# Feeding Unbalanced Loads by Controlling Four-Leg Inverters Using Resonant Controllers 

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#### Abstract

In recent years the number of distributed generators which are connected to a network or directly feed local loads is growing. Most of these generators need power inverters for their power transmission. On the other hand, in a power system most of the times, the loads are dynamic and change over time or even in some cases, they might become unbalanced. In this study, we have chosen a switching method presented for four-leg multilevel inverters. The method is based on the switching of two-level inverters which has a simple structure and generality for applying to any topology with any number of levels. Then, a technique for controlling inverters connected to unbalanced loads is presented based on resonant controllers. The presented method benefits from a very simple structure and can be implemented in a synchronous reference frame. At the end, a number of simulations are performed in MATLAB in order to validate the effectiveness of the proposed method and the results are presented.


$\underline{\text { Key words: Four-leg inverters, multilevel inverters, resonant controllers, unbalanced loads, validate }}$

## NTRODUCTION

In recent years, the number of distributed generators which are connected to a network or directly feed local loads is growing. Most of these generators need power inverters for their transmission. On the other hand in a power system most of the times, the loads are dynamic and change over time or even in some cases, they might become unbalanced. Recently, multilevel inverters have attracted considerable attention and their application in High Voltage Direct Current (HVDC) lines, renewable energies such as wind and photovoltaic, active filters, Flexible AC Transmission system (FACT) devices such as Static Var Compensator (STATCOM), Dynamic Voltage Restore (DVR) and Unified Power Quality Condition (UPQC) has increased impressively. Multilevel inverters have many advantages such as the possibility of switching in base frequency which can reduce switching losses and electromagnetic interferences (Patel, 2005; Ye et al., 2008). Additionally, they have low harmonic content which is due to the similarity of their output voltage wave form to sinusoid.

The stress on the switches in such inverters is lower than those with less number of levels which allows the use of switches with lower voltage rate. This can help in reducing cost and increasing efficiency due to the reduction of conduction resistance of the keys.

Furthermore, it is possible to increase the voltage of the output in multilevel inverters (Calais et al., 2001; Pires et al., 2011). To feed unbalanced loads, different topologies have been presented among which four-leg inverter topology is most commonly used because of its attaining a balanced three-phase output voltage by accurately regulating the neutral point voltage, decreased sensitivity to the type of the load and maximum using of DC voltage link.

In this study, we have used this structure to feed unbalanced loads. There exist different methods for switching four-leg multilevel inventors. Based on the modulation of two-level inverters, a method is presented that has the advantage of reducing the computational complexity, not needing trigonometric functions, the capability of being implemented in real time and universality so it can be used for any topology with any number of levels.

There are different techniques for controlling inverters connected to unbalanced loads. It is common to use synchronous reference system and PI controllers which are usually used for controlling inverters connected to balanced loads; however for unbalanced loads, controllers with proportional characteristics are required. The older method is sequence decomposition which has disadvantages such as multiple conversions between reference systems which increase calculations. Moreover,


Fig. 1: A four-leg switching network


Fig. 2: The four-leg inverter of a three-phase voltage source
it needs many PI controllers and requires additional computations for decomposition into sequence components (Pires et al., 2011). In this study, a new method based on resonant controller is investigated.

Four-leg inverter: Until now, four-leg inverters are used for many purposes. In (Steigerwald, 1979), the fourth leg has been exploited to forcibly commutate the inverters of thyristor current sources.

By replacing the three-leg switching network with a four-leg one, a four-leg inverter will be attained as shown in Fig. 1. By connecting the neutral point to the middle of the fourth leg of a four-leg inverter (Fig. 2).
we can form a path for the flow of the current of the zero component which is produced by unbalanced threephase or nonlinear single-phase loads. For such case a balanced three-phase output voltage can be acquired by accurately setting the neutral point voltage. A neutral inductor $L_{n}$ is optional for this case. The inductor can alleviate pulses caused by switching (Steigerwald, 1979; Patel et al., 2010; Julian et al., 1996).

To clarify the role of a four-leg inverter, the switching function is defined as follows:
$S_{\text {if }}=\left\{\begin{array}{cc}! & \text { if } S_{j b} \text { and } S_{f n} \text { areclosed } \\ \wedge & \text { if } S_{j b} \text { and } S_{f n} \text { or } S_{j b} \text { and } S_{f n} \text { areclosed } j=\{a, b, c\} \\ -, & \text { if } S_{j n} \text { and } S_{f n} \text { areclosed }\end{array}\right\}$

Consequently, the voltage of AC terminals will be (Jahns et al., 1993):

$$
\left[\mathrm{V}_{\mathrm{af}} \mathrm{~V}_{\mathrm{bf}} \mathrm{~V}_{\mathrm{cf}}\right]^{\mathrm{T}}=\left[\mathrm{S}_{\mathrm{af}} \mathrm{~S}_{\mathrm{bf}} \mathrm{~S}_{\mathrm{cf}}\right]^{\mathrm{T}} \mathrm{~V}_{\mathrm{pn}}
$$

Similarly, the DC link current $i_{p}$ is be expressed as:

$$
\mathrm{i}_{\mathrm{p}}=\left[\mathrm{S}_{\mathrm{af}} \mathrm{~S}_{\mathrm{bf}} \mathrm{~S}_{\mathrm{cf}}\right] \cdot\left[\mathrm{i}_{\mathrm{a}} \mathrm{i}_{\mathrm{b}} \mathrm{i}_{\mathrm{c}}\right]^{\mathrm{T}}
$$

The voltage of AC terminals, $\mathrm{V}_{\mathrm{af}} \mathrm{V}_{\mathrm{bf}}$ and $\mathrm{V}_{\mathrm{cf}}$ are oscillating voltages with a constant switching period $\left(\mathrm{T}_{\mathrm{s}}\right)$. Taking the average of Eq. 17-2 and 18-2 during a switching period, the average output voltage and average current are derived by the following equations:

$$
\begin{aligned}
& {\left[\overline{\mathrm{V}}_{\mathrm{af}} \overline{\mathrm{~V}}_{\mathrm{bf}} \overline{\mathrm{~V}}_{\mathrm{cf}}\right]^{\mathrm{T}}=\left[\mathrm{d}_{\mathrm{af}} \mathrm{~d}_{\mathrm{bf}} \mathrm{~d}_{\mathrm{cf}}\right]^{\mathrm{T}} \cdot \mathrm{~V}_{\mathrm{pn}}} \\
& \mathrm{I}_{\mathrm{p}}=\left[\mathrm{d}_{\mathrm{af}} \mathrm{~d}_{\mathrm{bf}} \mathrm{~d}_{\mathrm{cf}}\right] \cdot\left[\mathrm{I}_{\mathrm{a}} \mathrm{I}_{\mathrm{b}} \mathrm{I}_{\mathrm{c}}\right]^{\mathrm{T}}
\end{aligned}
$$

In these equations, $d_{a b} d_{b f}$ and $d_{c f}$ are duty cycles of the lines to neutral point. As a result, the model of the inverter can be expressed as in Fig. 3.


Fig. 3: The model of a four-leg inverter for the average of large signal


Fig. 4: General structure of the proposed method

## MATERIALS AND METHODS

Structure of the proposed control method: A resonant controller is similar to an integrating controller, except that the frequency at which the gain is infinite is altered. A resonant controller with infinite gain at frequency $\omega$ is obtainable as:

$$
\mathrm{G}_{\mathrm{AC}}(\mathrm{~s})=\frac{\mathrm{Y}(\mathrm{~s})}{\mathrm{E}(\mathrm{~s})}=\frac{{ }^{2} \mathrm{~K}_{\mathrm{i}} \mathrm{~s}}{\mathrm{~s}^{2}+\omega^{2}}
$$

This is the ideal structure of a resonant controller and if added to a proportional controller, the ideal structure of a Proportional Resonant (PR) controller would result. This controller has an infinite gain at the frequency $\omega$ and does not include any phase shift or gain in other frequencies. The coefficient of the proportional controller is regulated as before and as ordinary PI controller. This coefficient mostly determines the dynamics of the system and affects factors such as bandwidth, phase margin and gain margin. To avoid stability problems due to the infinite gain at resonance frequency, we can use non-ideal resonance control structure with a transform function as below:

$$
\begin{equation*}
\mathrm{G}_{\mathrm{AC}}(\mathrm{~s})=\frac{\mathrm{Y}(\mathrm{~s})}{\mathrm{E}(\mathrm{~s})}=\frac{2 \mathrm{~K}_{\mathrm{i}}\left(\omega_{\mathrm{c}} \mathrm{~s}+\omega_{\mathrm{c}}^{2}\right)}{\mathrm{s}^{2}+2 \omega_{\mathrm{c}} \mathrm{~s}+\left(\omega_{\mathrm{c}}^{2}+\omega^{2}\right)} \approx \frac{2 \mathrm{~K}_{\mathrm{i}} \omega_{\mathrm{c}} \mathrm{~s}}{\mathrm{~s}^{2}+2 \omega_{\mathrm{c}} \mathrm{~s}+\omega^{2}} \tag{1}
\end{equation*}
$$

This structure has a specific gain in the resonance frequency but this again is still high enough to remove steady-state error in frequency. Another benefit of a non-ideal structure of resonant controllers is the possibility of setting bandwidth by $\omega_{c}$. This reduces the sensitivity to very little alterations of network frequency.

At the conditions of load imbalance, the components of $\mathrm{d}_{\mathrm{q}}$, in addition to a DC component, have also an oscillating one. At this stage d and q have oscillating components with frequency that is twice of the synchronous frequency. As a consequence, we can use controllers with a resonant frequency which are adjusted to $2 \omega$ for removing the oscillating component. We can also use resonant controllers with frequency $\omega$ to remove the oscillations of the zero component. The structure of the proposed method is shown in Fig. 4.

As shown in Fig. 4, the general structure of the control method is implemented in a reference system. The values of the voltages of $d_{q}$ are measured and compared with the reference values and are input through a
resonant controller. The outputs of the controller are the reference values of current, which are compared with the measured values and the result is given to PWM modulator after transition into abc coordinates.

## RESULTS AND DISCUSSION

To examine the proposed method precisely, simulations in MATLAB software and in the simulation environment is performed. The values of the reference voltages are as follows which are also true for imbalance conditions:

$$
\left[\begin{array}{c}
\mathrm{v}_{\mathrm{r}}^{\mathrm{a}} \\
\mathrm{v}_{\mathrm{r}}^{\mathrm{b}} \\
\mathrm{v}_{\mathrm{r}}^{\mathrm{c}} \\
\mathrm{v}_{\mathrm{r}}^{\mathrm{n}}
\end{array}\right]=\left[\begin{array}{c}
1.4 \sin (\omega \mathrm{t}) \\
1.9 \sin (\omega \mathrm{t}+2 \pi / 3) \\
0.8 \sin (\omega \mathrm{t}-2 \pi / 3) \\
1.2 \sin (3 \omega \mathrm{t}+\pi)
\end{array}\right]
$$

The base frequency is 50 Hz for this situation. The switching frequency is chosen as 10 KHz . Switching simulation system structure is shown in Fig. 5. In this case, we Choose series inverter ( CHB ) as multilevel inverter. Since the number of the levels for output voltage is five for each leg two half-bridge inverters in series are used.

Performance for balanced loads: For balanced loads as previously mentioned, the components of $d_{q}$ are zero. Hence, only the conventional PI controllers operate and the resonant controller will not participate in the circuit. The whole structure of the simulated system is shown in Fig. 6.

In these simulations, we assume the load to be linear and resistive. CHB inverter type is used because the output will be in the form of a five-level voltage. The number of full-bridge modules in each branch is two. An LC filter is used at the output of the inverter to remove high frequency components or harmonics caused by switching. The values of these elements are given in Table 1.

Table 1: The parameters of the simulated system

| Parameters | Units |
| :--- | :--- |
| Load voltage (L-L) | 380 V |
| Load voltage frequency | 50 Hz |
| Load resistance | $1 \Omega$ |
| Output filter capacitance | $0.1 \mu \mathrm{~F}$ |
| Output Filter inductor | 1 mH |
| Sampling frequency | 20 kHz |
| Switching frequency | 10 kHz |
| DC-Link voltage | 360 V |
| $\mathrm{k}_{\mathrm{p}}$ | 10 |
| $\mathrm{k}_{\mathrm{i}}$ | 100 |



Fig. 5: The structure of the simulated system


Fig. 6: The structure of the simulated system


Fig. 7: Voltages of phase $A C$ of the output


Fig. 8: Load current

The steady-state performance: In this case, the value of the load is fixed. Voltage and current waveforms of load are shown in Fig. 7 and 8. As shown in Fig. 7, the output phase voltages are sinusoid and in equilibrium. The percentage of THD of load voltage is $1.65 \%$ in this case which is reasonable. Here, the reference voltages are assumed to be zero for q and 0 components. Since, the load is in equilibrium, no current flows in the fourth wire, i.e. the neutral one and this current is equal to zero.

Transient state performance: To evaluate the performance of the transient state, a step alteration was
imposed to the value of the load and the response of the PI controllers due to these changes were studied as shown in Fig. 8-10. For this case, a reduction of $50 \%$ in the load was examined.

A very small increase in the load voltage was inspected which was damped to zero with a specific time constant.

This is for a condition where load has been reduced by $50 \%$ which is rare in common situations. This increase in voltage is due to the parallel capacitors that are used in the filter where some of their energy is returned to the load at the moment of transition.


Fig. 9: Load voltage for a reduction of $50 \%$ in load


Fig. 10: Output current waveform

Performance for the case of load imbalance: As previously discussed in the circumstances of load imbalance, the sum of the currents of the three phases is not equal to zero and as a result a current from the fourth wire flows to ground.In these conditions, it must be possible for the ground current or we can say the current of the neutral component to flow through.

Otherwise, the lack of zero current leads to large fluctuations in the load voltage amplitude. If the possibility for the neutral component of current to flow is provided, the amplitude of the fluctuations diminish significantly however, it will not be zero and we must use a suitable control method to eliminate them.

To evaluate the performance of the control method for the conditions of load imbalance, analogous simulations in MATLAB are performed. Primarily, the performance of the methods based on conventional PI controllers are shown in Fig. 11.

As shown in Fig. 11, conventional PI controllers cannot compensate for the fluctuations in voltage amplitude and load voltage imbalance. To evaluate the performance of the proposed resonant controller, the structure of these controllers is designed and simulated in the next section.

Resonant controllers: In the case of load imbalance, in dq 0 system, d and q components in addition to a fixed


Fig. 11: Voltage amplitude fluctuations in the conditions of load imbalance


Fig. 12: Alterations of frequency response of non-ideal resonant controllers versus $\omega_{c}$
value, hold an oscillating component with frequency $2 \omega$. Similarly, the zero component has an oscillating component with frequency $\omega$. Therefore we can conclude that the resonance frequency of the controllers should be set between these values.

As mentioned in section I, resonant controllers can be used for both ideal and non-ideal cases. To avoid instability problems, generally non-ideal structure with high gain is used. The controller transfer function is expressed by Eq. 2 :

$$
\begin{equation*}
\mathrm{G}_{A C}(\mathrm{~s})=\frac{\mathrm{Y}(\mathrm{~s})}{\mathrm{E}(\mathrm{~s})}=\frac{2 \mathrm{~K}_{\mathrm{i}}\left(\omega_{\mathrm{c}} \mathrm{~s}+\omega_{\mathrm{c}}^{2}\right)}{\mathrm{s}^{2}+2 \omega_{\mathrm{c}} \mathrm{~s}+\left(\omega_{\mathrm{c}}^{2}+\omega^{2}\right)} \approx \frac{2 \mathrm{~K}_{\mathrm{i}} \omega_{\mathrm{c}} \mathrm{~s}}{\mathrm{~s}^{2}+2 \omega_{\mathrm{c}} \mathrm{~s}+\omega^{2}} \tag{2}
\end{equation*}
$$

In this equation $\omega$ is the resonance frequency of the controller which was mentioned for each component in the previous section. $\omega_{\mathrm{c}}$ expresses the bandwidth of the controller. The more we desire to reduce the sensitivity of resonance frequency and the frequencies at which the controller has high gain, the more we require to increase bandwidth. $\mathrm{K}_{\mathrm{i}}$ adjusts the amount of gain at the resonance frequency. This controller along with a proportional controller with a gain of $K_{p}$, provides a non-ideal PR controller.

Figure 12 shows the alterations in the frequency response of these controller versus $\omega_{c}$. In these cases, the resonance frequency is equal to 50 Hz and the values of $\mathrm{K}_{\mathrm{i}}$ and $\mathrm{K}_{\mathrm{p}}$ are equal to 20 and 1, respectively.


Fig. 13: Alterations of frequency response of-ideal resonant controllers versus $k_{i}$


Fig. 14: Frequency response with resonant controllers: load voltages


We can infer from Fig. 12 that by increasing the value of $\omega_{c}$, bandwidth increases and hence the sensitivity to frequency variations decreases. Setting $\omega_{c}$ to 10 , the variations of the frequency response versus $k_{i}$ is shown in Fig. 13. By increasing the coefficient of the integrator, the gain value in resonance frequency has increased. However, the bandwidth of system has also increased, which in turn affects the dynamics and the speed control of the system.

Performance of the proposed controller: To evaluate the performance of the proposed controller, some simulations have been done. The structure of the
simulated system is in accordance with the values of Table 1. The parameters of the resonant controller are given in Table 2.

The unbalanced load for which the response of the conventional PI controllers was evaluated, mentioned in section III.d is used again. The responses of the resonant controllers are shown in Fig. 14 and 15.

As indicated, the resonant controller is able to maintain the stability of the amplitude of the voltage across the load. The fluctuations of the voltage amplitude are not canceled totally in this case, though greatly reduced. If we take into consideration the currents of the load and the fourth leg, it is clear that load is highly unbalanced and the current of fourth leg is very large. In the rest of the study, the performance of the proposed controller for transient state and also for the transition from a state of imbalance to a balanced load is presented.


Fig. 15: Frequency response with resonant controllers: load currents


Fig. 16: Frequency response with resonant controllers: fourth leg current


Fig. 17: Frequency response with resonant controllers: inverter output voltage, phase a


Fig. 18: Frequency response with resonant controllers: inverter output voltage, phase b


Fig. 19: Frequency response with resonant controllers: inverter output voltage, phase c


Fig. 20: Proposed controller transient response: load voltage


Fig. 21: Proposed controller transient response: load current


Fig. 22: Proposed controller transient response; a) d component; b) q component; c) zero component

Here, a transition from an unbalanced load to balanced conditions has occurred at 0.5 s . From Fig. 20-22, it can be seen that the controller has retained a proper performance for both states. The components q, d and zero have a desirable constant value.

As indicated, the proposed controller only uses a rotating reference system for transformation of $q, d$ and zero components. This helps towards further simplifying the proposed controller. This is in contrast with the method of analysis of components in which two rotating systems are required and therefore needs calculation of two angles. Besides, the analysis of components method needs six PI controllers for implementation which in the proposed controller, it was achieved by using three resonant controllers. Another benefit is the capability of using a synchronous reference or dq0 system in proposed method.

## CONCLUSION

There is a variety of methods for controlling the inverters connected to unbalanced loads. The common method is using a synchronous reference system and PI controllers which is used to control the inverters connected to balanced loads. However for situations that lack balanced loads, we need controllers with specific characteristics.

The traditional method is analysis of components that has disadvantages such as the large number of reference conversions between systems which inherently increases the computations. Also, the large number of PI controllers needs additional calculations for analysis of components. In this study, a new method was proposed based on resonant controllers. The proposed method has the following advantages:

- Reduction in the number of PI controllers
- Low computational requirements
- Implementation in one reference system


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