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Round Flat Membrane at Great Deformations

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Abstarct: The problem of the extension in a circular membrane plane made of an isotropic elastic incompressible material is solved within the framework of thin shell nonlinear theory. The solution is represented in quadratures. An analytic solution is obtained for the Chernykh potential. It is shown that the solution can have a peculiarity at finite transverse dimensions of a deformed membrane.

Key words: Elastomers, membrane, strain, stress, elastic potential

INTRODUCTION

The products made from zlastomeric materials can experience large strain without breaking up to several hundred percent. According to the mentioned, it is necessary to implement some plans in way of use of nature of clean energies with approach of sustainable development and create some powerful foundations for this purpose through an overview of Iran's traditional architecture which has paid specific attention to climate and the designations and constructions have been based on climatic approaches (Ghasemi, 2017; Bochkareva and Kolpak, 1993). The calculation of products from such materials is carried out within the framework of geometrically and physically non-linear elasticity theory (Kabrits and Slevpenva, 1998; Malkov and Kabrits, 1999). Mathematical models are the boundary value problems for the systems of nonlinear partial differential equations. The solution of such problems can be a non-unique one (Bochkareva and Kolpak, 1993; Gent, 2005; Kanner and Horgan, 2007; Kolpak *et al*., 2015, 2016; Kolpak and Maltseva, 2015; Atafar *et al*., 2013; Wineman, 2005). Along with this, the solutions are also possible that have singularity at some points (Dal and Pronina, 1998; Yuan *et al*., 2006). The analytical solutions of non-linear equations can be modeled in exceptional cases. The numerical solutions do not always allow us to reveal the features in a stress-strain state (Kabrits *et al*., 1986). The example of an analytical solution existence with the peculiarity for a circular flat elastomeric membrane stretched in a plane is shown below.

Law of elasticity: The nonlinear relationship between stresses and strains in the non-linear theory of elasticity is determined by the means of an elastic potential $\Phi = \Phi(\lambda_1, \lambda_2, \lambda_3)$ which is an elongation ratio function for an isotropic material λ_1, λ_2 and λ_3. For an incompressible material, the following incompressibility condition must be satisfied: $\lambda_1 \lambda_2 \lambda_3 = 1$. The potentials of Muni-Rivlin, Bartenev-Khazanovich, Gent-Thomas, Ishihara, Biderman, Alexander, Hart-Smitt, Ogden are among the potentials for incompressible materials (Agostiniani and DeSimone, 2012; Feng *et al*., 2006). Some of them and their "modifications" are often used to solve specific problems (Lectez *et al*., 2014; Liu and Fatt, 2011). The problem of the potential selection remains to be open one (Albrecht and Chandar, 2014; Liu and Fatt, 2011; Nah *et al*., 2010) because each kind of elastomer has its own unique properties. Below they will use the power potential proposed by Kabrits *et al*. (1986):

$$\Phi = \frac{\mu}{n^2}\begin{bmatrix}(1+\beta)\left(\lambda_1^n + \lambda_2^n + \lambda_3^n - 3\right) + \\ (1-\beta)\left(\lambda_1^{-n} + \lambda_2^{-n} + \lambda_3^{-n} - 3\right)\end{bmatrix} \quad (1)$$

Where:
μ = The linear shear modulus
n = The parameter
β = The constant satisfying the constraint $-1 \leq \beta \leq 1$

From this potential the potentials of Bartenev-Khazanovich ($n = 1$, $\beta = 1$), Neogukov ($n = 2$, $\beta = 1$) and Muni-Rivlin ($n = 2$) potential follow.

MATERIALS AND METHODS

Membrane equilibrium equations: Let the circular membrane of the outer radius r_2 and with the radius of the inner hole r_1 is stretched in a plane by a uniform load applied to its outer contour. Let is the radius of the middle

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surface points before deformation, r after deformation; h^0 the thickness of the membrane in an undeformed state and h-the thickness of the membrane in a deformed state λ_1 and λ_2 elongation multiplicity, T_1 and T_2 the forces in the radial and circumferential directions. Then, for the case of an axisymmetric deformation the equilibrium equation is the following one (Kabrits *et al*., 1986; Kolpak *et al*., 2015):

$$r^0 \frac{dT_1}{dr^0} + T_1 - T_2 = 0 \quad (2)$$

and the equation, connecting elongation multiplicities λ_1 and λ_2:

$$r^0 \frac{dr}{dr^0} + \lambda_2 - \lambda_1 = 0,\ \lambda_1 = \frac{dr}{dr^0}, \quad (3)$$
$$\lambda_2 = r/r^0,\ \lambda_3 = h/h^0 = 1/\lambda_1\lambda_2$$

The relationship between the forces and the multiplicities of elongations is determined by the elastic potential $\Phi = \Phi(\lambda_1, \lambda_2, \lambda_3)$ (Kabrits *et al*., 1986):

$$T_1 = h^0\left(\lambda_1 \frac{\partial\Phi}{\partial\lambda_1} - \lambda_3 \frac{\partial\Phi}{\partial\lambda_3}\right),\ T_2 = h^0\left(\lambda_2 \frac{\partial\Phi}{\partial\lambda_2} - \lambda_3 \frac{\partial\Phi}{\partial\lambda_3}\right) \quad (4)$$

The following boundary conditions are accepted on the inner contour of the membrane at $r^0 = r_1$: $r = r_1$ or $\lambda_2 = 1$ and on the external contour at $r^0 = r_2$: r^*: or $\lambda_2 = r_*/r_2 > 1$.

These boundary conditions assume that the points of the inner contour are not displaced. The radius of the outer contour is increased to the valuer $r = r_*$. At that, the load stretching the membrane is calculated according to the following equation: $P = T_1$ ($r^0 = r_2$). According to the case of the membrane with constant thickness (h^0 = const) from Eq. 2:

$$\frac{\partial T_1}{\partial\lambda_1}\frac{d\lambda_1}{d\lambda_2} + \frac{\partial T_1}{\partial\lambda_2} = \frac{T_2 - T_1}{\lambda_1 - \lambda_2} \quad (5)$$

Since, the forces T_1 and T_2 are the functions of the elongation multiplicities only, then the dependence $\lambda_1 = \lambda_1(\lambda_2)$ is obtained from this equation. And the relationship between λ_2 and r^0 is obtained from Eq. 3:

$$\ln r - \ln C_0 = \int_{\lambda_2(r_1)}^{\lambda_2} \frac{d\lambda_2}{\lambda_1(\lambda_2) - \lambda_2}$$

where, C_0 is the integration constant. A homogeneous solution satisfies Eq. 2 and 3 for a continuous membrane $\lambda_1 = \lambda_2$ = const, $s = \lambda r$, $T_1 = T_2 = P$ where, P is the load acting on an external contour.

RESULTS AND DISCUSSION

Analytical solution: For the case of potential Eq. 1, it follows from Eq. 4 that:

$$T_1 = \mu h^0 \lambda_1^{-1}(\lambda_1 - \lambda_3)(1 + \beta + (1-\beta)\lambda_2)$$
$$T_2 = \mu h^0 \lambda_2^{-1}(\lambda_2 - \lambda_3)(1 + \beta + (1-\beta)\lambda_1)$$

and Eq. 5 takes the following form:

$$\frac{d\lambda_1}{d\lambda_2} = -\frac{\lambda_1}{2\lambda_2}\left[1 + \frac{1+\beta+(1-\beta)\lambda_1}{1+\beta+(1-\beta)\lambda_2}\right] \quad (6)$$

The quadratures at $\beta = 1$ follow from this equation:

$$\frac{1}{\lambda_1\lambda_2} = \lambda_3 = \text{const} = C \quad (7)$$

at $\beta = -1$:

$$\lambda_2^{-3/2} + 3\lambda_1^{-1}\lambda_2^{-1/2} = \text{const} = C \quad (8)$$

at $|\beta|<1$:

$$(1+\beta)\,\lambda_1^{-1} f(x) + (1-\beta)\varphi(x) = (1-\beta)C \quad (9)$$

$$x = (1-\beta)(1+\beta)^{-1}\lambda_2 f(x) = x^{-1}\sqrt{1+x}$$
$$\varphi(x) = \frac{1}{2}f(x) + \frac{1}{4}\ln\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}$$

The integration constant C contained in Eq. 7 and 8 must be a positive value. This constant and the constant C are found from the satisfaction of the boundary conditions. The function $\lambda_1 = \lambda_1(\lambda_2)$ as the solution of Eq. 6 at various values of the parameter β is determined from Eq. 7-9 explicitly.

Equation 8 and 9 allows the solution, at which λ_1 can take infinite values at some point of the interval (r_1, r_2). At that λ_2 will take finite values at this point. The finite values will take the effort T_1 the effort T_2 is infinitely large and the multiplicity of membrane thickness λ_3 deformation change turns to zero. That is, at finite transverse dimensions of the stretched membrane a peculiarity may appear at some point in a membrane.

In order to solve Eq. 8 on the inner contour, since at this point $\lambda_2 = 1$ the equality $1+3\lambda^{-1}_1 = C$ hall be performed. Since, λ_1 is a positive value, then the constant C should satisfy the inequalities $1 \le C \le 4$. The value C = 4 corresponds not a non-deformed membrane but to C = 1 the case when $\lambda_1 = \infty$ on the inner contour. In order to solve (9) at $\lambda_2 = 1$ since $0<\beta<1$ and $x = (1-\beta)(1+\beta)^{-1}<1$ the functions f(x) and $\Phi(x)$ take positive values in the considered range of parameter β variation. The values of the constant C will be positive and finite ones in this case at any values on the inner contour, also at $\lambda_1 = \infty$.

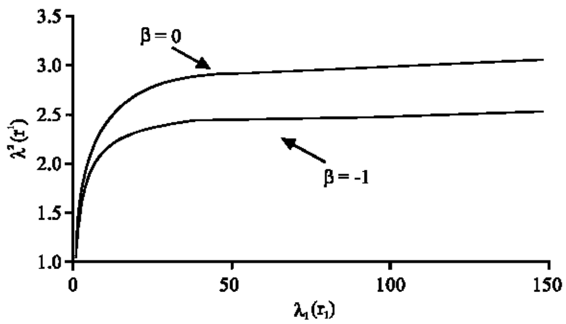

Fig. 1: The dependence of outer contour radius λ_2 relative change on the multiplicity of elongation λ_1 within the inner contour

Figure 1 shows the dependence of the outer contour radius λ_2 increase on the elongation multiplicity λ_1 on the internal contour for the case $r_1 = 0.01$ and $r_2 = 1$ at the values $\beta = 0$ and $\beta = -1$.

During the operation of specific products from elastomers the physical properties of material can be changed under the influence of external factors including vibrational loads (Gasratova and Stareva, 2016; Kabrits and Slepneva, 1998; Kolpak *et al*., 2016) chemical effects (Pronina, 2010, 2013; Rivlin, 2006) and temperature gradients. In the considered membrane model this can be related to the change of the parameter β in the elastic potential (1). And if under the influence of external causes in time $\beta \to 1$ then in this case the loss of equilibrium state stability (Tahmassebpour, 2016; Zubov, 2007) may take place with the subsequent destruction of the structure (Fig. 1).

CONCLUSION

The obtained solution with a peculiarity within the framework of thin shells theory should be considered only as one of the possible directions for deformation development. And the equations of thin shells theory in such variants of physical nonlinearity can not be used to describe the stress-strain state of thin membranes. In these cases it is necessary to use the theories that take into account the heterogeneity of the stress state along the thickness more accurately than the theory of thin shells.

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