Journal of Engineering and Applied Sciences 12 (12): 3194-3197, 2017
ISSN: 1816-949X

Image Decomposition on the Orthogonal Basis of Subband Matrices Eigenvectors

Evgeniy G. Zhilyakov, Andrey A. Chernomorets,
Evgeniya V. Bolgova and Anastasiya N. Kovalenko
Belgorod State University, Pobedy St., 85, 308015 Belgorod, Russia

Abstract: In this study, we propose a method of image decomposition on the orthogonal basis of eigenvectors of specially calculated subband matrices. We show the difference of so called basic images generated by eigenvectors of subband matrices corresponding to various subdomains of spatial frequencies. The accuracy of the images presentation on the specified basis is given.

Key words: Image, decomposition, orthogonal basis, eigenvector, subband matrix, subdomain of spatial frequencies

INTRODUCTION

In modern information and telecommunication systems, the main volume of the information is presented in the form of signals and images in digital form. Analysis of this information is performed in both the time (spatial) domain and the transform domain.

In solving many problems of digital image processing for example, filtration, compression (Gonzalez and Woods, 2012; Pratt, 1982; Yaroslavsky, 1979) information steganography embedding (Khonahovitch and Puzyrenko, 2006; Chernomorets *et al*., 2015; Zhilyakov *et al*., 2015) and other, the specialists often use the analysis of digital images in the transform domain also called image analysis in the frequency domain D_2 (domain of normalized spatial frequencies (u, v) (SF): $-\pi\leq u,\ v<\pi$). Now they widely use the presentation of images based on different systems of orthogonal basis functions (Gonzalez and Woods, 2012; Pratt, 1982; Atafar *et al*., 2013) such as trigonometric functions (exp -jnu), sin (nu), cos (nu)), Hartley function cas (nu), Walsh functions (wal (n, u) had (n, u), pal (n, u)) Haar function har (nu) and etc.

In this study, we propose a method of image decomposition on the basis of eigenvectors of specially calculated subband matrices (Tahmassebpour, 2017) which provides exact presentation of images in the specified basis.

MATERIALS AND METHODS

Main part

The method of image decomposition in the special orthogonal basis: Let us consider the digital image presented in form of real values matrix $\Phi = (f_{ij})$, i = 1, 2, ..., N, j = 1, 2, ..., M which elements correspond to the brightness of individual pixels of the image. Let us build a centrally Symmetric Subdomain of Spatial Frequencies (SSF) V of following type:

$$V:\ \{(u, v)\,|\,(u\in[\alpha_1, \alpha_2[v\in[\beta_1, \beta_2[)\cup(u\in[\alpha_1, \alpha_2[v\in[-\beta_2, -\beta_1[)\cup(u\in[-\alpha_2, -\alpha_1[v\in[-\beta_2, -\beta_1[)\cup u\in[-\alpha_2, -\alpha_1[v\in[\beta_1, \beta_2[)\}\ 0\leq\alpha_1, \alpha_2, \beta_1, \beta_2\leq\pi \tag{1}$$

and the corresponding square matrices $A = (a_{ik})$, i, k = 1, 2, ..., N and $B = (b_{jm})$, j, n = 1, 2, ..., M (Tahmassebpour, 2017) having dimensionality of N×N and M×M accordingly (we call them as subband matrices) whose elements are calculated using the following ratios:

$$a_{ik} = \begin{cases} \dfrac{\sin(\alpha_2(i-k)) - \sin(\alpha_1(i-k))}{\pi(i-k)}, & i\neq k, \\ \dfrac{\alpha_2-\alpha_1}{\pi}, & i = k, \end{cases}$$
$$b_{jm} = \begin{cases} \dfrac{\sin(\beta_2(j-m)) - \sin(\beta_1(j-m))}{\pi(j-m)}, & j\neq m, \\ \dfrac{\beta_2-\beta_1}{\pi}, & j = m \end{cases} \tag{2}$$

In study (Tahmassebpour, 2017; Zhilyakov *et al*., 2016) we shown that subband matrices A and B are symmetric and positive definite, therefore, they can be presented as the following decompositions:

$$A = Q^A L^A (Q^B)^T,\ B = Q^B L^B (Q^B)^T \tag{3}$$

Corresponding Author: Evgeniy G. Zhilyakov, Belgorod State University, Pobedy St., 85, 308015 Belgorod, Russia

Where:
Q^A = The columns of the matrices
Q^B = Are eigenvectors of matrices A and B, the main diagonals of matrices L^A and L^B contain eigenvalues of A and B

$$\begin{aligned} &Q^A = (\vec{q}_1^A, \vec{q}_2^A, ..., \vec{q}_{N_1}^A),\ Q^B = (\vec{q}_1^B, \vec{q}_2^B, ..., \vec{q}_{N_2}^B), \\ &L^A = \mathrm{diag}(\lambda_1^A, \lambda_2^A, ..., \lambda_{N_1}^A),\ L^B = \mathrm{diag}(\lambda_1^B, \lambda_2^B, ..., \lambda_{N_2}^B) \end{aligned} \tag{4}$$

Consider the matrix:

$$\Gamma^V = (\gamma_{ik}),\ i = 1, 2, ..., N,\ k = 1, 2, ..., M$$

$$\Gamma^V = (Q^A)^T \Phi Q^B \tag{5}$$

which elements:

$$\gamma_{ik},\ i = 1, 2, ..., N,\ k = 1, 2, ..., M$$

$$\gamma_{ik} = (\vec{q}_i^A)^T \Phi \vec{q}_k^B \tag{6}$$

can be considered as projections values of image Φ on the orthogonal eigenvectors $\{\vec{q}_i^A\}$, $i = 1, 2, ..., N$ and $\{\vec{q}_k^B\}$, $k = 1, 2, ..., M$ of subband matrices A and B, corresponding to the given SSF V.

It is obvious that the product of the projections matrix Γ^v to the matrices Q^A and Q^B, formed by the corresponding eigenvectors coincide with the image Φ:

$$\Phi = Q^A \Gamma^V (Q^B)^T \tag{7}$$

Lets us transform the right side of Eq. 7:

$$Q^A \Gamma^V (Q^B)^T = (\vec{q}_1^A \vec{q}_2^A ... \vec{q}_N^A) \begin{pmatrix} \gamma_{11}\ \gamma_{12} \cdots \gamma_{1M} \\ \gamma_{21}\ \gamma_{22} \cdots \gamma_{2M} \\ \cdots \\ \gamma_{N1}\ \gamma_{N2} \cdots \gamma_{NM} \end{pmatrix} \begin{pmatrix} (\vec{q}_1^B)^T \\ (\vec{q}_2^B)^T \\ \cdots \\ (\vec{q}_M^B)^T \end{pmatrix} = (\vec{s}_1 \vec{s}_2 ... \vec{s}_M) \begin{pmatrix} (\vec{q}_1^B)^T \\ (\vec{q}_2^B)^T \\ \cdots \\ (\vec{q}_M^B)^T \end{pmatrix}$$

Where:

$$\vec{s}_k = \sum_{i=1}^{N} \vec{q}_i^A \gamma_{ik}$$

Then, we have:

$$\Phi = \sum_{i=1}^{N} \sum_{k=1}^{M} \gamma_{ik} \vec{q}_i^A (\vec{q}_k^B)^T \tag{8}$$

Thus, the image Φ can be presented as the sum of several components (images) X_{ik}, $i = 1, 2, ..., N$, $k = 1, 2, ..., M$ with multipliers whose values are equal to the corresponding projections γ_{ik} of the image Φ:

$$\Phi = \sum_{i=1}^{N} \sum_{k=1}^{M} \gamma_{ik} X_{ik} \tag{9}$$

$$X_{ik} = \vec{q}_i^A (\vec{q}_k^B)^T,\ i = 1, 2, ..., N,\ k = 1, 2, ..., M \tag{10}$$

Images X_{ik}, $i = 1, 2, ..., N$, $k = 1, 2, ..., M$ of type (Eq. 10) which are formed by the eigenvectors $\vec{q}_i^A$ and $\vec{q}_k^B$ of subband matrices A and B respectively, will be called the basic images in the basis of subband matrices eigenvectors which correspond the given SSF V.

Thus, Eq. 9 and 10 determine the images decomposition using the images projections to the corresponding eigenvectors in the basis of orthonormal eigenvectors of subband matrices (Bolgova, 2017).

RESULTS AND DISCUSSION

Computational experiments: The aim of computational experiments is basic images visualization and the image presentation errors detection in the above basis.

Figure 1 shows an example of basic images in the subband eigenvectors basis of matrices A and B, size of 64×64 elements and which correspond to SSF V: (0, π/4) for each of the spatial frequencies. It should be noted that Fig. 1 shows the basic images formed by the paired products of the first four eigenvectors (Fig. 1) of given matrices.

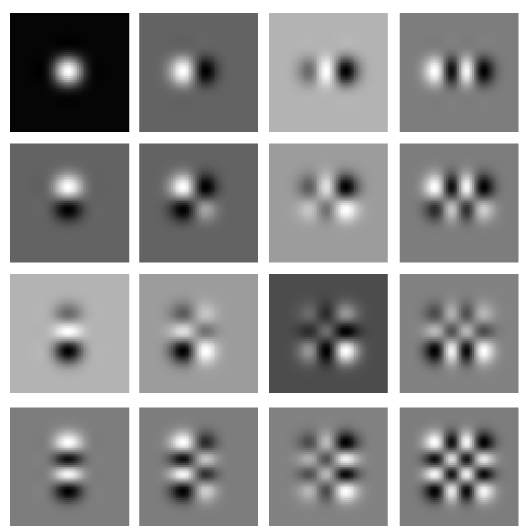

Fig. 1: An example of a basic images in the basis of the subband matrices eigenvectors which correspond to SSF V: (0, π/4)

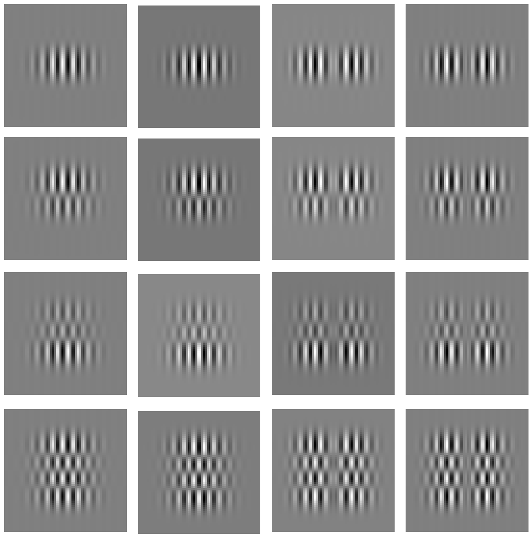

Fig. 2: An example of the basic images in the basis of eigenvectors of the subband the matrices which correspond to SSF $V_1 = (0, \pi/4)$ and $V_2 = (\pi/4, \pi/2)$

Let us show that the view of the basic images varies according to the basis of eigenvectors of subband matrices corresponding to different SSF.

Figure 2 shows an example of the basic images in the basis of eigenvectors of different subband matrices A and B, size of 64×64 elements. Thus, subband matrix A corresponds to SSF V_1: $(0, \pi/4)$ and subband matrix B corresponds to SSFV_2: $(\pi/4, \pi/2)$. Figure 2 shows the basic image formed by the first four pairs of the given matrices eigenvectors.

Basic images shown in Fig. 2 demonstrate that while using the eigenvectors of subband matrices corresponding to different SSF, the periodicity of displaying of presented images individual elements alters in comparison with the images in Fig. 1.

Computational experiments results have also shown that a significant number of image projections to the pairs of corresponding eigenvectors (Eq. 6) have a relatively small values. This property of images projection can be used in solving the problems of image filtration, compression and information embedding into the images.

It is also of interest the evaluating of the error of the image presentation in the form of decomposition on a basis of subband matrices eigenvectors.

Thus, while presenting the domain of spatial frequencies ($-\pi \leq u < \pi$, $-\pi \leq v < \pi$) as a union of equal subdomains of spatial frequencies V_{sr}, s = 1, 2, ..., S, r = 1, 2, ..., R (S = R = 4) the error values of studied images decomposition, calculated as the standard deviation of the expression (Eq. 9) right side with respect to the pixel values of the given image Φ are shown in Table 1. In Table 1, the error is calculated for decomposition on the basis of subband matrices eigenvectors for each of SSF V_{sr}, s = 1, 2, ..., S, r = 1, 2, ..., R.

Table 1: Error of image decomposition on the basis of subband matrices eigenvectors (S = R = 4) corresponding to different SSF V_{sr}

S/r	1	2	3	4
1	3.35E-15	3.37E-15	3.98E-15	3.96E-15
2	2.68E-15	2.85E-15	3.76E-15	3.07E-15
3	2.94E-15	3.24E-15	4.29E-15	2.62E-15
4	3.55E-15	3.15E-15	3.08E-15	5.00E-15

Summary: Thereby, it was shown that a significant amount of images projections on the corresponding subband matrices eigenvectors has a relatively small values, thus images decomposition on the orthonormal basis of eigenvectors can be used in solving the tasks of images filtration, compression, information embedding into images, etc.

CONCLUSION

The results given in the table, show that the values of the error of image presentation in the form of decomposition on the basis of subband matrices eigenvectors are insignificant and these errors have values into the range of error of real numbers presentation in the computer. The results of computational experiments show that the presentation of images in the form of a decomposition on the orthogonal basis of subband matrices eigenvectors can be used as an effective tool in dealing with image processing tasks.

REFERENCES

Atafar, A., M. Shahrabi and M. Esfahani, 2013. Evaluation of university performance using BSC and ANP. Decis. Sci. Lett., 2: 305-311.

Bolgova, E.V., 2017. About the eigenvalues of cosine transform subinterval matrices: Belgorod State University scientific bulletin economics. Inf. Technol., 2: 92-101.

Chernomorets, A.A., E.G. Zhilyakov, E.V. Bolgova, M.A. Petina and A.N. Kovalenko, 2015. On subband embedding resistance to data compression. Res. J. Appl. Sci., 10: 403-406.

Gonzalez, R.C. and R.E. Woods, 2012. Digital Image Processing. Tehnosfera, Moscow, Russia, Pages: 1104.

Khonahovitch, G.F. and A.Y. Puzyrenko, 2006. Computer Steganography: Theory and Practice. MK Press & Publishing Systems, Moscow, Russia, Pages: 288.

Pratt, W.K., 1982. Digital Image Processing. Mir Publisher, Moscow, Russia, Pages: 312.

Tahmassebpour, M., 2017. A new method for time-series big data effective storage. IEEE Access, 2007: 10694-10699.

Yaroslavsky, L.P., 1979. Introduction to Digital Image Processing. Sov. radio Publisher, Moscow, Russia, Pages: 312.

Zhilyakov, E.G., I.S. Konstantinov and A.A. Chernomorets, 2016. Decomposition of images into additive components. Intl. J. Imaging Rob., 15: 1-8.

Zhilyakov, E.G., I.S. Konstantinov, A.A. Chernomorets and E.V. Bolgova, 2015. Image compression subband method. Intl. J. Soft Comput., 10: 442-447.