

## MIMO STBC Multi Node Selective C(0) Protocol Based Cooperative Wireless Communication over Nakagami-m Fading Channel Considering the Effect of Channel Estimation Error

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**Abstract:** This study provides an insight into the selective Decode-Forward (DF) relaying based Multiple-Input-Multiple-Output (MIMO)-Space-Time Block-Coded (STBC) cooperative wireless communication system with single and multiple relays over Nakagami-m fading channels, considering the effect of channel estimation error. Firstly, we examine the Symbol Error Rate (SER) performance at equal power allocation. Secondly, we have analyzed the SER performance at the optimal power allocation case and compare the Symbol Error Rate (SER) performance at equal power allocation and optimal power allocation case. Lastly, we have analyzed the symbol error rate performance with increasing number of relays.

**Key words:** C(0), Nakagami-m fading channel, relay selection, SER, STBC, MIMO, Decode and Forward (DF) protocol

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### INTRODUCTION

Multipath fading degrades the performance of the wireless communication system (Tse and Viswanath, 2005). Installation of multiple antenna on small communication terminal is a very difficult task. Cooperative communication has resolved this problem to a great extent as it mitigates the effect of multipath fading by creating the virtual MIMO system. Various cooperation (Laneman *et al.*, 2004) protocols have been introduced such as Decode-Forward (DF) protocol, Amplify-Forward protocol (AF).

In selective decode-forward protocol, source broadcasts information to all the relays and destination in the 1st phase. In the 2nd phase the relay sends information to the destination only if it decodes the symbol correctly and Signal to Noise Ratio (SNR) threshold value is defined in this case. If the SNR value of the received signal at the relay is greater than the SNR threshold value, then only it will forward the signal to destination otherwise it will remain idle. In Amplify-Forward (AF) relaying, the source sends information to relay, amplification of the received signal takes place at the relay and then it forwards the amplified signal to the destination.

To improve the Symbol Error Rate (SER) performance, we have combined MIMO and Space Time Block Code (STBC) with selective Decode and Forward (DF) cooperation scheme. At the receiver Maximal Ratio Combining (MRC) is used to detect the signal. As in the case of Alamouti space-time block coded system,

Channel State Information (CSI) is not required. Selective decode-forward multiple input multiple output cooperation system with space-time block code based transmission also does not require the knowledge of channel state information. Full diversity has been achieved and drawbacks of beam forming based multiple input multiple output cooperative system have been overcome as well.

In most of the analysis fading channel is the rayleigh type without considering the channel estimation error condition. Liu (2009) a class of cooperative communication C(m),  $1 \leq m \leq N$  where N is the number of relay has been proposed. In this protocol each relay combines the signals from all the previous m relays ( $1 \leq m \leq N$ ) as well as from sources. Exact symbol error rate expression has been derived for both the MPSK and MQAM signal and relay selection algorithm has been explained over Rayleigh fading channel.

A two phase (Varshney *et al.*, 2015; Wang *et al.*, 2016) MIMO-STBC-DF cooperative system protocol C(0) has been analyzed over the nakagami-m fading channel. The closed form expression has been derived for exact symbol error rate and upper bound of symbol error rate. Also, optimal power allocation is done using the convex optimization method.

### MATERIALS AND METHODS

**Multiple relay selective DF C(0) protocol without relay selection:** In the case of C(0) protocol, relay receives

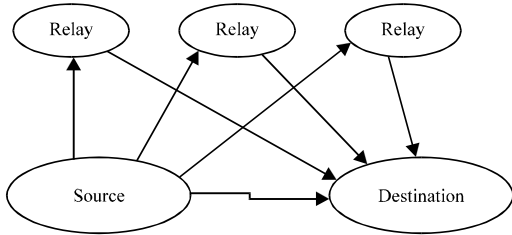


Fig. 1: C(0) protocol cooperation model

signals directly from the source. In the case of C(0) protocol without relay selection, the source node transmits signal to destination as well as all relay nodes. Relays which decode the signal correctly will be able to transmit signals to the destination otherwise it will remain idle. So, we can say that C(0) based protocol is basically a 2-phase system model. In the 1st phase, the received signal at the source and the R relays is given Fig. 1:

$$Y_{SD} = \sqrt{P_s} h_{SD} X + \eta_{SD} \quad (1)$$

$$Y_{SR_i} = \sqrt{P_s} h_{SR_i} X + \eta_{SR_i}, i = 1, \dots, R \quad (2)$$

where,  $h_{XY}$  is a nakagami-m distributed random variable for any  $XY \in \{SD, SR_1, SR_2, \dots, R_1D, R_2D, \dots, R_RD\}$  and  $\eta_{XY} \sim C(0, N_0)$  is a zero Mean Complex AWGN. In 2nd Phase the signal received becomes:

$$Y_{R_iD} = \sqrt{\tilde{P}_{R_i}} h_{R_iD} X + \eta_{R_iD}, i = 1, \dots, R$$

where,  $\tilde{P}_{R_i} = P_{R_i}$ , that the  $i$ th relay decodes correctly, otherwise  $\tilde{P}_{R_i} = 0$ . Using the concept of maximal ratio combining, destination signal can be written as:

$$Y_D = \sqrt{P_0} h_{SD} Y_{SD} + \sum_{i=1}^R \sqrt{\tilde{P}_{R_i}} h_{R_iD} Y_{R_iD} \quad (3)$$

The SNR at the destination is given as:

$$SNR_D = \frac{P_0 |h_{SD}|^2 + \sum_{i=1}^R \tilde{P}_{R_i} |h_{R_iD}|^2}{N_0}$$

Probability of error is given in Eq. 4:

$$\psi(SNR_{XY}) = \frac{1}{\pi} \int_{\theta=0}^{(M-1)\pi} \exp\left(-\frac{bSNR_{XY}^{(K)}}{\sin^2 \theta}\right) d\theta \quad (4)$$

The instantaneous of probability of symbol error for PSK system.  $\xi_z^J$  is the set of correctly decoding relays and

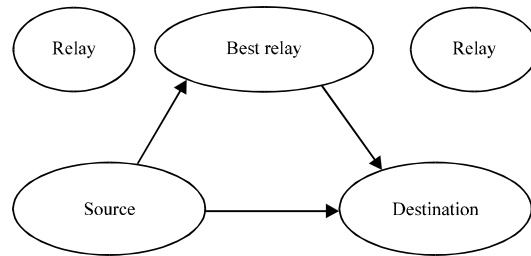


Fig. 2: Multiple relay selective C(0) protocol with relay selection

$\xi_z^J$  denotes its complement. For simplicity, we have taken  $m_{SD} = m_{SR} = m_{RD} = m$ . Symbol error rate expression is given in Eq. 4. The final expression for SER is given in Eq. 5. For optimum power allocation, the optimization problem can be equation as:

$$\text{minimize} \rightarrow P_R(E) \quad (5)$$

$$\text{s.t.} \left( P_s + \sum_{M=1}^R P_{R_M} \right) = P \quad (6)$$

The optimization problem is Geometric program and it is solved efficiently by using CVX (Boyd and Vandenberghe, 2004) convex optimization problem solving diversity order is given as:

$$D.O = - \lim_{SNR \rightarrow \infty} \left\{ \frac{-2m(R+1)\log(2SNR) + \log \sum_{J=0}^R \sum_{Z=1}^{C_J} F(J,Z)}{\log(SNR)} \right\} = R+1 \quad (7)$$

**Multiple relay selective df C(0) protocol with relay selection:**

In the case of multiple relay DF C(0) protocol with relay selection, there are 2 phases. In the 1st phase source calculate the metric  $\beta_m$  here  $\beta_m$  is the best possible source relay destination link. Let  $\alpha$  be the cooperation threshold for the event  $\phi: (\beta_{SD} > \alpha \beta_m)$  and  $\beta_{SD} = |h_{SD}|^2$  then the source will use only the direct link from source to destination and transmit the full available power  $P_s + P_R = P$  system equation is given as Fig. 2:

$$Y_{SD}^\phi = \sqrt{P_s} h_{SD} X + \sqrt{(\delta P_s + N_0)} \eta_{SD} \quad (8)$$

If,  $\phi^c: (\beta_{SD} \leq \alpha \beta_m)$  then the relay cooperation will be taken into effect and transmission is done using the best source relay-destination link. The system equation is given:

$$Y^{\phi^c}_{SD} = \sqrt{P_s}h_{SD}X + \sqrt{(\delta P_s + N_0)}\eta_{SD} \quad (9)$$

$$Y^{\phi^c}_{SR} = \sqrt{P_s}h_{SR}X + \sqrt{(\delta P_s + N_0)}\eta_{SR} \quad (10)$$

In the next phase given that the relay cooperative system mode is selected in the 1st phase, the relay retransmits decoded symbol in the 2nd phase if decoded correctly. The system equation is given:

$$Y^{\phi^c}_{RD} = \sqrt{P_r}h_{RD}X + \sqrt{(\delta P_r + N_0)}\eta_{RD} \quad (11)$$

where,  $\tilde{P}_r = P_r$  when decoding is done correctly, otherwise the relay will remain idle and  $\tilde{P}_r = 0$ ,  $h_{XY}$  is nakagami-m distributed random variable and  $\eta_{XY} \sim C(0, N_0)$  is ZMCS (Zero Mean Circular Shift) Complex AWGN.  $XY = \{SD, RD, SR\}$  denote the Nakagami-m faded channel for various links:

$$\beta_m = \max(\beta_{SR_1D}, \beta_{SR_2D}, \dots, \beta_{SR_R D})$$

$$\beta_{SR_iD} = \frac{2q_1q_2\beta_{SR_i}\beta_{R_iD}}{q_1\beta_{R_iD} + q_2\beta_{SR_i}}, \quad i = 1, 2, 3, \dots, R \quad (12)$$

The error at the destination is given as:

$$P_R(E) = P_R\left(\frac{E}{\phi}\right)P_R(\phi) + P_R\left(\frac{E}{\phi^c}\right)P_R(\phi^c) \quad (13)$$

- $P_R(E/\phi)$  = Direct mode probability of error :  $\Psi(SNR_{SD})$
- $P_R(\phi)$  = Error probability given direct mode is chosen:  $P_R(\beta_{SD} \geq \alpha\beta_m)$
- $P_R(E/\phi^c)$  = Probability of error given cooperation mode is being selected:  $\Psi(SNR_{SD} + SNR_{RD})$
- $P_R(\phi^c)$  = Error probability given cooperation mode is chosen:  $P_R(\beta_{SD} \leq \alpha\beta_m)$

Assuming  $m_{SD} = m_{RD} = m_{RD} = m$ . SER expression is given in Eq. 14. SER can be summarized as:

$$P_R(E) \leq (1/SNR)^{2m(R+1)} F(r) \quad (14)$$

Diversity order:

$$D.O = - \lim_{SNR \rightarrow \infty} \left\{ \frac{-2m(R+1)\log(2SNR) + \log F(r)}{\log(SNR)} \right\} = R + 1 \quad (15)$$

The optimum power allocation of the multi-node relay-selective DF cooperative scenario with R relays is:

$$\Gamma_{opt} = \frac{1 - \frac{RX_1}{2(R+1)X_2} + \sqrt{1 + \frac{(R+2)X_1}{(R+1)X_2} + \left(\frac{RX_1}{2(R+1)}\right)^2}}{2 - \frac{RX_1}{2(R+1)X_2} + \sqrt{1 + \frac{(R+2)X_1}{(R+1)X_2} + \left(\frac{RX_1}{2(R+1)}\right)^2}} \quad (16)$$

$$X_1 = (A^{2R+2}I(2R+2) + A^3B^RI(2R))\Omega_{SR} \quad (17)$$

$$X_2 = (A^{2R}BI(2R+2) + AB^RI(2R)) \quad (18)$$

$$I(P) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^p \theta d\theta \quad (19)$$

$$A = I(2) = \frac{M-1}{2M} + \frac{\sin\left(\frac{2\pi}{M}\right)}{4\pi}, \quad (20)$$

$$B = I(4) = \frac{3(M-1)}{8M} + \frac{\sin\left(\frac{2\pi}{M}\right)}{4\pi} - \frac{\sin\left(\frac{4\pi}{M}\right)}{32\pi}$$

From Eq. 18, the optimal power allocation ratio does not depend on the parameter m and the channel variance (source-relay and relay-destination). The Optimum cooperation threshold is given as:

$$\max_{\alpha} CG \times W \quad (21)$$

$$\Pr(e) \leq \left( CG \times \frac{P}{N_0} \right)^{-(R+1)} \quad (22)$$

Where:

CG = The cooperative gain

W = The bandwidth efficiency and is defined as

$$W \approx \frac{1}{2} \left[ 1 + \sum_{i=0}^R \binom{R}{i} (-1)^i \frac{2\alpha}{2\alpha + \left( \frac{1-r}{B\Omega_{SR}} + \frac{r}{A^2\Omega_{RD}} \right) r\Omega_{SD}^i} \right] \quad (23)$$

## RESULTS AND DISCUSSION

Figure 3-6 we have taken the optimal power allocation scenario in C(0) without relay selection. Following cases arise. Case 1:

$$\Omega_{SR} = \Omega_{SD} = \Omega_{RD} = 1$$

$$P_s(\text{source}) = 0.4832, P_{R1}(\text{relay1}) = 0.2584,$$

$$P_{R2}(\text{relay2}) = 0.2584$$

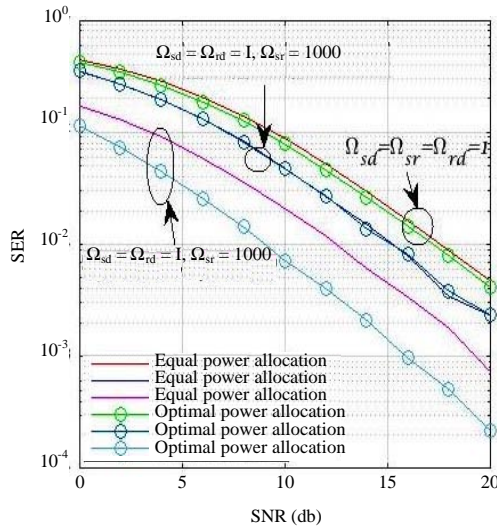


Fig. 3: SNR dB vs. SER performance at optimal power allocation over the uncorrelated Nakagami-m fading channel of selective C(0) protocol without relay selection for  $m = 0.50$

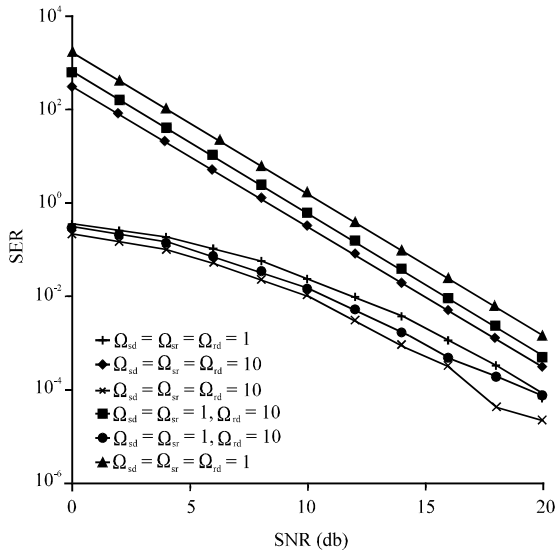


Fig. 4: SNR dB vs. SER performance over uncorrelated Nakagami-m fading channel of selective C(0) protocol without relay selection for various channel variances for 4PSK symbols and  $m = 0.50$

The above power allocation produces improvement of the SER performance over equal power allocation. As source relay links are of the same strength as a result equal power is allocated to both the relays. Power allocation depends upon the actual ratio of channel strengths rather than individual channel strengths. Case 2:

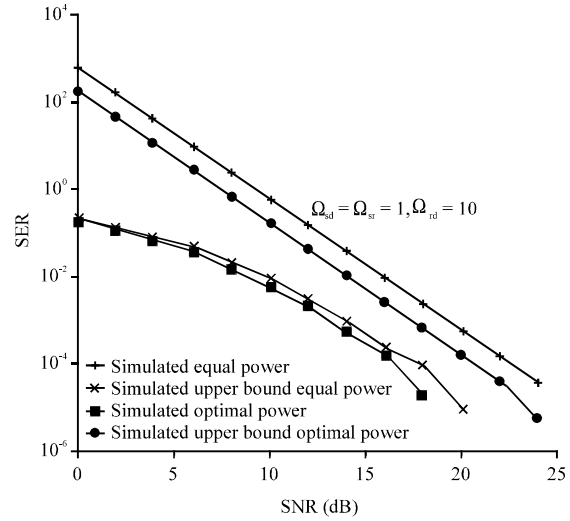


Fig. 5: SNR dB vs. SER performance at optimal allocation for C(0) with relay selection

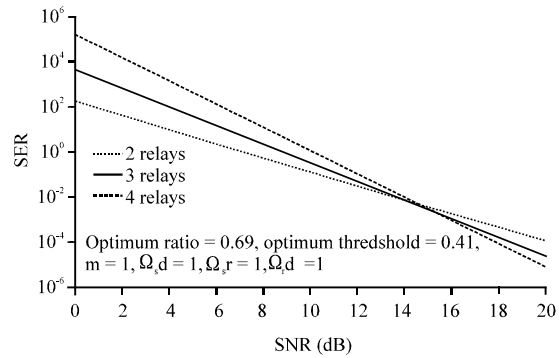


Fig. 6: SNR dB vs. SER performance with optimum ratio and threshold for increasing number of relays for relay selection C(0) protocol

$$\begin{aligned} \Omega_{SR} &= 1, \Omega_{SD} = 1, \Omega_{RD} = 1000 \\ P_S(\text{source}) &= 0.8712, P_{R1}(\text{relay1}) = 0.0644, \\ P_{R2}(\text{relay2}) &= 0.0644 \end{aligned}$$

The above power allocation produces significant improvement of the SER performance over equal power allocation. As relay destination link is very strong as compared to source to relay link, the relay needs little power to forward the symbol to the destination as a result source has maximum power. The performance difference between optimal and equal power allocation is maximum in this case (Table 1). Case 3:

$$\begin{aligned} \Omega_{SR} &= 1000, \Omega_{SD} = 1, \Omega_{RD} = 1 \\ P_S(\text{source}) &= 0.3368, P_{R1}(\text{relay1}) = 0.3316, \\ P_{R2}(\text{relay2}) &= 0.3316 \end{aligned}$$

Table 1: Optimum power ratio and cooperation threshold values for unit channel variances

| No. of relays | Power ratio | Cooperation threshold |
|---------------|-------------|-----------------------|
| 2             | 0.6900      | 0.41                  |
| 3             | 0.6787      | 0.35                  |
| 4             | 0.6724      | 0.32                  |

Optimal power allocation will tend to equal power allocation in this scenario. As the source relay link is very strong, the probability of the symbol being correctly decoded at the relay becomes high even with low powers. Thus, resulting in more power to the relay. Figure 4, we have shown SER performance of C(0) protocol for various channel variances. The SER result shows that higher the value of  $\Omega_{RD}$  is more important than higher  $\Omega_{SR}$  value. Figure 5 shows the SER performance at optimal allocation for C(0) protocol with a relay selection for  $m = 1, \delta = 0.001$  and  $\Omega_{SR} = \Omega_{SD} = \Omega_{RD} = 10$ . From the results we can say that the optimal power out performs the equal power allocation case. Figure 6 analyzes the optimum ratio and optimum threshold for increasing number of relays.

**CONCLUSION**

Following conclusion arises in the case of optimal power allocation SER performance improves as compared to the equal power allocation. As we increase the value of the shape parameter, m performance improves. With increasing the value of the  $\delta$ , SER performance decreases. In case of C(0) protocol when  $\Omega_{SR} = \Omega_{SD} = \Omega_{RD} = 1000$ , we get the highest performance difference between equal power allocation and optimal power allocation. Also in case of C(0) relay selection protocol optimum threshold and optimal ratio produces improvement in SER performance. Also, as we increase the number of relays the symbol error rate improves:

$$P_R(E) = \sum_{j=0}^R \sum_{Z=1}^{C_j} \left[ \psi \left( \text{SNR}_{SD} + \sum_{M \in \xi_Z^j} \text{SNR}_{R_{MD}} \right) \times \left\{ \prod_{M \in \xi_Z^j} (1 - \psi(\text{SNR}_{SR_M})) \right\} \left\{ \prod_{T \in \xi_Z^j} \psi(\text{SNR}_{SR_T}) \right\} \right] \tag{24}$$

$$P_R(E) \leq \sum_{j=0}^R \sum_{Z=1}^{C_j} \left[ \frac{\left( \frac{2N_0}{b} \right)^{2m(R+1)} \frac{(I_0)^{2m(R-j)} (I_j)^{2m}}{(P_S m^{-1})^{2m(R-j+1)} (\Omega_{SD})^{2m}} \times 1}{\left( \prod_{M \in \xi_Z^j} (P_{R_M} m^{-1}) \Omega_{R_{MD}} \right)^{2m} \times \left( \prod_{T \in \xi_Z^j} \Omega_{SR_T} \right)^{2m}} \right] \tag{25}$$

$$I_j = \frac{1}{\pi} \int_0^{\pi} (\sin^2 \theta)^{j+1} d\theta \tag{26}$$

$$P_R(E) \leq \frac{(R!)^{2m} \left( \frac{2(N_0 + \delta P_S)}{bP} \right)^{2m(R+1)} I(2R+2)^{(2m)} \left( \frac{t_2}{2\alpha} \right)^{2mR}}{(\Omega_{SD} m)^{2m}} + \frac{(R!)^{2m} \left( \frac{2(N_0 + \delta P_S)}{b} \right)^{2m(R+1)} (t_2)^{2m(R-1)} \left( \frac{q_1 \Omega_{RD} + q_2 \Omega_{SR}}{P_S \Omega_{SD}} \right)^{2m}}{(\Omega_{SD} m)^{2m}} \times \left( \left( \frac{q_1}{P_R} \right)^R I(2R+2) + \left( \frac{q_2}{P_S} \right)^R A I(2R) \right)^{2m} \tag{27}$$

$$q_1 = \frac{A^2}{r^2}, q_2 = \frac{\beta}{r(1-r)}, r = P_S/P, b = \sin^2(\pi/M), \tag{28}$$

$$A = I(2), B = I(4), I(j) = (1/\pi) \int_0^{\frac{(M-1)\pi}{M}} \sin^j \theta d\theta$$

**REFERENCES**

Boyd, S. and L. Vandenberghe, 2004. Convex Optimization. Cambridge University Press, Cambridge, England, UK.,

Laneman, J.N., D.N.C. Tse and G.W. Wornell, 2004. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. IEEE Trans. Inform. Theory, 50: 3062-3080.

Liu, K.R., 2009. Cooperative Communications and Networking. Cambridge University Press, Cambridge, England, UK.,

Tse, D. and P. Viswanath, 2005. Fundamentals of Wireless Communication. 1st Edn., Cambridge University Press, Cambridge, UK., ISBN-13: 9780521845274.

Varshney, N., A.V. Krishna and A.K. Jagannatham, 2015. Selective DF protocol for MIMO STBC based single/multiple relay cooperative communication: End-to-end performance and optimal power allocation. IEEE. Trans. Commun., 63: 2458-2474.

Wang, Y., Y. Xu, N. Li, W. Xie and K. Xu *et al.*, 2016. Relay selection of full-duplex decode-and-forward relaying over Nakagami-m fading channels. IET. Commun., 10: 170-179.