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Estimating the Number of Paediatric Patients Using Least Square and Conjugate Gradient Methods

[1]Nur Syarafina Mohamed, [1]Mustafa Mamat, [1]Nur Hamizah Abdul Ghani, [1]Norhaslinda Zull, [1]Nurul Aini, [2]Mohd Rivaie and [2]Nur Syairah Shahrul Nizam
[1]Fakulti Informatik dan Komputeran, Universiti Sultan Zainal Abidin, 21300 Terengganu, Malaysia
[2]Fakulti Sains Komputer dan Matematik, Universiti Teknologi Mara, Kampus Chendering, Terengganu, Malaysia

Abstract: Paediatric is one of the branches in medicine dealing with health and medical care for infants, children and adolescents. Recent findings show that the number of paediatric patients is increasing year by year. In this study, a data of paediatric patients from 2005 until 2015 is collected from Specialist Clinic HSNZ, Kuala Terengganu. By using discrete least square method of numerical analysis and conjugate gradient method in unconstrained optimization, an estimation of 2016 data can be forecasted. These methods have been chosen based on its simplicity and accuracy. The calculations are based on linear and quadratic approach together with their errors. Results showed that the conjugate gradient method is comparable with the quadratic least square method.

Key words: Least square method, conjugate gradient method, medicine dealing, medical care, Kuala Terengganu

INTRODUCTION

Regression analysis is a well known approach used in economics, finance, trade, meteorology and medicine (Moyi *et al*., 2014). The classical regression model is defined by:

$$y = h\left(x_1, x_2, ..., x_p + \varepsilon\right) \quad (1)$$

Where:
- y = The response variable
- x_i = The predictor variable with $i = 1, 2, ..., p$, $p>0$ is an integer constant
- ε = The error term

The function $h(x_1, x_2, ..., x_p)$ describes the type of relationship between y and $x = (x_1, x_2, ..., x_p)$. Thus, the following linear regression model:

$$y = a_0 + a_1x_1 + a_2x_2 + ... + a_px_p + \varepsilon \quad (2)$$

is the simplest regression model where $a_0, a_1, ..., a_p$ are the regression parameters. In regression analysis, the most important task is to estimate the parameters $a = (a_0, a_1, ..., a_p)$. For that, the least square method is often used which is defined as:

$$\min E(a) = \sum_{i=1}^{n}\left(h_i - a_0 + h_1x_{i1} + h_2x_{i2} + ... + h_px_{ip}\right)^2 \quad (3)$$

Where:
- h_i = The data estimation of the ith response variable
- $x_{i1}, x_{i2}, ..., x_{ip}$ = p data evaluation of the response variable

Given m is the number of data. Then, if the dimension of p and m is small, the parameters $a = (a_0, a_1, ..., a_p)$ can be acquired from a multivariate value of calculus (Yuan and Wei, 2009). From Syarafina consider an unconstrained optimization problem:

$$\min_{x \in R^n} f(x) \quad (4)$$

where, $f: R^n \rightarrow R$ is a continuously differentiable function which is bounded from below. Starting from an initial guess at point x_0 an iterative Eq. 5 is generated as:

$$x_{k+1} = x_k + \alpha_k d_k, \ k = 0, 1, 2, ... \quad (5)$$

Where:
- x_k = The current iterate point
- $a_k>0$ = A step size which is obtained by one dimensional search

In this study, the exact line search is used:

$$f(x_k + \alpha_k d_k) = \min_{\alpha \geq 0} f(x_k + \alpha d_k) \quad (6)$$

Corresponding Author: Nur Syarafina Mohamed, Fakulti Informatik dan Komputeran, Universiti Sultan Zainal Abidin, 21300 Terengganu, Malaysia

The search direction d_k is defined by:

$$d_k = \begin{cases} -g_k & \text{if} \quad k = 0 \\ -g_k + \beta_k d_{k-1} & \text{if} \quad k \geq 1 \end{cases} \quad (7)$$

where, g_k is the gradient of f(x) at the point x_k. From Fletcher and Reeves (1964), Polak and Ribiere (1969) and Rivaie *et al.* (2012), the β_k used are:

$$\beta_k^{PR} = \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2} \quad (8)$$

$$\beta_k^{FR} = \frac{g_k^T g_k}{\|g_{k-1}\|^2} \quad (9)$$

$$\beta_k^{RMIL} = \frac{g_k^T(g_k - g_{k-1})}{\|d_{k-1}\|^2} \quad (10)$$

g_k and g_{k-1} denote the gradients of f(x) at the point x_k and x_{k-1}, respectively. Detailed explanation can be found from Hamoda *et al.* (2015), Zoutendijk (1970), Mohameda *et al.* (2016) and Abidin *et al.* (2014).

Derivation process: The least square method involves determining the best approximating line by comparing the total least square error (Narula and Wellington, 1977). Consider a set of data (x_i, y_i) where x is said to be exact values if only y values have errors. The error is defined as:

$$E_i = (a_0 + a_1 x_i) - y_i \quad (11)$$

The strategy for fitting the "best" line through the data would be to minimize the sum of the residual error squares for all the available data:

$$\min \sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} ((a_0 + a_1 x) - y_i)^2 \quad (12)$$

Differentiate Eq. 12 with respect to a_0 and a_1 and solve it simultaneously, then, the general Eq. 13 to find the discrete least square models can be described as:

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \dots & \sum_{i=1}^{n} x_i^m \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \dots & \sum_{i=1}^{n} x_i^{m+1} \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \dots & \sum_{i=1}^{n} x_i^{m+2} \\ \sum_{i=1}^{n} x_i^m & \sum_{i=1}^{n} x_i^{m+1} & \sum_{i=1}^{n} x_i^{m+2} \dots & \sum_{i=1}^{n} x_i^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \\ \sum_{i=1}^{n} x_i^m y_i \end{bmatrix} \quad (13)$$

Algorithm 1 (Least square method):

Step 1: Identifying formula from Eq. 13 both linear and quadratic
Step 2: Identifying variables and data summation based on Eq. 13
Step 3: Calculation of a_0 and a_1
Step 4: Generating equations
Step 5: Calculate error by using| (Exact value-Approximate value)/exact value|and Eq. 12
Step 6: Model estimation

According to Buonaccorsi and Elkinton (2002) one can observe that there exists a linear relationship between the year and the number of paediatric patients, the regression equation is given by $y = a_0 + a_{1x}$ with a_0 and a_1 denoting the regression parameters. Thus:

$$\min_{x \in R^3} f(a) = \sum_{i=1}^{n} [y_i - a(1, x_i, x_i^2)^T]^2 \quad (14)$$

Algorithm 2 (Conjugate gradient method):

Step 1: Initialization. Set k = 0 and select x_0. Identifying formula from Eq. 12 for both linear and quadratic
Step 2: Compute β from Eq. 8-10
Step 3: Compute search directions d_k based on Eq. 7. If $\|gk\| = 0$, then stop
Step 4: Solve α_k using the exact line search, α_k based on Eq. 6
Step 5: Updating new initial point using Eq. 5
Step 6: Convergence test and stopping criteria. If $f(x_{k+1})<f(x_k)$ and $\|gk\| \leq \varepsilon$ then stop. Otherwise go to Step 2 with k = k+1

MATERIALS AND METHODS

Data analysis: The data collection is from Paediatric Specialist Clinic, Hospital Sultanah Nur Zahirah (HSNZ), Kuala Terengganu. It represents the number of paediatric patients visiting the paediatric specialist clinic (HSNZ) from year 2005 until 2015 (Table 1 and 2). From the

Table 1: The number of paediatric patients

Years	No. of paediatric patients (y)
2005	8535
2006	8331
2007	9127
2008	9574
2009	10235
2010	10988
2011	12513
2012	12681
2013	12951
2014	14864
2015	15590

Table 2: Data summation

Years	x_{ii}	y_i	x^2	$x_i y_i$
2005	1	8535	1	8535
2006	2	8331	4	16662
2007	3	9127	9	27381
2008	4	9574	16	38296
2009	5	10235	25	51175
2010	6	10988	36	65928
2011	7	12513	49	87591
2012	8	12681	64	101448
2013	9	12951	81	116559
2014	10	14864	100	148640
Σ	55	109799	385	662215

data, x-variable denotes the years whilst y-variable denotes the number of paediatric patients. For calculation purpose, only the data from 2005-2014 is considered. The 2015 data is reserved for error calculation

RESULTS AND DISCUSSION

In this study, the number of paediatric patients is estimated both from the Discrete Least Square method and the Conjugate Gradient method. Two different trend lines are applied in order to form an approximation function for each method. They are linear trend line and quadratic trend line. The result of trend line is constructed by using Microsoft Excel 2013 and all of the calculations obtained are from MAPLE 16 Software (Table 3).

Approximation function

Trend line: Based on the actual data in Table 1 a trend line has been identified via linear and quadratic approach. The data is plotted automatically from $y = a_0+a_{1x}$ for linear and $y = a_0+a_1x+a_2x^2$ for quadratic. The number of paediatric patients denoted by y is represented in the y-axis. The x-axis represents the year which is denoted as x in the raw data. The trend line below is from linear and quadratic approach for least square method only. Figure 1 and 2 show that the graph for quadratic fit well for the data compared to the linear graph.

Average sum of error: Once obtaining these models of Least Square method and Conjugate Gradient method, the relative error and sum of error for each data are computed by comparing the actual data and approximation data. All calculations of errors are made by using Microsoft Excel 2013 and Maple 16 (Table 4). The model that gives the smallest sum of relative error is considered as the best model to estimate the number of paediatric patients for year 2016.

From Table 4, the result shows that the quadratic model is the best approximation function in estimating the number of pediatrics patients. Quadratic least square model gives the smallest number of relative error which is 0.0071210 compared to other methods. The average of sum relative error for quadratic models are also smaller than linear model which is 0.01930748 for quadratic model and 0.03885144 for linear model for both methods. Therefore, the quadratic least square is chosen as the best model estimator.

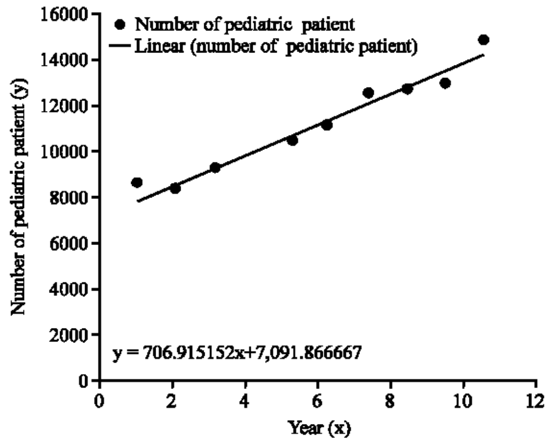

Fig. 1: The number of paediatric patients for linear LS

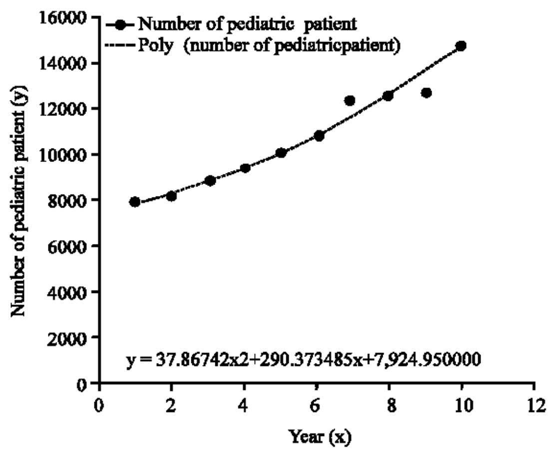

Fig. 2: The number of paediatric patients for quadratic LS

Table 3: Approximation function

Methods	Linear approach	Quadratic approach
Least Square method (LS)	7091.866667+706.9151515x	$7924.95+290.3734848x+37.6742424x^2$
Conjugate method (FR)	7091.86666666667+706.915151515102x	$906.447888+1910.871244x+48.8333331x^2$
Conjugate method (PRP)	7091.86666756220+706.915151386411x	$906.447889+1910.871245x+48.833331x^2$
Conjugate method (RMIL)	7091.86666756221+706.915151386409x	$906.447788+1910.871224x+48.833331x^2$

Table 4: Sum of error

	Linear		Quadratic	
Methods	Error	Relative error	Error	Relative error
Least square	14867.93333	0.04856530	15701.01667	0.0071210
FR	14867.93333	0.04856530	16017.19851	0.0274020
PRP	14867.93333	0.04856530	16017.19853	0.0274020
RMIL	14867.93336	0.04856540	16017.19820	0.0274020
Trend line	14867.99000	0.04856130	15701.01700	0.0072104
Average sum error	0.038851440	0.03885144	-	-

CONCLUSION

From the study, it is found that the quadratic Least Square method is the best method to estimate the number of paediatric patients for year 2016 where the approximation function obtained is $y = 7924.95+290.3734848x+37.86742424x^2$. For 2016 where $x = 12$, the value from the obtained function is given as $y = 16862$ which is the closest value to the real data. In conclusion, the quadratic model is the best fit model compared to linear for both methods with the best overall is quadratic least square.

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