Journal of Engineering and Applied Sciences 11 (Special Issue 2): 3136-3141, 2016
ISSN: 1816-949X

To the Study of the Motion of a Cylinder with Variable Mass in Flow: The Dynamics of a Free-Flow Micro Hydropower Plant

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Abstract: The working characteristics of a longitudinal flow hydraulic installation is largely determined by the interaction of operating elements with hydraulic flow. A salient feature of the studied device is the operative parts in the form of hollow cylinders with through-holes. Entering the flow, they fill with water, with their mass increasing. Leaving the flow, the cylinder becomes empty and its weight is equal to its proper weight. The process of increasing in the cylinder's weight in its interaction with the flow can be regarded as manifestation of the "pseudo-reactive force". The research considers the features of this interaction for the movable operating elements of variable mass. An original solution is suggested for this problem of practical value and theoretical significance.

Key words: Kinetic energy, velocity, flow, differential equation, function, movable operating element

INTRODUCTION

Free-flow micro hydroelectric plants use the kinetic energy of the flow. The flow's energy is received by the device's movable operating element. The efficiency of energy transfer to the movable operating element is defined by the well-known formula for determining the resistance force at the flow around the body:

$$F = c_x \times \frac{v^2}{2} \times S \qquad (1)$$

The indicated force is determined by the three quantities: c_x is resistance coefficient, v is flow velocity, S is the cross-section of the operating element. Since, a change in the mass of the movable element is time-related in a complex non-linear way, to determine the velocity of the movable part as the function of time is a challenging, non-standard task, that is significant for both practical application and theory of variable-mass body motion.

Literature review: The most rational value of v_c and c_x is achieved in a device, the movable operating elements of which have the form of ladles and located on the free surface of the flow.

In the devices converting the driving members' translational motion into rotational, the work efficiency is additionally determined by the kinetic energy value:

$$K = \sum m_0(mv) \qquad (2)$$

It follows from Eq. 1 that the angular momentum magnitude is dependent, among other factors, on mass (Singh *et al*., 2007). In the device of the patent (Krasnov *et al*., 2015), the movable operating element is designed as a cylinder. It has become customary to classify such devices as 'free flow/unimpeded flow' ones (Obozov and Botpav, 2010); body motion in compressible media was explored by Basmat (2005).

The cylinder's constructional feature is that it is hollow and has through slits. Upon its immersion into the flow (Fig. 1), it acquires a velocity and increases its mass during filling its volume with water (Stosser and Nikora, 2008).

Filling of the movable parts of the driving strand (Fig. 2) with water increases the latter's total weight (Stosser, 2014). When coming out of the flow, the cylinders are being emptied and the weight of the return strand considerably diminishes which leads to a reduced resistance to its motion.

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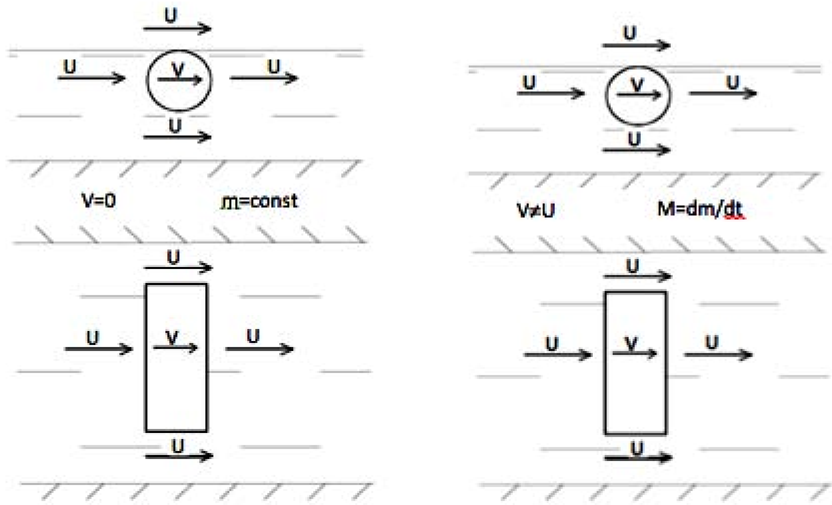

Fig. 1: Flow around the cylinder

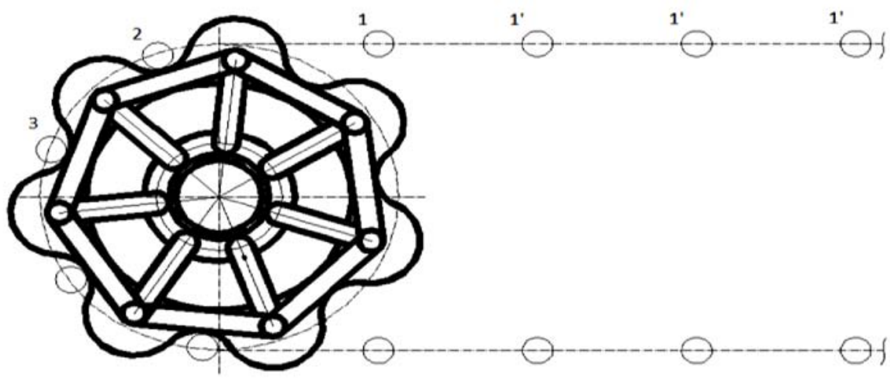

Fig. 2: Hydropower plant's strands

These features predetermine the roller movement in the flow under the influence of the flow resistance forces dependent on velocity and mass.

MATERIALS AND METHODS

$$p_x = m_x(U - U_x) \quad (3)$$

The momentum transferred by water (flowing into the hole) of the mass m_x. Pseudo-reactive force (Fig. 3):

$$F_p = \frac{\partial \rho_x}{\partial t}$$

Resultant force:

$$(m_0 + m_x)a$$

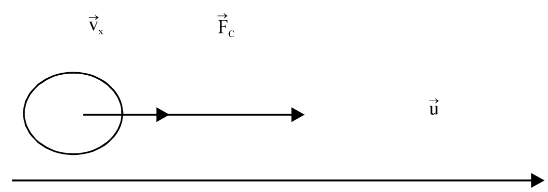

Fig. 3: Design model

$$(m_0 + m_x)a = \Gamma_p + \Gamma_c = \frac{\partial \rho_x}{\partial t} + C_x \frac{\rho(U - VX)^2}{2} S$$

$$(m_0 + m_x)\frac{\partial V_x}{\partial t} = \frac{\partial (U - V_x)m_x}{\partial t} + C_x \frac{\rho(U - V_x)^2}{2} S$$

$$m_0 \partial V_x + m_x \partial V_x = U \partial m_x - V_x \partial m_x - m_x \partial V_x + C_x \frac{\rho(U - V_x)^2}{2} S$$

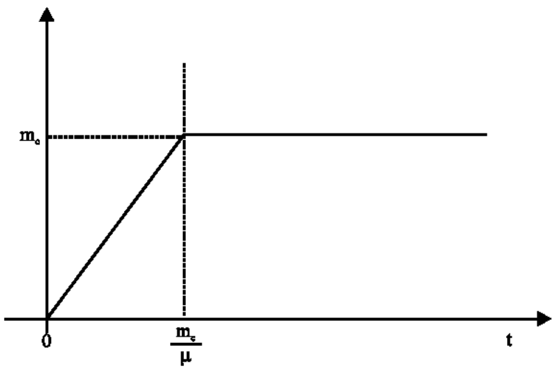

Fig. 4: Function on the half-time t>0

$$
\begin{aligned}
m_0 V_x + m_x V_x &= U - V_x m_x - m_x V_x + \frac{\rho(U-V_x)^2}{2} S + \\
V_x(m_0 + m_x + m_x + m_x) \\
&= C_x \frac{\rho(U-V_x)^2}{2} S - V_x(m_0 + 3m_x) \\
&= C_x \frac{\rho(U-V_x)^2}{2} S + A \\
\frac{V_x}{(U-V_x)^2} &= \frac{C_x pSt}{2(m_x + 3m_x - Um_x)} + A \\
\frac{(U-V_x)^2}{V_x} &= \frac{C_x pSt}{2(m_x + 3m_x - Um_x)} + A
\end{aligned} \tag{4}
$$

Let us assume:

$$
m_c(t) = \begin{cases} \mu t & 0 < t < \dfrac{m_c}{\mu} \\ m_c & t > \dfrac{m_c}{\mu} \end{cases}
$$

where, m_c/μ is the time to fill the pipe completely. Below, there is a graph of the function $m_c(t)$ on the half-line $t>0$ (Fig. 4).

For simplicity, let us substitute the variables: $x = u\text{-}v_x$ into the part $0<t<m_c/\mu$ of Eq. 4 and $y = u\text{-}v_x$ into the part $t>m_c/\mu$ of the same equation. The decomposition of the forces $F_p(t)$, $F_c(t)$ gives two differential equations:

$$\frac{dx}{dt} + \frac{\mu}{m_0 + 2\mu t} x = -\frac{C_x \rho S}{2(m_0 + 2\mu t)} x^2 \tag{5}$$

$$\frac{dy}{dt} + \frac{\mu}{m_0 + 2m_c} y = -\frac{C_x \rho S}{2(m_0 + 2m_c)} y^2 \tag{6}$$

These are Bernoulli-type differential equations. Equation 5 is to be solved first. The variable x is replaced by the variable z:

$$z = \frac{1}{x} \tag{7}$$

whence it follows that:

$$\frac{dz}{dt} = -\frac{1}{x^2}\frac{dx}{dt} \tag{8}$$

Let us multiple both parts of Eq. 5 by the value $-1/x^2$. The result is:

$$-\frac{1}{x^2}\frac{dx}{dt} - \frac{\mu}{m_0 + 2\mu t}\frac{1}{x} = \frac{C_x \rho S}{2(m_0 + 2\mu t)} \tag{9}$$

The substitution of the replacement Eq. 7 in Eq. 8 gives the following:

$$\frac{dz}{dt} - \frac{\mu}{m_0 + 2\mu t} z = \frac{C_x \rho S}{2(m_0 + 2\mu t)} \tag{10}$$

This is a linear differential equation of first order, which can be solved, for example, by the Lagrangian method. This method is that solution is sought in the product of two functions, namely:

$$z = u \cdot v \tag{11}$$

from whence:

$$\frac{dz}{dt} = \frac{du}{dt} v + u \frac{dv}{dt} \tag{12}$$

Let us substitute this replacement into Eq. 10:

$$\frac{du}{dt} v + u \frac{dv}{dt} - \frac{\mu}{m_0 + 2\mu t} uv = \frac{C_x \rho S}{2(m_0 + 2\mu t)} \tag{13}$$

To make the calculations simple, we denote:

$$A = \frac{\mu}{m_0 + 2\mu t}, \; B = \frac{C_x \rho S}{2(m_0 + 2\mu t)}$$

In this case, Eq. 10 has the following form:

$$\frac{du}{dt} v + u \frac{dv}{dt} - Auv = B \tag{14}$$

By means of transformations, there is:

$$\ln v = \int \frac{u}{w}\frac{dw}{2\mu} = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln w \tag{15}$$

i.e.:

$$v = \sqrt{w} \quad (16)$$

Then let us substitute the obtained function into Eq. 14. The result is the second auxiliary equation:

$$\frac{du}{dt}\sqrt{w} = B \quad (17)$$

or in detail:

$$\frac{du}{dt}\sqrt{m_0 + 2\mu t} = \frac{C_x\rho S}{2(m_0 + 2\mu t)} \quad (18)$$

Then, we solve this auxiliary Eq. 18 for the function u and re-write it as:

$$\frac{du}{dt} = \frac{C_x\rho S}{2(m_0 + 2\mu t)^{3/2}} \quad (19)$$

Let us integrate both members of this equality:

$$u = \frac{C_x\rho S}{2}\int \frac{dt}{(m_0 + 2\mu t)^{3/2}} \quad (20)$$

and make a similar substitution again:

$$w = m_0 + 2\mu t,\ t = \frac{w - m_0}{2\mu},\ dt = \frac{dw}{2\mu}$$

Taking into account that $z = u{\cdot}v$, there is the equation solution (Eq. 10):

$$z = \left(C - \frac{C_x\rho S}{2\mu\sqrt{m_0 + 2\mu t}}\right)\sqrt{m_0 + 2\mu t} \quad (21)$$

After removing parenthesis, we have:

$$z = C\sqrt{m_0 + 2\mu t} - \frac{C_x\rho S}{2\mu} \quad (22)$$

From the replacement $z = 1/x$, we find:

$$x(t) = \frac{2\mu}{2C\mu\sqrt{m_0 + 2\mu t} - C_x\rho S} \quad (23)$$

The constant C is to be searched based on the initial condition:

$$v_x(0) = 0$$

or:

$$x(0) = u\text{-}v_x(0) = u$$

Therefore:

$$x(0) = \frac{2\mu}{2C\mu\sqrt{m_0} - C_x\rho S} \quad (24)$$

Equating these expressions gives:

$$u = \frac{2\mu}{2C\mu\sqrt{m_0} - C_x\rho S} \quad (25)$$

From the above, we deduce that:

$$2C\mu\sqrt{m_0} = C_x\rho S + \frac{2\mu}{u} \quad (26)$$

Hence:

$$C = \frac{C_x\rho Su + 2\mu}{2\mu u\sqrt{m_0}} \quad (27)$$

Let us substitute the obtained constant into (Eq. 28):

$$x(t) = \frac{2\mu u\sqrt{m_0}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2\mu t} - C_x\rho Su\sqrt{m_0}} \quad (28)$$

From the previously imposed replacement $x = u\text{-}v_x$, we find the velocity v_x. Therefore, the velocity $v_x(t)$ in the part $0<t<m_c/\mu$ obeys the following law:

$$v_x = u - \frac{2\mu u\sqrt{m_0}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2\mu t} - C_x\rho Su\sqrt{m_0}} \quad (29)$$

Let us solve Eq. 6 now:

$$\frac{dy}{dt} + \frac{\mu}{m_0 + 2m_c}y = -\frac{C_x\rho S}{2(m_0 + 2m_c)}y^2 \quad (30)$$

Using a similar replacement:

$$z = \frac{1}{y} \quad (31)$$

We get:

$$-\frac{1}{y^2}\frac{dx}{dt} - \frac{\mu}{m_0 + 2m_c}\frac{1}{y} = \frac{C_x\rho S}{2(m_0 + 2m_c)} \quad (32)$$

and then:

$$\frac{dz}{dt} - \frac{\mu}{m_0 + 2m_c}z = \frac{C_x\rho S}{2(m_0 + 2m_c)} \quad (33)$$

The solution of this linear differential equation can be also found in the form: z = u·v. Let us substitute this replacement into Eq. 33:

$$\frac{du}{dt}v+u\frac{dv}{dt}-\frac{\mu}{m_0+2\mu t}uv=\frac{C_x\rho S}{2(m_0+2\mu t)} \quad (34)$$

As previously, for simplicity, we denote:

$$A=\frac{\mu}{m_0+2m_c},\ B=\frac{C_x\rho S}{2(m_0+2m_c)}$$

Then, Eq. 34 has the following form:

$$\frac{du}{dt}v+u\frac{dv}{dt}-Auv=B \quad (35)$$

Solving the first auxiliary equation in relation to v leads to:

$$\ln v=\int\frac{\mu}{m_0+2m_c}dt=\frac{\mu t}{m_0+2m_c} \quad (36)$$

By potentiating this equality, we have:

$$v=\exp\frac{\mu t}{m_0+2m_c} \quad (37)$$

The found function is substituted into Eq. 34. There is the second auxiliary equation:

$$\frac{du}{dt}\exp\frac{\mu t}{m_0+2m_c}=B \quad (38)$$

or:

$$\frac{du}{dt}=\exp\left(-\frac{\mu t}{m_0+2m_c}\right)\frac{C_x\rho S}{2(m_0+2m_c)} \quad (39)$$

We solve this auxiliary Eq. 39 with regard to the function u. Then, the solution of Eq. 35 relatively the axillary variable z has the form:

$$z=\left(C-\frac{C_x\rho S}{2\mu}\exp\left(-\frac{\mu t}{m_0+2m_c}\right)\right)\exp\frac{\mu t}{m_0+2m_c} \quad (40)$$

Or to put it simple:

$$z=C\exp\frac{\mu t}{m_0+2m_c}-\frac{C_x\rho S}{2\mu} \quad (41)$$

Keeping in mind the replacement z = 1/y, we get:

$$y(t)=\frac{2\mu}{2C\mu\exp\frac{\mu t}{m_0+2m_c}-C_x\rho S} \quad (42)$$

The constant C in Eq. 42 is to be found based on the natural condition of continuity of functions x(t), y(t) in the point m_c/μ:

$$x\left(\frac{m_c}{\mu}\right)=y\left(\frac{m_c}{\mu}\right) \quad (43)$$

Namely:

$$x\left(\frac{m_c}{\mu}\right)=\frac{2\mu u\sqrt{m_0}}{(C_x\rho Su+2\mu)\sqrt{m_0+2m_c}-C_x\rho Su\sqrt{m_0}} \quad (44)$$

$$y\left(\frac{m_c}{\mu}\right)=\frac{2\mu}{2C\mu\exp\frac{m_c}{m_0+2m_c}-C_x\rho S} \quad (45)$$

After equating these values, we have:

$$\frac{2\mu u\sqrt{m_0}}{(C_x\rho Su+2\mu)\sqrt{m_0+2m_c}-C_x\rho Su\sqrt{m_0}}=\frac{2\mu}{2C\mu\exp\frac{m_c}{m_0+2m_c}-C_x\rho S} \quad (46)$$

After the cancellation of 2μ, there is:

$$2C\mu u\sqrt{m_0}\exp\frac{m_c}{m_c+2m_c}-C_x\rho Su\sqrt{m_0}=(C_x\rho Su+2\mu)\sqrt{m_0+2m_c}-C_x\rho Su\sqrt{m_0} \quad (47)$$

The next step is:

$$2C\mu u\sqrt{m_0}\exp\frac{m_c}{m_0+2m_c}=(C_x\rho Su+2\mu)\sqrt{m_0+2m_c} \quad (48)$$

Hereof:

$$2\mu C=\frac{(C_x\rho Su+2\mu)\sqrt{m_0+2m_c}}{u\sqrt{m_0}}\exp\left(\frac{-m_c}{m_0+2m_c}\right) \quad (49)$$

Let us substitute the expression into Eq. 41:

$$y(t)=\frac{2\mu u\sqrt{m_0}}{(C_x\rho Su+2\mu)\sqrt{m_0+2m_c}\exp\frac{\mu t-m_c}{m_0+2m_c}-C_x\rho Su\sqrt{m_0}} \quad (50)$$

Based on the replacement $y = u-v_x$, there is:

$$v_x(t) = u - \frac{2\mu u\sqrt{m_0}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2m_c}\exp\frac{\mu t - m_c}{m_0 + 2m_c} - C_x\rho Su\sqrt{m_0}} \quad (51)$$

So, for $0<t<m_c/\mu$:

$$v_x = u - \frac{2\mu u\sqrt{m_0}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2\mu t} - C_x\rho Su\sqrt{m_0}} \quad (52)$$

and for $t>m_c/\mu$:

$$v_x = u - \frac{2\mu u\sqrt{m_0}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2m_c}\exp\frac{\mu t - m_c}{m_0 + 2m_c} - C_x\rho Su\sqrt{m_0}} \quad (53)$$

It is easy to notice that the function $v_x(t)$ asymptotically approaches a constant value at the part $t>m_c/\mu$ with $t\to\infty$. Let us denote it as Ω. In particular:

$$\lim_{t\to\infty}\left(u - \frac{2\mu u\sqrt{m_0}/\exp\frac{\mu t - m_0}{m_0 + 2m_c}}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2m_c} - C_x\rho Su\sqrt{m_0}/\exp\frac{\mu t - m_c}{m_0 + 2m_c}} \right) \quad (54)$$

$$= u - \frac{1}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2m_c}}$$

Thus:

$$\Omega = u - \frac{1}{(C_x\rho Su + 2\mu)\sqrt{m_0 + 2m_c}} \quad (55)$$

The resulting dependence establishes the correlation of the change in the cylinder velocity with respect to the flow velocity and the change in its mass. The results of the study can be used for designing longitudinal flow hydraulic power plants and, in particular, for determining the cross sections of through slots in the cylinders.

RESULTS AND DISCUSSION

The findings of the study can be used both for designing longitudinal flow hydropower plants (in particular, for determining cross-sections of the cut-through slots in cylinders and for achieving the cavity in-fill time to be equal to the time of the cylinder's reaching the speed of the flow) and for further theoretical researches.

CONCLUSION

It is known from the previous experience that a cylindrical cavity with a longitudinal slot, when submersed in a flow of constant velocity, starts speeding up and after a while reaches a speed of the flow. Since the authors of the present study could not find any known solution to this problem, they developed their own original solution which accords well with experimental data. At reference points and even more importantly, at asymptotes, the solution perfectly matched expected results.

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