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On Special Case of Smoothness in the Viviani's Curve

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Abstract: In mathematics, a curve is generally speaking, an object similar to a line but which is not required to be straight. This entails that a line is a special case of curve, namely a curve with null curvature. Often curves in two-dimensional (plane curves) or threedimensional (space curves). Euclidean space are of interest. Intuitively, the smooth curve is collection of points that we can without lifting the pen from the study to draw it. Viviani's curve is the space curve giving the intersection of the sphere and the cylinder. In this study, we want to show that the Viviani's curve with special parameter is a smooth curve.

Key words: Viviani's curve, smooth curve, regular curve, parameters, Iran

INTRODUCTION

Viviani's curve, sometimes also called Viviani's window, firstly was studied by Viviani in 1692. A disciple of Galileo Galilei, proposed a problem of the construction of four equal windows cut out of a hemispherical cupola such that the remaining surface area can be exactly squared (Caddeo *et al*., 2001). The solution to this problem is given by an intersection of the hemisphere and a cylinder whose diameter equals the radius of the hemisphere. This intersection curve of a sphere and a cylinder tangent to the diameter of the sphere and its equator is the famous Viviani curve which can be generalized to an intersection with an arbitrary cylinder. For a recent extended study of such curves see (Raghavan *et al*., 1999; Kroll, 2005; Graefe *et al*., 2014) analyzed the relevance of the Viviani curve for the quantum Bose-Hubbard dimer. Babaeian *et al*. (2015a, b) and Smerzi *et al*. (1997) showed and proved that using a curvature constraint, we can capture the geodesic distances in manifold in higher dimensions. Based on Viviani's cirve and Mehrian proposed a novel model that can be employed in coupled thermoelasticity and fracture mechanics (Nowruzpour Mehrian *et al*., 2013a, b, 2014; Vaziri *et al*., 2015; Mehrian *et al*., 2016).

MATERIALS AND METHODS

Preliminaries: Viviani's curve is creates from intersection of the sphere with center (0,0,0) and radius (a) and the cylinder with equation. We want write a parametric equation $x^2+y^2 = ax$ of Viviani's curve. We write the equation of cylinde as follow:

$$\left(x-\frac{\alpha}{2}\right)^2+y^2=\frac{\alpha^2}{4} \quad (1)$$

$$x-\frac{\alpha}{2}=\frac{\alpha}{2}\cos\theta \quad (2)$$

$$y=\frac{\alpha}{2}\sin\theta \quad (3)$$

$$z=\pm\sqrt{\alpha^2-\left(x^2+y^2\right)}=\pm\sqrt{\alpha^2-\left(\frac{a\alpha}{4}(\cos\theta+1)^2+\frac{\alpha^2}{4}\sin^2\theta\right)}=\pm a\sin\frac{\theta}{2} \quad (4)$$

Therefore, for:

$$0\le\theta\le 2\pi,\quad \alpha(\theta)=\left(\frac{\alpha}{2}\right)(\cos\theta+1),\frac{\alpha}{2}\sin\theta,\pm\alpha\sin\frac{\theta}{2} \quad (5)$$

Is the parametric representation of Viviani's curve.

Remark: If $\alpha(t) = (x(t), y(t), z(t))$ is a curve with parameter t, then this curve is regular whenever $\alpha'(t)\neq 0$. Now, we show that Viviani's curve is regular. It is enough to show. So, we have:

$$\alpha'(\theta)=\left(-\frac{\alpha}{2}\sin\theta,\frac{\alpha}{2}\cos\theta,\pm-\frac{\alpha}{2}\cos\frac{\theta}{2}\right)\neq 0 \quad (6)$$

Therfore Viviani's curve with parametric equation $\alpha(\theta)$ is regular. Now we express the following lemmas.

Lemma 1: A regular curve with curvature $k\neq 0$ is a smooth curve if only if torsion of curve is zero ($\tau = 0$) (Babaeian *et al*., 2015).

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Lemma 2: If $\alpha(t)$ is a regular curve with parameter t, then curvature (k) and torsion (τ) of $\alpha(t)$ is as following (Babaeian *et al*., 2015):

$$k = \frac{|\alpha'(t) \wedge \alpha''(t)|}{|\alpha''(t)|^3} \quad (7)$$

$$\tau = \frac{|(\alpha'(t) \wedge \alpha''(t))\alpha'''(t)|}{|\alpha'(t) \wedge \alpha''(t)|^2} \quad (8)$$

RESULTS AND DISCUSSION

By use of lemma 2, we calculate curvature and torsion of curve $\alpha(\theta)$. According to Babaeian *et al.* (2015) if:

$$\alpha(\theta) = \left(\frac{\alpha}{2}(\cos\theta + 1), \frac{\alpha}{2}\sin\theta, \alpha\sin\frac{\theta}{2}\right)$$

Then:

$$\alpha'(\theta) = \left(-\frac{\alpha}{2}\sin\theta, \frac{\alpha}{2}\cos\theta, \frac{\alpha}{2}\cos\frac{\theta}{2}\right) \quad (9)$$

$$\alpha''(\theta) = \left(-\frac{\alpha}{2}\cos\theta, -\frac{\alpha}{2}\sin\theta, -\frac{\alpha}{4}\sin\frac{\theta}{2}\right) \quad (10)$$

$$\alpha'''(\theta) = \left(\frac{\alpha}{2}\sin\theta, -\frac{\alpha}{2}\cos\theta, -\frac{\alpha}{8}\cos\frac{\theta}{2}\right) \quad (11)$$

$$\alpha'(\theta) \wedge \alpha''(\theta) = \left(-\frac{\alpha^2}{8}\sin\theta\frac{\theta}{2}\cos\theta, -\frac{\alpha^2}{4}\cos^2\theta\right) \quad (12)$$

Thus:

$$k = \frac{\left|\left(-\frac{\alpha^2}{8}\sin\frac{\theta}{2}\cos\theta, 0, -\frac{\alpha^2}{4}\cos^2\theta\right)\right|}{\left|\left(-\frac{\alpha^2}{2}\sin\theta, -\frac{\alpha}{2}\cos\theta, -\frac{\alpha}{8}\cos\frac{\theta}{2}\right)\right|^3} \neq 0 \quad (13)$$

Now by lemma 2, we calculate torsion of viviani curve with equation $\alpha(\theta)$. We have:

$$(\alpha'(\theta) \wedge \alpha''(\theta))\alpha'''(\theta) = -\frac{\alpha^2}{16}\sin\theta\sin\frac{\theta}{2}\cos\theta - \frac{\alpha^3}{32}\cos\frac{\theta}{2}\cos^2\theta \quad (14)$$

$$|\alpha'(\theta) \wedge \alpha''(\theta)|^2 = \frac{\alpha^4}{64}\sin^2\frac{\theta}{2}\cos^2\theta + \frac{\alpha^4}{16}\cos^4\theta \neq 0 \quad (15)$$

We can calculate Eq. 14 as following:

$$= \frac{\alpha^3}{32}\cos^3\frac{\theta}{2} - \frac{\alpha^3}{32}\cos\frac{\theta}{2} - \frac{\alpha^3}{64}\cos^3\frac{\theta}{2} + \frac{\alpha^3}{64}\cos\frac{\theta}{2} = \alpha^3\cos^3\frac{\theta}{2} - \alpha^3\cos\frac{\theta}{2} \quad (16)$$

And so:

$$\tau = \frac{\alpha^3\cos^3\frac{\theta}{2} - \alpha^3\cos\frac{\theta}{2}}{\frac{\alpha^4}{64}\sin^2\frac{\theta}{2}\cos^2\theta + \frac{\alpha^4}{16}\cos^4\theta} \quad (17)$$

If in the above relation, $\theta = 0, \pi, 2\pi$ then relation (Eq. 16) = 0 and according to Eq. 15 and 16, we conclud that $\tau = 0$. As such in Eq. 5 if $\alpha(t) = (\alpha/2(\cos\theta+1), \alpha/2\sin\theta, -\alpha\sin\theta/2)$ then for $\theta = 0, \pi, 2\pi$, we conclude that $\tau = 0$. Since, it proof that the Viviani's curve is a regular curve and as such according to lemma 1, therefore it follows that the Viviani's curve with this parametric equation is smooth curve.

CONCLUSION

We know that study propeties of curves is interesting. Therefore, in this study, we confined our attention tothis spacial case. We have studied smoothness of the Viviani's curve with properties of torsion and curvature. It seen that we can review other curve. As such we use other parametric equations for Different curves. We proof being regular and calculate curvature and torsion of curves and finally with use of torsion and curvature of curve, we verify smoothness of curves.

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