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Algorithms of Digital Calculations of Optimal Flight Characteristics and Rational Design Parameters

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Abstract: Computation method of emphasized flight velocities calculation oflight prop engine airplane with taking into account nominal engine performanceis suggested. Algorithm of airscrew blade geometry (twist angle, chord and optimal angle of attack along blade) numerical calculation is developed. Numerical calculations of design parameters include computation of rational blade angle of attack and remove arbitrary presetting these parameters. Automatic choiceof airscrew design parameters matched to wide range of flight velocities for prop engine light planebecome enable. Features comparison of high-altitude airplane and airscrew characteristics calculated by impulse-blade-element theory allow us to state that our algorithm can be effectively used to thrust vs. flight speed design calculations. Design calculation includes several tasks: efficient algorithms of airscrew geometry design calculations, more efficient airscrew operation dew to optimal rate of rotation and changing blade incidence angle. Simple enough algorithm to piston-engine airplane performances calculation by using of engine characteristics power RPM per hour fuel consumption is suggested. Feasible rational versions of plane design parameters are discussed.

Key words: Computation methods, flight performance, airscrew, blade twist, design performance, high-altitude

INTRODUCTION

Some aircraft flight velocities didn't depends from engine performance. They are V_{stall} stall speed, V_{bg} velocity of maximum range (best glide); V_{md} velocity of maximum flight duration (minimum descend).

Velocities V_{bg} and V_{md} are defined by equation of parabolic drag polar equation $C_d = C_{d0}+KC_L^2$ and can be expressed by following equation:

$$V_{bg} = \left(\frac{4K}{C_{d0} \cdot \rho^2} \left(\frac{W}{S} \right)^2 \right)^{1/4} \quad (1)$$

$$V_{md} = \left(\frac{4}{3} \frac{K}{C_{d0} \rho^2} \left(\frac{W}{S} \right)^2 \right)^{1/4} = 0.758 \cdot V_{bg} \quad (2)$$

Where:

C_{d0}	=	Coefficient of parasitic drag
W	=	Airplane mass (weight)
ρ	=	Air density
S	=	Wing reference area
K	=	Polar induced drag correction factor (Gainutdinov *et al*., 2009)

Engine thrust vs. speed dependence is required to calculate cruise velocity V_c, V_y velocity of maximum climb rate; V_x velocity of economical climb; V_m minimal speed (stall speed); V_m maximum velocity of level flight, time and distance of take-off run. For example, following thrust vs. speed dependence $T = T_0 - aV^2$ is used to time and distance of take-off run calculations where T_0 is static thrust, a is some factor that can be equal zero or bigger (less) then 1. Cruise speed is usually defined by V_{bg} for example, there is recommendation to define V_C by following (Carson, 1980) Eq. 2:

$$V_C = 1.32\, V_{bg} \text{ and } \left(\frac{C_L}{C_d} \right)_c = \frac{\sqrt{3}}{2} \left(\frac{C_L}{C_d} \right)_{max} = 0.866 \left(\frac{C_L}{C_d} \right)_{max} \quad (3)$$

where, C_L and C_d are lift and drag coefficient correspondently.

MATERIALS AND METHODS

Algorithm to piston-engined airplane performances calculation: Equation of required an available power balance is used in flight performance design calculation for light plane with prop engine:

$$P_a.\eta = C_{d0} \frac{\rho^{V^3} M}{2} S + \frac{2KW^2}{\rho^V M^S} \quad (4)$$

Where:

P_a	=	Available engine power
η	=	Engine efficiency coefficient

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Table 1: Engine performance according to rotational speed

Rotational speed	----------Engine performance----------					
gPα (Wt)	7340	14680	22020	29360	33030	36700
n (rev/min)	3500	4500	5250	5750	6000	36700
$\bar{m}_f$ (kg/h	5.964	7.7	9.23	10.5	11.0	12.2

Table 2: Emphasized velocity

V (m/sec)	Values	M_f (kg)	P_{r_SL} (Wt)	T_r (H)	n (γev/min)	m_{fuel} (kg/h)
V_{md} = 35.1	35.3*	51.0	9602	141	3808 (1523)	6.50
V_{bg} = 46.4	46.3*	40.9	10933	123	3990 (1596)	6.80
V_C = 61.2	64.8*	36.7	18766	151	4927 (1971)	8.57
V_M = 85	85.0	39.7	36200	223	6140 (2456)	12.12

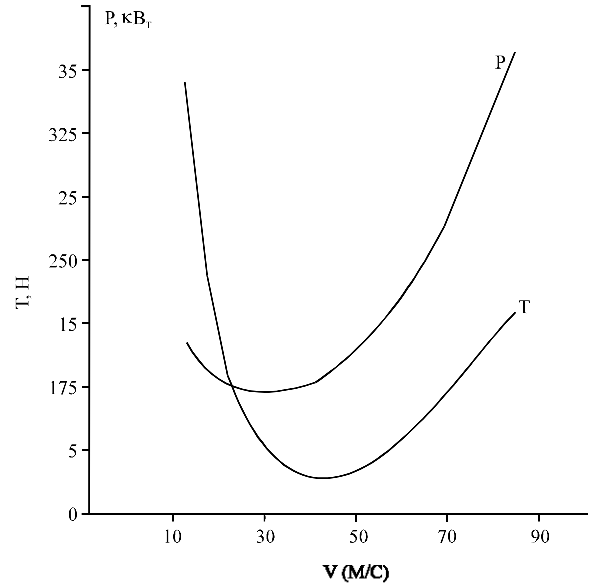

Fig. 1: Required power P and thrust vs. flight velocity V

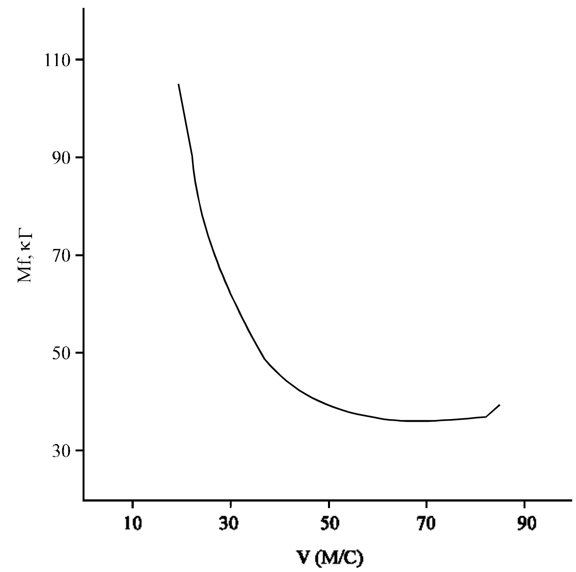

Fig. 2: Total fuel mass vs. flight velocity to R = 1000 km

Decreasing of engine power due to height of flight is defined by relative air density for example, $P_a = P_{a\_SL} \cdot (\rho/\rho_0)$ where P_{a_SL} is sea level available engine power.

Thereby we suggest to use simple enough algorithm to piston-engined airplane performances calculation by using of engine characteristics power-rpm-per hour fuel consumtion. Each engine has table of engine performance that bind available power P_{a_SL} rotational speed of the rotor n and per hour fuel consumptionm $\bar{m}_f$. Example of that data is shown in Table 1.

It is possible to calculate dependences of required power P_{r_SL} and rotational speed of the rotor n to flight velocity V. If the flight range R is preset we can calculate required total fuel mass M_f that also depends from flight velocity V. Examples of that dependences are shown in Fig. 1 and 2.

Table 2 contains data of M_f, P_{r_SL}, n, $\bar{m}_f$ and required Thrust T_r vs. emphasized velocity V_{md}, V_{bg}, V_C and V_M. Marked by asterisk velocities is defined by above described method. Revolution per minute n (in parenthesis) correspond rate of revolution after reducer. We can see that V_C to V_{bg} ratio is defined by interrelationship $V_C^* = 1.4\ V_{bg}$ versus of recommended $V_C = 1.32\ V_{bg}$ and required to range R total fuel mass change slightly if flight velocity is between 60 and 80 m sec^{-1} despite fuel consumption per hour $\bar{m}_f$ increase proportional to flight velocity. For this reason cruise speed can be chosen bigger then $V_C = 1.32\ V_{bg}$ and even bigger then $V_C^* = 1.4\ V_{bg}$. Certainly required total fuel mass will be the least for $V_C^* = 1.4\ V_{bg}$ but flight time to target decrease if the flight velocity is bigger then 1.4 V_{bg}.

That results are obtained for following input data: $P_{SLmax} = 36500$ Wt, $\eta_P = 0.7$, $\rho/\rho_0 = 0.742$ (H = 3000 m), S = 3 m^2, W = 2000 H, K = $1/\pi \cdot e \cdot AR$ where e = 0.7 is Oswald coefficient, AR = 12 wing aspect ratio.

RESULTS AND DISCUSSION

Airscrew blade geometry calculations: There is necessity to calculate thrust vs. speed dependences that includes airscrew characteristics. Moreover, we must design airscrew blade geometry to obtain required aircraft performance at all mode of flight. Thereby the problem of design calculation includes several tasks: efficient algorithms of airscrew geometry design calculations, more efficient airscrew operation dew to optimal rate of rotation and changing blade incidence angle.

Features comparison of high-altitude airplane Strato 2C five-blades airscrew (Wald, 2006) and airscrew

Table 3: Comparison of flight features for different airplanes

Flight height	I	II	III	IV
Altitude H (m)	12000	18500	22000	24000
Air demsity ρ (kg/m^3)	0.30	0.11	0.064	0.047
Flight velocity V (m/sec)	62.2	101.3	131.3	153.8
Airscrew Thrust (H)				
Calculated (ideal airscrew)	2760	2808	2194	1896
Experimental	2502	2556	1909	1252
Calculated (our scheme)	2660	2620	1980	1680

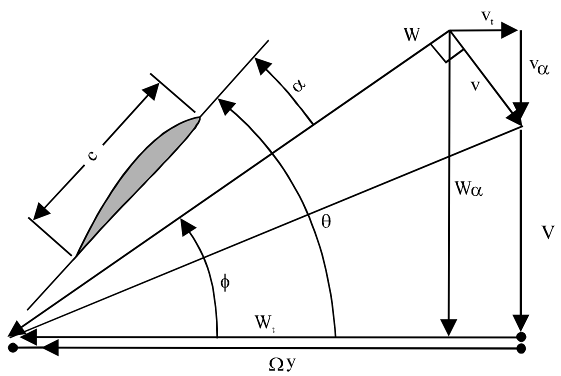

Fig. 3: Velocities components of airflow around cylindrical blade cross section

characteristics calculated by impulse-blade-element theory allow us to state that our algorithm can be effectively used to thrust vs. flight speed design calculations. Flight features (practical and calculated) published in Wichmann and Koster is shown in Table 3.

There are simple and effective algorithms of blade geometry design calculations on the base of element blade theory and impulse theory equation (Alexandrov, 1951; Kravetc, 1941; Juriev, 1948; Shaidakov and Maslov, 1995). Input data for that calculations are available engine power, shaft revolution per minute, blade section airfoil characteristics, etc. Consider briefly main equations of this theory.

The first equation is obtained by analysis of velocity components and emphasized angles of airflow around cylindrical blade cross section that is shown in Fig. 3. This equation can be written by following expression:

$$\cos\phi + \zeta = x\sin\phi \tag{5}$$

Here, V is velocity of axial inflow (advanced velocity), Ωy is velocity caused by rotation of the blade, V_a and V_t are axial and tangent component of induced velocity-velocity of incoming flow (W_a and W_t are axial and tangent component), $\overline{W} = W/R$ (R isblade radius), c is blade cross section chord, α is angle of attack, θ is blade pitch, α is angle of incoming flow:

Table 4: Blade airfoil characteristics

NACA	C_{lmax} Re = 3×10^6	α_s (grad)	C_{m0}	(C_l/C_d) max.	C_{li}	C_{dmin}	(t/c) max (%)
2415	1.4	14	-0.050	102	0.3	0.0065	15
23012	1.6	16	-0.013	120	0.3	0.0060	12

$$\zeta = \frac{v}{V}, x = \frac{\Omega y}{v} = \frac{\Omega R}{v}\frac{y}{R} = \frac{1}{\lambda_V}r$$

The second equation defines equality blade element lift and annular radius force of momentum theory:

$$N_b \bar{c} C\iota b \overline{W}^2 = 8\pi r F(\zeta + \zeta^2 \cos\phi) \tag{6}$$

Where:

N_b	=	Number of airscrew blades
$\gamma = y/R$	=	non-dimensional radial distance along blade
$\bar{c} = c/R$	=	Relative blade chord
C_{1b}	=	Lift coefficient of blade element airfoil
$\overline{W}^2 = 3\zeta^2 + x^2 + 1$, F	=	Prandtl tip loss function (Wald, 2006)

The third group of equations defines rotor thrust T_a and power P_a:

$$C_T = \frac{T_a}{\rho\pi R^2 (\Omega R)^2} = 4\int_0^1 F\lambda_V^2 (\zeta + \zeta^2 \cos\phi)$$
$$\cos\phi(1-\varepsilon \cdot \tan\phi) r dr)$$

$$C_P = \frac{P_a}{\rho\pi R^2 (\Omega R)^3} = 4\int_0^1 F\lambda_V^2 (\zeta + \zeta^2 \cos\phi) \tag{7}$$
$$\sin\phi(1-\varepsilon/\tan\phi) r^2 dr)$$

Where:

$\varepsilon = C_d/C_l$ = Drag to lift ratio of blade element airfoil
C_t and C_P = Rotor thrust and power coefficients

Chosen blade airfoil characteristics is defined by values enumerated in Table 4 and by relationships (Eq. 8-10):

$$C_d = C_{dmin} + K(C_l - C_{lmin})^2 \tag{8}$$

$$K = \frac{C_{d1} - C_{d\,min}}{(C_{l1} - C_{l\,min})^2} \tag{9}$$

$$C_l = C_l^\alpha (\alpha - \alpha_0) \tag{10}$$

where, C_{l1} and C_{d1} are values taken from polar graph C_l-C_d. For example, airfoil NACA 2415 have following data C_l^α = 6.156 1/рад and α_0 = -2.25 град, K = 0.0083.

Numerical calculation allow us to obtain followings design values: θ(r) blade pitch θ(r), relative blade chord $\bar{c}$(r) and rational angle of attack α(r). Blade pitch θ(r) and relative blade chord $\bar{c}(r)$ are defined as a results of calculation using relationships (Eq. 5-7). Special numerical algorithm must be developed to $\alpha(r)$ calculation. We use clause of maximum thrust (rotor thrust coefficient) corresponding available power (rotor power coefficient). Distribution of $\alpha(r)$ along blade represents by cubic function $\alpha(r) = Ar^3+Br^2+Cr+D$, expressed by vector $\alpha(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ where α_i are volume of $\alpha(r)$ at four points along blade. Iterative procedure of n+1 step of Newton maximum thrust searching scheme can be defined by following matrix equation:

$$\alpha_{(n+1)} = \alpha(n) - \left[\frac{d^2T(\alpha)}{d\alpha^2}\right]^{-1} \frac{dT(\alpha)}{d\alpha} \quad (11)$$

Each iteration require repetition of calculations trust tusing using relationships (Eq. 5-7) to calculate elements of matrix Eq. 11:

$$\frac{dT}{d\alpha_j} \approx \frac{\left[T(\alpha_j+\Delta\alpha_j)-T(\alpha_j-\Delta\alpha_j)\right]}{2\Delta\alpha_j}$$

$$\frac{d^2T}{(d\alpha_j)^2} \approx \frac{\left[T(\alpha_j+\Delta\alpha_j)-2T(\alpha_j)+T(\alpha_j-\Delta\alpha_j)\right]}{(\Delta\alpha_j)^2}$$

$$\frac{d^2T}{(d\alpha_i d\alpha_j)} \approx \frac{\left[T(\alpha_i+\Delta\alpha_i, \Delta\alpha_j)-2T(\alpha_j)+T(\alpha_j-\Delta\alpha_j)\right]}{(\Delta\alpha_j)^2} \quad (12)$$

$$\frac{d^2T}{(d\alpha_i d\alpha_j)} \approx \frac{\begin{bmatrix} T(\alpha_i+\Delta\alpha_i, \alpha_j+\Delta\alpha_j)-T(\alpha_i+\Delta\alpha_i, \alpha_j-\Delta\alpha_j)- \\ T(\alpha_i-\Delta\alpha_i, \alpha_j+\Delta\alpha_j)+T(\alpha_i-\Delta\alpha_i, \alpha_j-\Delta\alpha_j) \end{bmatrix}}{(4\Delta\alpha_i\Delta\alpha_j)}$$

$$(i, j = 1,\ 4)$$

Perform computation of blade parameters taking into account flight velocity V, power P_{r_SL} and rotational speed n from Table 2. Calculation results is shown in Fig. 4. Dashed linesdepict volume of pitch angle θ and relative blade chord $\bar{c}$ for flight velocity V_{bg} that differ from ones obtained for velocities V_C, V_M. Rational angle of attack $\alpha(r)$ practically is the same for velocities V_{bg}, V_C, V_M. As we can see difference of $\theta(r)$ that correspond flight velocity V_{bg}. It can be compensate by blade angle changing because nearly constant difference of $\theta(r)$ along blade. As for blade chord $\bar{c}(r)$ it is difficult to change this geometry parameter.

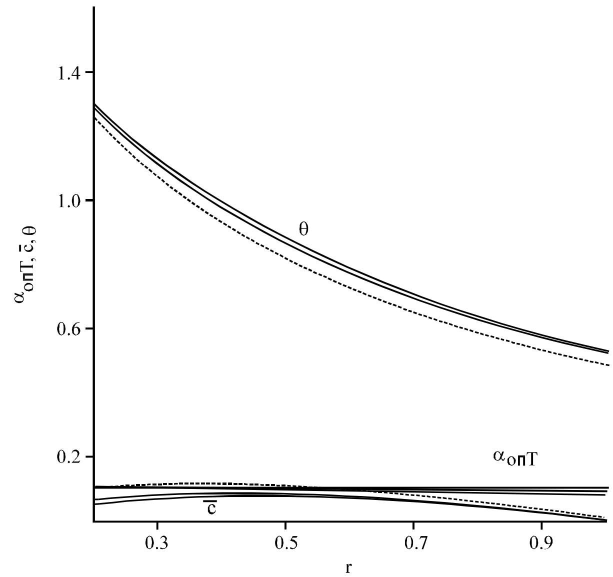

Fig. 4: Volume of $\theta(r)$, $\bar{c}(r)$, $\alpha(r)$ along blade

Obtained rotor parameters can be considered as satisfactory parameters only for cruise velocity V_C and maximum velocity V_M.

But not always it is possible to use variable-pitch propeller to provide appropriate thrust to other flight velocity. It is very difficult for this to change blade chord too. We can only vary rotational speed of the rotor and output engine power in constant pitch airscrew. Table 1 have engine certificate data "output power-rotational speed-per hour fuel consumption" that correspond engine rated power setting. Table 2 shows thatmaximum power and rotational speed is assignedto maximum velocity V_M. Therefore it is possible to change engine power setting to the flight velocity less then V_M. New blade design parameters $\theta(r)$, $\bar{c}(r)$ are shown in Fig. 4 that was calculated for two flight velocities V_{bg}, V_M and differ from one another shown in Fig. 5. The diameter of airscrew is D = 1.4 m. We can see that it is possible to chose output power and correspond airscrew rotational speed to make $\theta(r)$ more close.

Those airscrew parameters $\theta(r)$, $\bar{c}(r)$ is near to rational design parameters to flight velocities from V_{bg} to V_M. Lower flight velocities required variable pitch propellerusing ornon optimal engine settings in case of constant pitch ones. Required thrust T_{req} and available thrust T_{avl} for variable pitch (solid line) and constant pitch airscrew (dashed line) versus flight velocity are shown in Fig. 6.

Corresponding pitch step variation γ of variable pitch propeller and rotational speed n of the constant pitch ones are shown in Fig. 7. Dashed line depicts airscrew

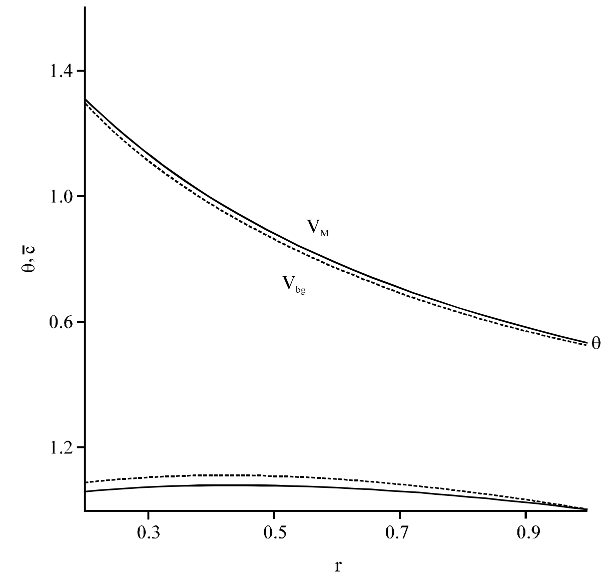

Fig. 5: Volume of θ(r), $\bar{c}$(r) for velocity V_{bg} и V_M

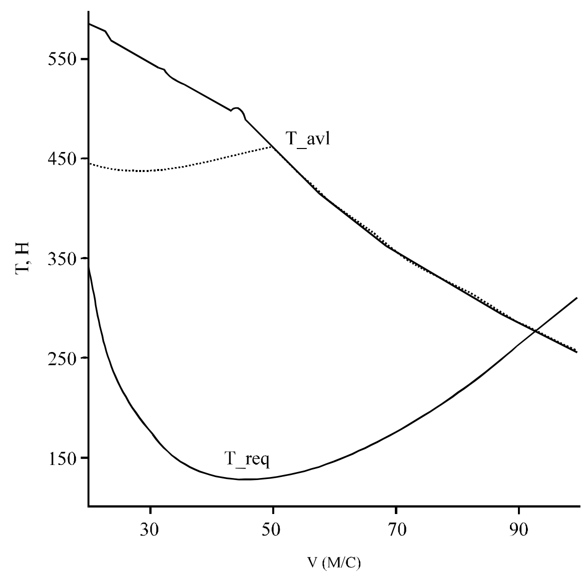

Fig. 6: Required and available thrust vs. flight velocity

rotational speed that recommended for optimal engine settings. We use iterative procedure of discrete Newton scheme to calculate (or n):

$$\gamma_{(n+1)} = \gamma_{(n)} - \left[\frac{dp(\gamma)}{d\gamma}\right]^{-1} \left[P^{*}-P(\gamma)\right] \quad (13)$$

where, P^*-P(γ) is residual between preset power P^* and calculated ones P(γ). If compare Fig. 6 and data of Table 2 we can see that maximum velocity in this case are higher because above stated screw computation method

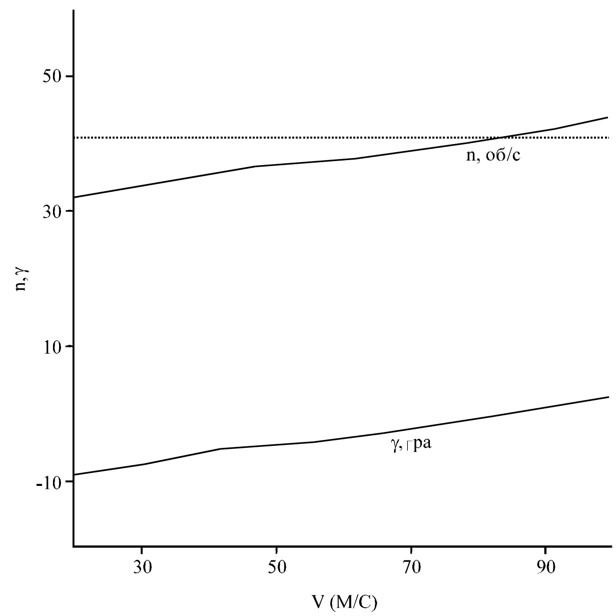

Fig. 7: Incidence blade angle γ and RPM n vs. flight velocity

do not take into account all aerodymamic losses and airscrew design parameters a closer to ones of so called ideal propeller. On base of above-stated it is possible to come to following conclusions:

- Light prop engine plane's cruise speed V_C that computed with taking into account nominal engine performance may differ from recommended V_C = 1.32 V_{bg} and can be calculated by using above stated algorithm
- Numerical calculations of airscrew design parameters include computation of rational blade angle of attack along blade α(r) and remove arbitrary presetting this parameters in design calculations
- On base above stated algorithm it is possible to chose airscrew design parameters matched to wide range of flight velocities for prop engine light plane

CONCLUSION

Suggested algorithm do enable to develop effective built in computing unit for light prop plane performance optimization.

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