removing and cleaning ballast are costly and such maintenance interrupts the regular use of the track. Ballast problems can allow vibrations to deform the railway track structure and it is important ensure that the ballast functions well to minimise track vibrations.

Track can remain in good order by ensuring the quality and suitability of its individual components, here broadly categorized as the superstructure and the substructure. The superstructure thus includes all non-granular components of the track while the substructure includes its granular components.

To withstand axle loads higher than 250 kN, rails need to be of high strength, joint less and massive, preferably with a weight greater than 60 kg m^{-1} (Zongliang, 1993). It is therefore desirable to connect the rails using elastic fasteners to pre-stress or reinforced concrete sleepers or steel sleepers which are more durable than traditional wooden sleepers. The ballast layer, for which the main functions are to ensure the train load is evenly distributes and to keep the sleepers in their required positions, needs to have a nominal thickness greater than 300 mm to provide sufficient resilience, strength and energy absorption (Zongliang, 1993). These structural criteria, however have not yet led to a general consensus about the requirements of ballast materials.

Variations in the ballast's support condition and differences in its stiffness influence the risk of damage to the track and trains. There are several ways to measure the vertical stiffness of the substructure components (Suiker, 2002). If stiffness is evaluated for a track system, it could be used as a parameter to optimize maintenance through allowing the identification of local weaknesses along the track and hanging sleepers.

Literature review: Gomes studied different commercial finite element codes (DIANA, PLAXIS and ANSYS) to model the dynamic performance of high-speed railway track. Surface displacements, velocities and accelerations along the model width were used for comparison at times 0.140, 0.175, 0.301, 0.335, 0.373 and 0.407s which correspond to the first local peaks of the applied load. Calculation was stopped at 0.5 sec when the load stabilized at zero.

Nguyen studied two and three dimensional dynamic analysis models for a vehicle-track system formulated by finite element numerical simulation. The wheel-rail contact was included as a nonlinear Hertz's spring. Track irregularity was modelled as a stationary ergodicprocess. Their work considered the contact force between the wheel and the rail and the force in the primary suspension and the envelope of dynamic amplification of these two forces with respect to the speed.

Szurgott and Bernys (2013) investigated the effects of vehicle speed and static wheel load on the impact forces at the wheel-rail contact and at the top of the sleeper. The wheel-rail impact force is the basic incremental dynamic wheel-rail force; the impact force of the train on the sleeper is transferred from the wheel-rail impact contact, through the rail and the rail pad, into the rail seat of the sleeper. The complete model of the wheel-track system in this case included a wheel, rail, three rail pads, three sleepers, a ballast layer, a capping layer and a formation layer. All components were created in the design modeller of the ANSYS Workbench.

Jin-Chun proposed a method for predicting the traffic-load-induced settlement of a road on soft subsoil with a low embankment foundation. The plastic vertical strain in the subsoil was calculated by an empirical equation with constants related to the physical and mechanical properties of the subsoil. Comparisons of the calculated values with field data indicate that the proposed method can provide a reasonable prediction of permanent settlement in the given context.

Nguyen developed effcient models for calculating the dynamic responses due to traffc loads on railway tracks. Their ballasted railway-track settlement prediction model evaluated settlement using a previously selected vehicle-track dynamic model and a track settlement law. The calculations were based on the dynamic response of high-speed finite element models with direct time integration, contact between the wheel and rail and interaction with railway cars. An initial irregularity profile was used in the prediction model. The results included the growth of track irregularity and the contact force in the final interaction of the numerical simulation.

Tao and Chen (2011) developed a ballast-less railway track subgrade. They used the finite element software ANSYS to examine 2D finite element models based on the subgrade with a plate ballast-less track-multiplayer system. The computation parameters were determined by selecting a rational train load and using the mechanical behaviour of the selected materials. They also studied the lateral distribution of vertical displacement, stress, strain and the vertical acceleration of the subgrade cross-section directly beneath the train load at different times to analyse the dynamic response of the subgrade to the load.

Gaps in existing research: Our literature search revealed no sufficiently in-depth report of the dynamic responses of ballasted railway track specifically studied using the LS-DYNA finite element computer code, a program capable of modelling the dynamic forces approximately as they occur in the real world. Most previous studies have focused on finite element simulations of static loads and have not sufficiently addressed dynamic effects. Besides contact between the wheel and rail, rail and sleeper, the sleeper and the ballast and the sub-ballast and subgrade were not studied perviously. However, these forces can be considered using the LS-DYNA program and this paper presents the first use of this program to model these forces.

Problem statement: Increasing economic growth requires improving the transport infrastructure in a way that is environmentally sustainable with regard to factors such as noise and emissions. Railways are a promising means of transport in this respect and railway owners have sought to increase the capacity of their networks through increasing the capacity of each train and/or making the train faster.

Railways require constant improvement and development to remain competitive against air, road and sea transport. This calls for highly technical yet affordable technologies to allow faster trains to carry more goods and people efficiently, safely and comfortably without requiring expensive maintenance or causing environmental damage.

Ballasted railway track settlement occurs as a result of the dynamic loading of high-speed trains and fatigue load. The dynamic loading of a train degrades and deforms the track substructure, leading to track settlement. The severity of settlement depends on the quality and behaviour of the substructure components. Any initial degeneration of the track structure significantly increases the variations of the dynamic train-track interaction forces, thereby accelerating track deterioration.

The finite element method is a powerful and generally applicable method for structural analysis, being able to model and simulate nearly any engineering structural system. It is particularly useful for modelling structural analysis problems.

Finite element analysis is conducted using the LS-DYNA finite element computer code which has the advantage of incorporating a wide range of element types and material models. This code is particularly effective for transient and impact load analysis. Explicit finite element methods were originally formulated to solve problems in wave propagation and impact engineering but currently find many other engineering applications such as sheet metal forming, underwater simulations, failure analysis, glass forming, metal cutting, pavement design and earthquake engineering (LSTC, 2014).

Ballasted high-speed railway track generally requires a strong substructure to withstand the dynamic responses that can potentially cause its deterioration and settlement. The dynamic responses of the track should be considered during the design and feasibility studies of the ballasted structure with numerical simulation using a finite element computer code such as DOT (2007) being potentially applicable for this purpose.

The objective of the study is to perform a finite element simulation of a high-speed train load on ballasted railway track using the LS-DYNA computer code. The analysis models the stress and plastic strain state, vertical displacement, internal and kinetic energy and sliding energy.

MATERIALS AND METHODS

Finite element method: The finite element method is a leading structural tool; however, several factors should be considered. First, the method is an approximate analytical procedure whose accuracy usually depends on the level of discretisation of the mesh. Second, the accuracy also depends on whether the major influences on the problem behaviour are included in the analytical idealization. Finally, the simulations must be properly interpreted to ensure the results are meaningful.

Geometry: The track geometry is constructed in two main parts: the superstructure and the substructure. Of the superstructure components, each rail is considered as a prismatic beam element as per Timoshenko Beam Theory; the sleepers are modelled as rectangular prisms and are considered as elastic beam elements; the fasteners

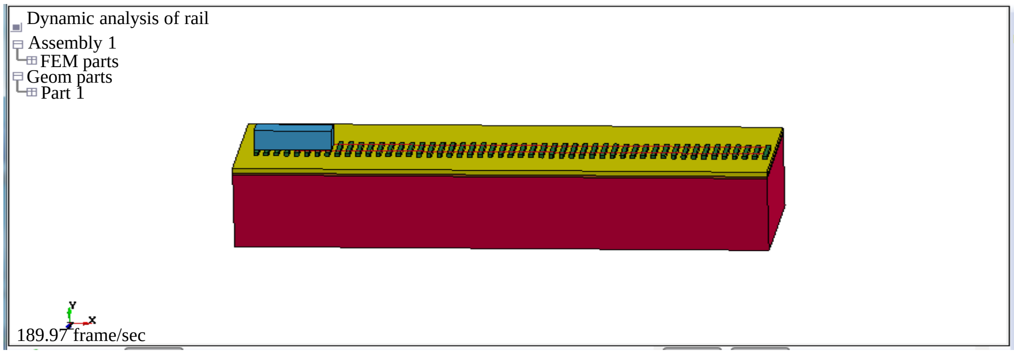

Fig. 1: Train load on ballasted railway track

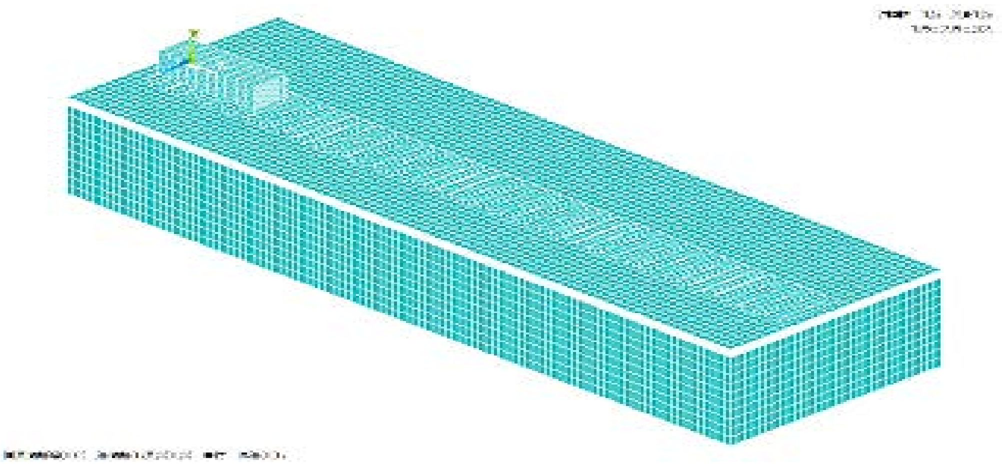

Fig. 2: Three-dimensional finite element model of ballasted railway track

and rail pads are considered as discrete spring and damper elements.The substructure comprises ballast and sub-ballast modelled as rectangular prisms that transfer the load toward the subgrade foundation, another rectangular prism. Figure 1 shows the train load on ballasted railway track.

Material model: Finite element analysis of a high-speedtrain on ballasted track is performed in the LS-DYNA code with a focus on the load distribution along successive sleepers and the underlying substructure. The material model includes two parallel rails as prismatic beam elements of a plastic-kinematic material deformable in flexure and shear. The sleepers, spacedat 0.6 m are modelled as rectangular prismatic elastic beams vibrating only vertically and laterally using beam finite elements and respective constraints. The ballast and sub-ballast are each modelled as a rectangular prism of linearly elastic material bounded by unrotatable and unmovable side and bottom boundary conditions. Soft subgrade soil is considered as a rectangular prism of linearly viscoelastic material bounded by unmovable side and bottom boundary conditions. The train is modelled as a rigid body. Table 1 lists the material properties of each component (Fig. 2).

Table 1: Material properties of each component of the high-speed railway model

Components	Parameters	Values
Rail	Density	7850 g/mm^3
	Young modulu	2×10^5 MPa
Sleeper	Density	2.4×10^{-3} g/mm^3
	Young modulus	70MPa
	Poison ratio	0.3
Ballast	Density	0.16×10^{-2} g/mm^3
	Young modulus	150MPa
	Poison ratio	0.35
Sub ballast	Density	0.19×10^{-2} g/mm^3
	Young modulus	80 MPa
	Poison ratio	0.35
Subgrade	Density	0.2×10^{-3} g/mm^3
	Young modulus	10 MPa
	Poison ratio	0.4
High speed train (rigid)	Density	7.67×10^5 g/mm^3
	Young modulus	2.1×10^5 MPa
	Poison ratio	0.25

These are raw data for the Yujiatou ballasted railway track from the Hubei design and Construction Limited Company

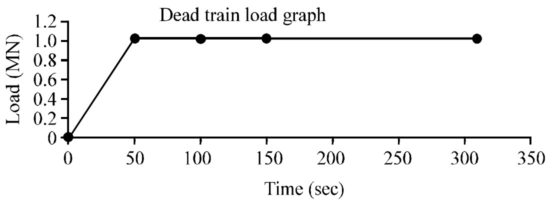

Fig. 3: Plot of train load with respect to time

Finite element model: The LS-DYNA code can simulate a model as close to the actual physical system as possible. Here, a three-dimensional finite element model of a high-speed train on ballasted railway track is created using solid brick elements to represent the substructure components and beam elements to represent rails. As an initial step, all finite element analysis requires meshing of the model, whereby the model is divided into a number of small elements. After loading, stress and strain are calculated at integration points of these small elements. Selecting an appropriate mesh density is important. A mesh size of 10 mm is used here as it has previously provided more satis factory results than other mesh sizes. Figure 2 shows the three-dimensional finite element model of the ballasted railway track including the boundary conditions and a moving train.

Boundary conditions: Modelling boundary conditions is often the most critical factor in achieving sensible, reliable data from a finite element analysis (LSTC, 2014). To ensure a good representation of the physical conditions, several tools are used to provide proper boundary conditions such as fixing the ends to be unmovable. The load application is also important and is discussed in detail below.

Load application: The dynamic load on the track component is modelled using the prescribed motion rigid finite element LS-DYNA computer code and the static load component of the train-rail contact is modelled as an axial load node set command. Details of the prescribed motion and static axial load protocol are discussed here. For both the dynamic and static load, the displacement direction and rotation are restrained in the X, Y and Z-axes.

Static load: The static train load is applied to the wheel-rail contact. Each axel comprises two wheels sets (an axle with two wheels). The total load on both axles is equivalent to 320 ton, distributed equally at the four wheels (i.e., 80 ton per wheel). The load is defined and prescribed in LS-DYNA as the LOAD-NODE SET command. Figure 3 shows the distributed static load with respect to time.

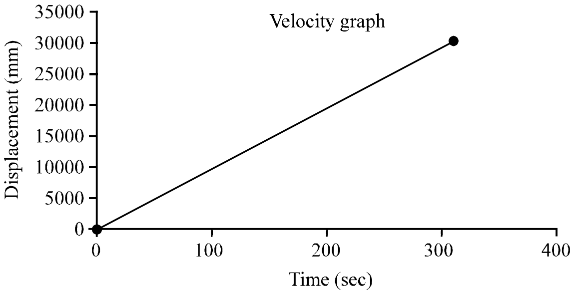

Fig. 4: Plot of the train's displacement with respect to time

Dynamic load: The dynamic load is prescribed by the LS-DYNA Software in its BOUNDARY_PRESCRIBED_ MOTION command. The finite element model in LS-DYNA defines the movement of the train in terms of displacement with respect to time. A maximum permissible static axle load of 160 kN is assumed throughout. The train is moving at 350 km h^{-1}. The axle spacing and bogie wheelbase as well as the train speed, corresponded to the dimensions of the track model, particularly the 0.6 m sleeper spacing to ensure that each wheel set is located exactly above a sleeper at each considered moment of time. Figure 4 depicts the train velocity in terms of its displacement with respect to time.

RAIL_TRACK and RAIL_TRAIN codes: The contact between the vehicle and the track is important in the simulation and LS-DYNA is the only software appropriate for this. Its RAIL_TRACK and RAIL_TRAIN commands are used here.

Contact algorithms: The contact algorithm is unique to LS-DYNA. It models the contact between parts, segments and nodes. It is defined and applied here for the 1st time on ballasted railway track. This contact algorithm is defined as the commands CONTACT_AUTOMATIC_SINGEL SURFACE, CONTACT SURFACE_SURFACE and CONTACT TRANDUCER PENALITY which are identified here as being applicable to model the contact between the rail, sleeper, ballast, sub-ballast and subgrade. The algorithm uses a penalty method to model the contact interface of the different track components.

RESULTS AND DISCUSSION

The finite element analysis results for a high-speed train on ballasted railway track are calculated and presented here. They yield interesting information regarding the global and local dynamic responses of the track to the load.

Explicit versus implicit finite element method: Explicit finite element analysis was originally developed to solve engineering problems in wave propagation and impact engineering but is now widely employed in many other applications.

Implicit finite element analysis is another type of problem solving mechanism. It requires thousands of time steps to be taken to solve a dynamic engineering problem and is computationally expensive due to the cost of inverting stiffness matrices to solve the large sets of nonlinear equations, especially for models with thousands of degrees of freedom or those with nonlinearities in the materials and geometry. An explicit method can find a solution without forming a global stiffness matrix by adopting element-by-element basis and thus can treat large three-dimensional models (thousands of degrees of freedom) without being unreasonably expensive. Other advantages include its easy implementation and accurate treatment of general nonlinearities. However, explicit methods are conditionally stable and thus require small time steps. Stable computation requires the time step to be less than the time required for a stress wave to travel through the shortest element, meaning this could result in excessive run times as the level of discretisation increases.

Ballasted railway track deformation: In general, implicit methods have the form:

$$U^{n+1} f = \left(u'^{n+1}, u''^{n+1}, u^{n}, \ldots\right) \tag{1}$$

and therefore the computation of the current nodal displacements requires knowledge of the time derivatives, which are unknown, meaning that simultaneous equations need to be solved. On the other hand, explicit methods have the form:

$$u^{n}, u'^{n}, u''^{n}, u^{n+1}, \ldots\ldots, U^{n+1} = f \tag{2}$$

Meaning the current nodal displacements in the model can be determined completely from the previous known displacements and their time derivatives.

The total track deformation is calculated in LS-DYNA along successive sleepers as shown in Fig. 5. The maximum deformation due to the dynamic train load is 0.3 mm. The calculated results suggest that the train causes significant track settlement and thus regular track maintenance is required. Dynamic loading degrades only the ballast and underlying layers if the track is suitably maintained and the damage can thus be controlled. Otherwise, settlement follows from permanent deformation of the ballast and the underlying soil and is detrimental to the railway. Of course the severity of the settlement depends on the quality and behaviour of the substructure components and any damage mitigation plan must consider these factors. Once the track structure starts to degenerate, the variations in the dynamic train-track interaction forces increase, accelerating the deterioration. Very often the ballast bed becomes unevenly distributed which because the sleepers are fastened to the rails, results in the loss of full contact between the sleepers and the ballast, leaving some sleepers hanging from the rails. The occurrence of hanging sleepers can be investigated by finite element analysis. Overall, the dynamic interaction of a high-speed train with ballasted track significantly deteriorates and deforms the track.

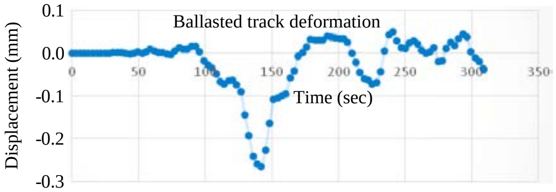

Fig. 5: Track displacement due to the dynamic load of a high-speed train

Stress-strain state of the track: Modelled contours of the stress and plastic strain are presented in Fig. 6a and b, respectively. They give information about the behaviour of the track and the stress-strain state of the foundation under a dynamic load.

Sliding energy: The sliding energy measures the penetration of one part into other components. Figure 7 shows the calculated sliding energy and shows positive values which means that the system is properly designed and modelled without any of the components penetrating each other.

Contact forces: The contact between the wheel and track is the basic element of railway vehicle dynamics. The geometric or kinematic and dynamic relations of this contact and its mechanical behaviour are crucial in calculating the contact forces. They can be properly considered within a completely linear multi-body formalism that takes into account kinematic nonlinearities by dynamic linearization. From the short-term dynamic calculations, a periodic non-harmonic motion can be obtained in terms of the generalised displacements. A

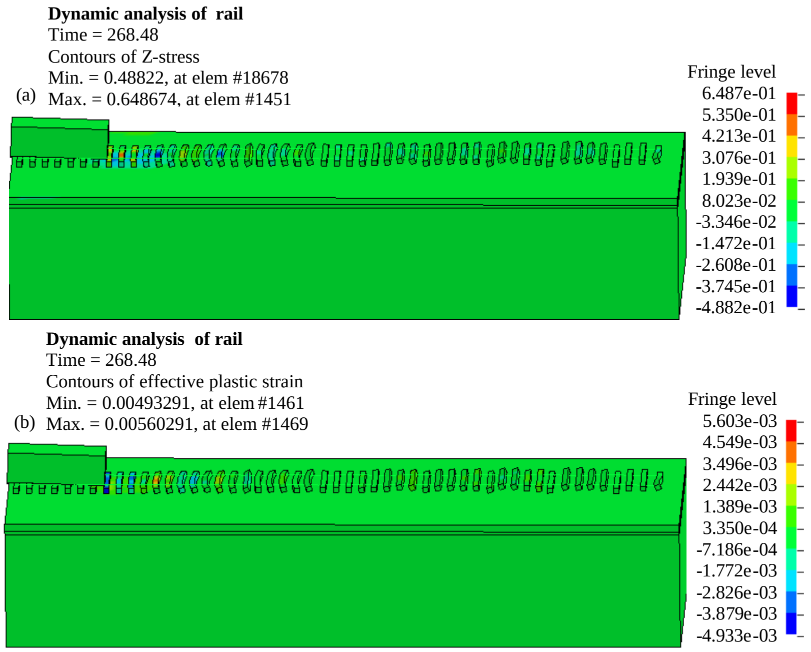

Fig. 6: a) stress state of ballasted railway under high-speed train load and b) plastic strain state of ballasted railway under high-speed train load

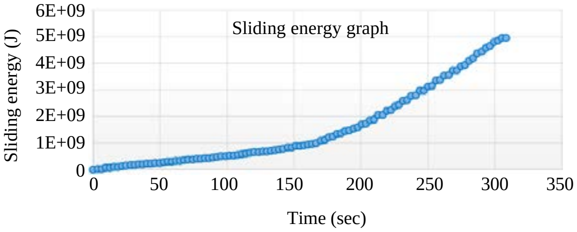

Fig. 7: Sliding energy

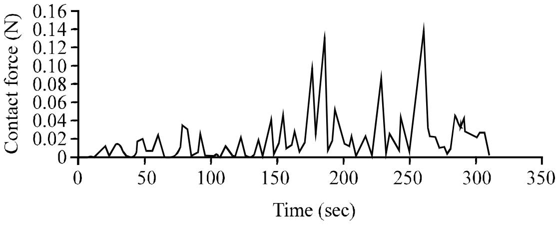

Fig. 8: Contact forces

train running on tracks has forces transmitted through the wheel-rail contact area which depends on the geometry of the wheel and the rail and the relationship between force and compression and tension as represented by the Hertzian contact force.

Hertz formulated a mathematical model to determine the area of contact and the pressure distribution at the surface of contact between the rail and the wheel. This model has the rail and wheel contact as similar to that of two cylinders (the circular wheel and the curved head of the rail) whose axes are perpendicular. The contact area between the two surfaces is bound by the ellipse as follows (Chandra and Agarwal, 2007):

$$F = 4.13\left(\frac{P}{R}\right)^{1/2} \tag{3}$$

Where:

F = The maximum shear stress
R = The radius of the fully worn out wheel
P = The static wheel load on the curve

Contact modelling by finite element analysis in LS-DYNA uses commands including CONTACT_AUTOMATIC_SINGEL SURFCE, CONTACT_SURFACE_SURFCE and CONTACT_TRANDUCER PENALITY successful contact modelling by this method as displayed in Fig. 8 has not been attempted in other works. Therefore, the present results demonstrate a method of finite element modelling that might interest other researchers.

Energy balance of dynamic analysis: An energy balance fequation can be used to evaluate whether a simulation yields an appropriate dynamic response. The kinetic energy of the deforming material should be calculated using internal energy throughout the majority of a dynamic analysis. This is generally not possible in the early stages of the analysis because the deformable body will be moving before it develops any significant deformation.

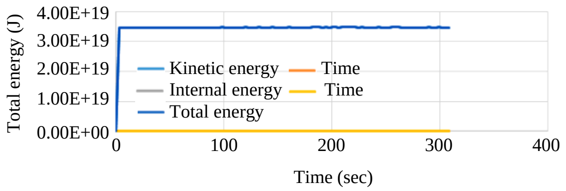

Fig. 9: Energy balance

In a physical test, the energy balance equation balances the work applied by the external forces in stressing the specimen (i.e., the Total Energy, TE) with the summation of its Internal Energy (IE) and Kinetic Energy (KE) as follows:

$$IE + KE = TE \tag{4}$$

This concept of energy conservation is central to structural engineering calculations as is the related concept of mass conservation. The results show that the total energy of the train and track is conserved, demonstrating that this numerical simulation obeys the fundamental principle of energy balance and mass balance throughout the whole dynamic analysis. Figure 9 shows the internal and kinetic energies and the total energy to demonstrate the energy balance.

CONCLUSION

This research selected and analysed a finite element model to simulate the dynamic response of a high-speed train on ballasted railway track. A typical track profile was modelled in the LS-DYAN program using 3D finite elements. The dynamic load of the train on the rail was also simulated in this program. Computation parameters were determined using a rational trainload and appropriate material properties. The distribution of vertical displacement, stress, strain, contact forces and sliding energy directly beneath the train load were also analysed to give the full dynamic response of the train-track system.

The dynamic response to the train load resulted in the substructure showing the greatest stress and ground vibration. It bore the majority of the vertical displacement, stress, strain and shear failure.

Equivalent stress and minimum principal strain occurred at the edge of the bearing plate of the super face of the ballast structure where they were most likely to generate stress concentration, resulting in plastic deformation and thus damage to the track.

Overall, most ballasted high-speed railway track requires a high-strength substructure to resist the dynamic response of passing trains which can potentially degrade the track and cause it to settle. This dynamic response should be considered and given more attention during design and feasibility studies. One such method to do this is numerical simulation using finite element computer code such as LS-DYNA.

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