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Oblique Compression Shock Wave and Shock Wave Polars

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Abstract: This research presents the history of research on the dynamic compatibility conditions on the oblique shock wave that define correlation of values of gas-dynamic variables before discontinuity and immediately after it. All relations can be used in numerical methods and applied for both compression shock waves and isentropic waves. The notion of shock waves polar is introduced. A research on their properties is conducted. Particular attention is paid to special points on shock wave polars and their significance for the research of shock wave properties and interference. Using step-by-step research works as an example, the researchers show the history of solving issues on interference of oblique shock wave between each other and other types of gas-dynamic discontinuities. A graphic method for solving problems of gas-dynamic discontinuity interference by using shock wave polars is provided.

Key words: Gas-dynamic compatibility conditions, gas-dynamic discontinuity, oblique compression shock wave, shock wave polar, discontinuity, interference

INTRODUCTION

The aim of the research is to reduce the relations on oblique compression shock waves to universal form that can be used to isentropic waves; to research dependency properties of gas-dynamic variables after discontinuity on flow parameters before the discontinuity; to provide a graphic method of solving problems of gas-dynamic discontinuity interference and to provide necessary graphical material for them.

Though, the gas-dynamic calculation method is widespread in set of cases the problem of direct compression shock calculation is still relevant, especially if optimal solution is required.

In numerous available studies, the calculation methods on the subject are usually presented in a form that is difficult to use for problems of supersonic flow optimization and control.

This state is further complicated by the fact that equations conjugated to shock calculation often have few solution, computational quirks or simply cannot be solved for a searched variable. In order to filter out the solutions that correspond to physically realizable shock wave configurations, additional reasoning is required.

On the other hand, there a minimal set of the most important shock characteristics for which problem can be stated in a convenient form. Knowing special and terminal shock parameters allows splitting solutions into classes easily.

In the current research, we present an approach that allows one to easily solve 90% problem of practical importance, related to calculation of singular oblique compression shocks.

A detailed analysis of gas-dynamic waves (an isentropic rarefaction and compression wave) and oblique compression shock waves that occur in plane stationary flows of non-viscous gas has been publish by Meyer (1908). In the same research, the parameters of an oblique compression shock that generates during flow over plane acute angle are described. In 1929-1937, Busemann in series of his researches (Busemann and Dynamics, 1931) laid a foundation for graphic solution methods for problems about gas-dynamic discontinuity interference by using shock wave polars that conjugate compression oblique shock wave intensity with flow's turn angle on shock wave. Since then, the shock wave polars are named Busemann polars after him. Because of their look they are also called heart-shaped curves. Another name is Isomach because each shock polar is built at specific Mach number of flow. The solution methods for discontinuity interference problems by using shock polars were developed by Courant and Friedrichs (1948).

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In 1940's, the tasks of designing supersonic aircraft instigated research of compression shock waves, interaction between waves and discontinuities. During first experiments involving shock wave pipe the one-dimensional interactions were studied. The theory of gas flow in shock wave pipe in one-dimensional setting was proposed by Schardin (1932). In 1950's at the University of Toronto, a series of experimental and theoretical research on interaction of one-dimensional running waves and discontinuities were conducted:

- Refraction of running shock wave on contact discontinuity (Bitondo *et al*., 1950; Bitondo and Lobb 1950; Ford and Glass,1956)
- Interaction of overtaking shock waves (Gould, 1952)
- Shock wave with rarefaction wave (Nicholl, 1951; Gould, 1952) and rarefaction wave refraction (Billington and Glass, 1951; Billington, 1955)

The theoretical results during these years were humbler. Taub (1947)'s research the shock wave's propagation through two initially at rest gases split by division surface (contact discontinuity) was studied. Molder (1960) developed an analytical theory of regular interaction of counter directed waves. The two and three-dimensional problems were solved exclusively by numerical methods.

Uskov has made a major investment into development of stationary gas-dynamic discontinuity theory. In modern form, its main statements were formulated in 1980 (Uskov, 1980). In the collection of scientific papers (Uskov, 1980; Kunchur *et al*., 2015), the conditions of dynamic consistency for main problems of discontinuity interference are presented. Results of the relations on the shock and properties of various shock wave structure analysis are presented in the monograph (Adrianov *et al*., 1995).

These results were developed later for cases of one-dimensional running waves and oblique shock waves (Uskov *et al*., 2002; Omelchenko *et al*., 2002). In these works, convenient formulas for calculating parameters of oblique compression shocks and oblique shock wave are provided. Uskov (1980) and Adrianov *et al*. (1995) conducted a research of heart-like curve (shock-wave polar) which allowed to define their important properties: presence of an envelope curve, terminal angle of flow deflection on discontinuity, points corresponding to discontinuities, after which Mach numbers are equal to one. It can be noted that presence of an envelope curve is important for problem of aircraft aerodynamics (Uskov and Chernyshov, 2014a, b; Kunchur *et al*., 2013; Rahmani *et al*., 2011; Raad *et al*., 2016) because it corresponds to pressure extremes on the sides of the body that flies with a set attack angle but variable velocity.

The mathematical apparatus commonness of non-stationary and two-dimensional problems about shock waves and shocks interaction that was demonstrated in researches by Arkhipova and Uskov (2012, 2013) allowed Chernyshov to solve a set of practically important problems (Uskov and Chernyshov, 2010; Tahmassebpour and Otaghvari, 2016; Uskov and Chernyshov, 2014a, b; Uskov *et al*., 2002; Seyedhosseini *et al*., 2016; Silnikov *et al*., 2014) of interaction of oblique shock with Prandtl-Meyer wave. The next step was to research interaction between shock wave and straight compression shock wave (Omelchenko and Uskov, 2002; Uskov and Mostovykh, 2008; Kunchur *et al*., 2015; Kunchur *et al*., 2013; Rahmani *et al*., 2011) as well as oblique shock wave (Omelchenko and Uskov, 2002) and non-stationary triple configurations (Uskov and Mostovykh, 2008).

MATERIALS AND METHODS

An oblique compression shock wave mathematical model: A shock wave model is a surface of the first-type mathematical discontinuity by passing through which the gas-dynamic variables get discontinued $[f] = f_2\text{-}f_1$. In general case, the shock wave can migrate in space. A stationary wave is called standing or compression shock. If there is an angle between a shock and oncoming flow, then the shock is called oblique. A ratio between variables f_2 and f_1 at different sides of gas-dynamic discontinuity is called Dynamic Compatibility Conditions (DCC) on shock wave.

Shock's slope angle σ, intensity J which is assumed to be a relation between pressure after shock wave P_2 and pressure before shock wave P_1 and flow's deviation angle on shock wave β (Fig. 1a) at set flow parameters before compression shock wave (M_1, P_1, P_{01}, ρ) have a mutual unique dependency. Setting one of the parameters allows for calculation of others. For instance, if flow's turn angle β is known (Fig. 1b), so when it is equal to wedge's angle that is flown over by a supersonic flow, then it is possible to find an intensity and slope angle of a generating oblique shock. If intensity J is known for instance in case of over-expanded jet where it equals to a ratio between environmental pressure and pressure on supersonic nozzle's cut (point A on Fig. 1c) then it is possible to find a shock's slope angle and flow's turn angle (jet's boundary) on shock. In cases when shock is a result of other discontinuitie's interference, its slope angle is usually known. This slope angle can be used to find intensity and flow's turn angle.

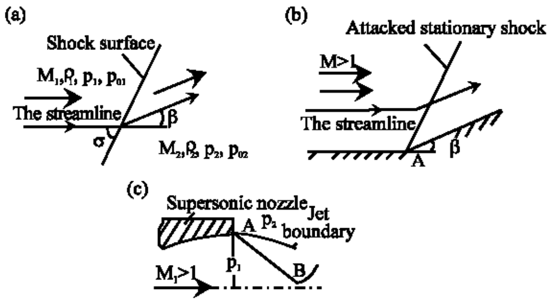

Fig. 1: A model of multi-piped PDE: parameters before shock wave; parameters after shock wave; M: Mach number; p: pressure; p_0: total pressure; β: flow's turn angle; s: slope angle of compression shock

The compression shock wave's parameters are dependent on thermo-physical properties of gas which are defined by a heat capacity ratio $\gamma = c_p/c_v$ (c_p is specific heat of gas in thermodynamic processes occurring at constant pressure, c_v specific heat of gas in thermodynamic processes occurring at constant volume) and by its molecular weight. In an ideal gas the heat capacity ratio depends on number of freedom degrees $\gamma = (j+2)/j$. If gas is monatomic, then it has 3 degrees of freedom and heat capacity ratio is 5/3 or 1.666. If gas is diatomic, then it has 5 degrees of freedom and heat capacity ratio is 7/5 or 1.4. A triatomic gas has 6 degrees of freedom and its heat capacity ratio is 8/6 or 1.33. In additional, $\gamma = 1.1$ for hydrocarbon fuel-air mixture, $\gamma = 1.2$ for hydrocarbon fuel-oxygen mixture, $\gamma = 1.251$ for combustion products. In real gas γ depends on temperature and pressure but it can be ignored at t<600 K.

The DCC on stationary discontinuities is a zero equation of the following shock parameters $[f] = f_2 - f_1$.

- Flow of substance:

$$[\rho\upsilon_n] = \rho_2\upsilon_{n2} - \rho_1\upsilon_{n1} = 0 \quad (1)$$

- Motion of impulse in projection on a normal to shock wave's surface:

$$\left[p + \rho\upsilon_n^2\right] = 0 \quad (2)$$

- Motion of impulse in projection on a tangent to shock waves surface:

$$[\rho\upsilon_n\upsilon_\tau] = 0 \quad (3)$$

- Energies:

$$\left[i + \upsilon_n^2/2\right] = 0 \quad (4)$$

Where:

υ_n and υ_τ = The vector's projection on discontinuity plane
I = Enthalpy
p = Pressure
T = Temperature

Density are conjugated with an ideal gas law:

$$\frac{p}{\rho T} = \text{const} = \frac{8340}{\mu} \quad (5)$$

which for an ideal gas (molecular weight and heat capacity ratio are constant, enthalpy i is proportional to T) can be rewritten in a form:

$$i = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} \quad (6)$$

The flow's compression rate in shock wave process is usually characterized by the density ratio $E = \rho_1/\rho_2$ which when external heat supply is absent, are called an adiabat. If E>1, there is an expansion of the flow, if E<1 the compression. In isentropic process, the E is defined by Laplace-Poisson adiabat (is entrop):

$$JE^\gamma = 1 \quad (7)$$

On compression shock wave by using Eq. 6 and system (Eq. 2-3), the energy Eq. 4 can be written in a form of Rankine-Hugoniot shock waves adiabat:

$$i_2 - i_1 = \frac{1}{2}\frac{p}{\rho}(J-1)(1+E) \quad (8)$$

The rarefaction shock do not exist, i.e., on shock always E<1. Instead of γ the following variable is often used in equations:

$$\varepsilon = (\gamma - 1)/(\gamma + 1) \quad (9)$$

which represents limit E at J→∞. One can observe that on the shock it is finite, i.e., its density cannot increase infinitely. The Rankine-Hugoniot adiabat can be written in the form of dependency from shock wave intensity:

$$E = \frac{1 + \varepsilon J}{J + \varepsilon} \quad (10)$$

Let us introduce Mach number M = v/a, where a is a local speed of sound:

$$a^2 = \gamma p/\rho \quad (11)$$

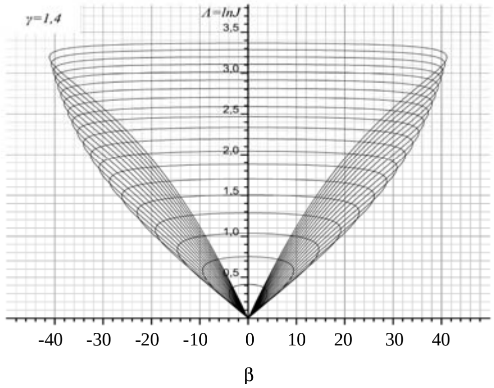

Fig. 2: A shock wave polar at γ = 1.4, a match number ranges from 2-5 with step 0.2

Then after simple transformation from Eq. 1-4 taking into account Eq. 10 and 11 and equation for oblique compression shock wave can be obtained:

$$J_\sigma = (1+\varepsilon)M^2 \sin^2\sigma - \varepsilon \quad (12)$$

and relation between flow's turn angles β and shock wave slope σ:

$$tg\beta = \frac{M^2\sin^2\sigma - 1}{\frac{1}{1-\varepsilon}M^2 - (M^2\sin^2\sigma - 1)} ctg\sigma \quad (13)$$

Equations 12 and 13 at set value M define a polar ln J-β (Fig. 2) in a parametric form with parameter σ which can vary in a rage from Mach angle α = arc sin (1/M) to 90°. It is noticeable that for each Mach number there is a maximum intensity:

$$J_m = (1+\varepsilon)M^2 - \varepsilon \quad (14)$$

using which the angle β can be expressed in the form:

$$tg\beta = \sqrt{\frac{J_m - J}{J+\varepsilon}} \frac{(1-\varepsilon)(J-1)}{(J_m+\varepsilon)-(1-\varepsilon)(J-1)} \quad (15)$$

If shock wave is set by intensity J then to calculate angle β then it is convenient to use (Eq. 15), if it is set by turn angle σ, then it is convenient to use (Eq. 13).

If a shock is set by turn angle β, then it is easier to solve Eq. 13 and 15 numerically, however there are cubic equations relative to J that explicitly conjugate J-β. For each β, there are two solutions for shock wave: with supersonic flow after it and with subsonic one. The relation between parameters on a shock can be written by using intensity J and generalized adiabat E:

$$M_2^2 = \frac{M^2 - (1-E)(J+1)}{EJ} \quad (16)$$

Relation of temperatures:

$$\frac{T_2}{T} = EJ \quad (17)$$

Relation of sound velocities:

$$\frac{a_2}{a} = \sqrt{EJ} \quad (18)$$

Total pressure recovery coefficient:

$$I_0 = \frac{P_{02}}{P_{01}} = (E^\gamma J)^{\frac{-1}{\gamma-1}} \quad (19)$$

Relation of densities:

$$\frac{\rho_2}{\rho_1} = \frac{1}{E} \quad (20)$$

If E in Eq. 16-20 is substituted with Laplace-Poisson adiabat Eq. 7 we get relations for simple and centered isentropic waves. If we substitute it with Rankine-Hugoniot adiabat Eq. 10 then we get an equation for shock waves. All variables after compression shock in Eq. 16-20 monotonously change depending on shock wave intensity J. Relations written in such form are true for any type of waves: simple, shock wave and detonation wave.

RESULTS AND DISCUSSION

Results of shock wave polars analysis: Figure 2-5 show shock wave polars for different γ at M = 2-5. A smaller polar corresponds to a lower Mach number.

For each M and γ there is a terminal angle β to which an oblique shock can deviate the flow. Thus, a flow picture showed on Fig. 1b is only possible at small wedge angles β). If it exceeds some terminal value for given M value of which is usually labeled as β_l, then a deviated curved compression shock is generated (Fig. 6).

Intensity of the shock that is capable of turning flow to a maximum possible angle β_l is expressed by Eq. 23:

$$J_l = \frac{M^2-2}{2} + \sqrt{\left(\frac{M^2-2}{2}\right)^2 + (1+2\varepsilon)(M^2-1) + 2} \quad (23)$$

By introducing Eq. 23 into Eq. 15, we get a value of flow turn terminal angle. Thus, the shock polar J_l-β_l can be built (Fig. 7).

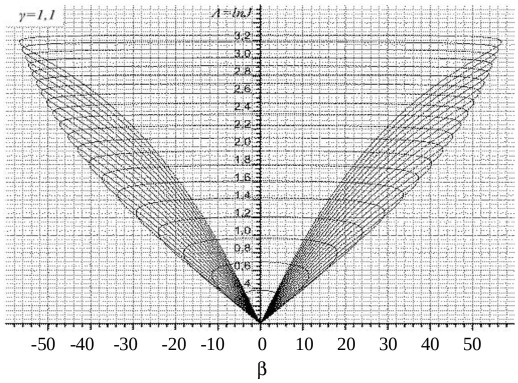

Fig. 3: Shock wave polar at γ = 1.1, Mach number ranges from 2-5 with step 0.2

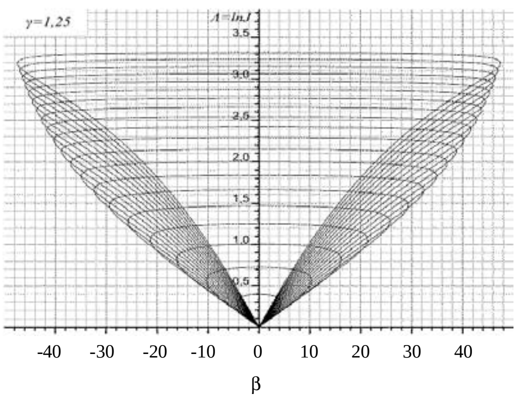

Fig. 4: Shock wave polar γ = 1.25, Mach number ranges from 2-5 with step 0.2

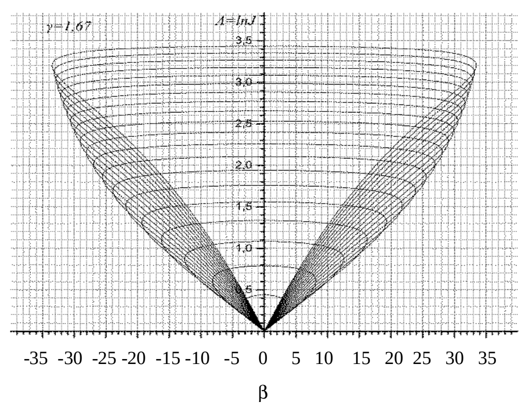

Fig. 5: Shock wave polar at γ = 1.67, Mach number ranges from 2-5 with step 0.2

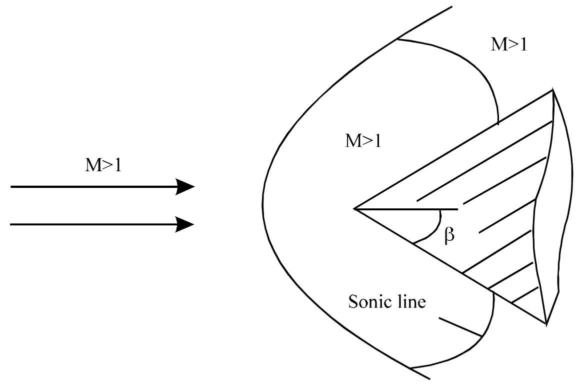

Fig. 6: Flow picture at wedge's angle larger than β

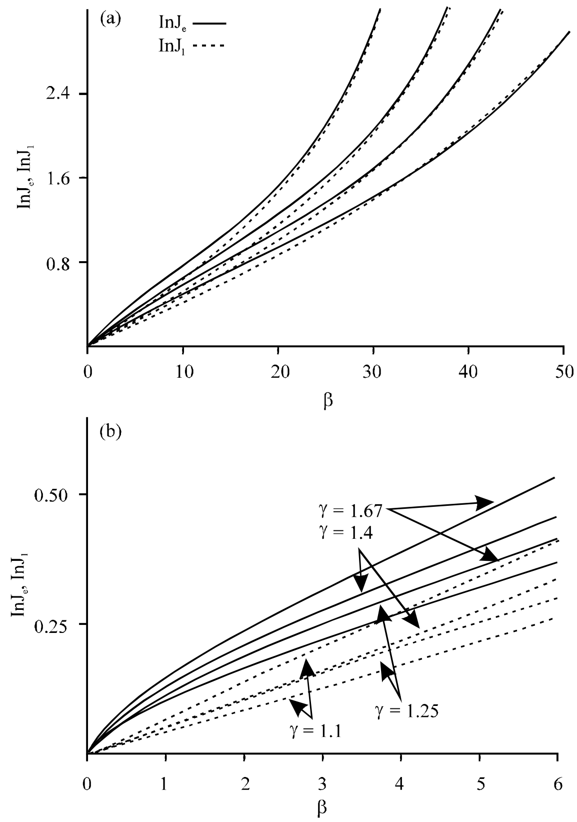

Fig. 7: Dependency for flow turn terminal angle J_l-β_l and envelope curve for a family of polars J_e-β_e

The point on heart-like curve corresponding to J_l divides a polar into two parts. A part located below that point corresponds to attached shocks and the part above corresponds to detached shocks. A terminal angle of deviation β_l increases with increases of M and at $M\to\infty$ is equal 48.58° for γ = 1.4 (Fig. 8). The slope angle of shock wave σ_l, at which the limit angle of flow deviation is reached β_l is non-monotonously dependent on Mach number.

Two shock wave polars that correspond to different Mach numbers can pass trough an arbitrary point in

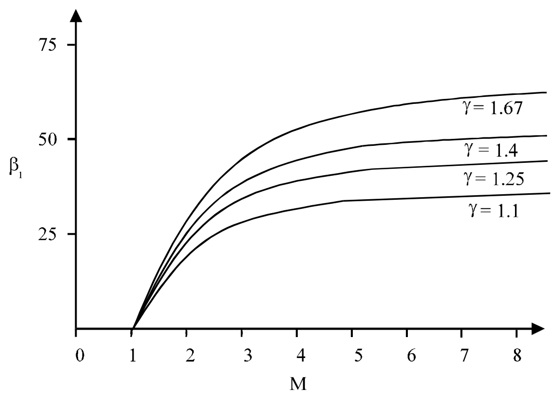

Fig. 8: Dependency of terminal angle of deviation J_1 on Mach number

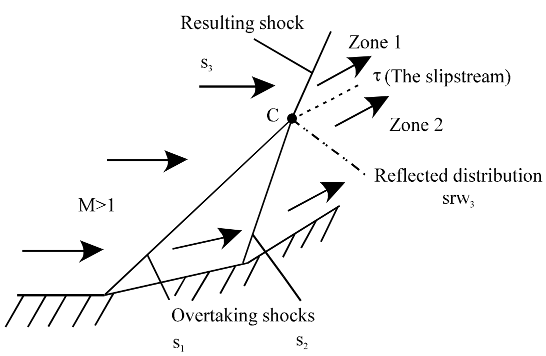

Fig. 9: An interjection of two shock (s_1 and s_2) with same direction, this results generation of a third shock s_3

coordinate plane {J; β}. This defines presence of envelope shock wave polars that limit area on plane {J; β} that is occupied by shock wave polars at $1<M<\infty$. In parametrical form the equation for envelope curves looks like this:

$$J_e = M^2 - 1 \tag{24}$$

$$tg\beta_e = \frac{M^2 - 2}{2\sqrt{\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{\gamma+1}{2}M^2 - 1\right)}} \tag{25}$$

Since, on compression shocks the J>1 then from Eq. 24 follows that shock wave polars at $M<2^{1/2}$ do not have an envelope curve. Flow's turn angle on a shock wave with intensity J_e is maximal, compared to all other shock waves of same intensity that occur at other Mach numbers. The envelope curve is shown on Fig. 7. Special points e, s, l, can be defined out on any polar, in addition the inequality $J_e<J_s<J_l$ is always true.

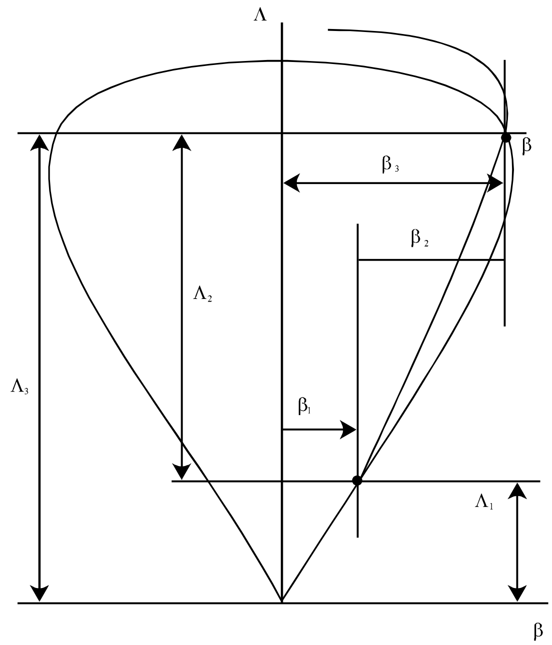

Fig. 10: A solution on plane of polars for problem about interference of two compression shock waves with same direction

By using graphs shown above, it is possible to solve graphic problem about gas-dynamic discontinuity interference. Let us demonstrate it by using example of interjection of two-compression shock wave with same direction (Fig. 9).

An interjection of two shock wave polars (Fig. 10) on plane of shock polars corresponds to this case. On the main polar that corresponds to Mach number M the point with coordinates Λ_1-β_1 is marked. From this point, a second polar shoots that was built by Mach number after the shock s_1. The polars intersect at a point 1-3, coordinates of which define intensities Λ_2, Λ_3 and flow turn angles β_2, β_3 for shocks s_2, s_3.

CONCLUSION

Universal formulas for calculating parameters after the shock wave are presented. Formulas are written by using a generalized adiabat and can be applied to simple waves and detonation waves (with use of corresponding equation for adiabat). These formulas allow calculating shock parameters if at least one parameter after the shock is known.

If flow parameters before the shock and shock's intensity are known, then these equations allow calculating all parameters after the shock. The results of calculating dependencies of the most important shock characteristic on Mach number and specific heat ration are presented in a convenient form for direct use.

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REFERENCES

Adrianov, A.L., A.L. Starykh and V.N. Uskov, 1995. Interference of Stationary Gasdynamic Discontinuities. Nauka Company, Moscow, Russia,.

Arkhipova, L.P. and V.N. Uskov, 2012. Refraction of the rarefaction riemann wave from the vertical solid wall. Her. Spbgu, 1: 62-65.

Arkhipova, L.P. and V.N. Uskov, 2013. Universal solution for problems of reflected moving waves from a solid wall and its analysis for one-dimensional shock and compression waves. Her. Spbgu, 1: 77-81.

Billington, I.I. and I.I. Glass, 1951. On the one-dimensional refraction of a rarefaction wave at a contact surface. Master Thesis, University of Toronto Institute of Aerophysics (UTIA), Ontario, Canada.

Billington, I.I., 1955. An experimental study of one-dimensional refraction of a rarefaction wave at a contact surface. MSc Thesis, University of Toronto Institute of Aerophysics (UTIA), Ontario, Canada.

Bitondo, D. and R.K. Lobb, 1950. Experiments on the amplification of a plane shock wave. Master Thesis, University of Toronto Institute of Aerophysics (UTIA), Ontario, Canada..

Bitondo, D., I.I. Glass and G.N. Patterson, 1950. One dimensional theory of absorption and amplification of a plane shock wave by a gaseous layer. Master Thesis, University of Toronto, Toronto, Ontario.

Busemann, A. and G. Dynamics, 1931. [Manual of Experimental Physics]. Academic Publishing, Leipzig, Germany, (In German).

Busemann, A., 1937. [Hodograph method of gas dynamics (In German)]. ZAMM. J. Appl. Math. Mech., 17: 73-79.

Courant, R. and K.O. Friedrichs, 1948. Supersonic Flow and Shock Waves. Inderscience Publishers Company, Geneva, Switzerland,.

Ford, C.A. and I.I. Glass, 1956. An experimental study of one-dimensional shock-wave refraction. J. Aeronaut. Sci., 23: 189-191.

Gould, D.G., 1952. The head-on collision of two shock waves and a shock and rarefaction wave in one-dimensional flow. Master Thesis, University of Toronto Institute of Aerophysics (UTIA), Ontario, Canada.

Kunchur, M.N., C. Dean, M. Liang, N.S. Moghaddam and A. Guarino et al., 2013. Depairing current density of Nd 2-x Ce x CuO 4-d superconducting films. Phys. C. Supercond., 495: 66-68.

Kunchur, M.N., C.L. Dean, N.S. Moghadam, J.M. Knight and Q.L. He et al., 2015. Current-induced depairing in the Bi2Te3-FeTe interfacial superconductor. Phys. Rev. B., Vol. 92,

Meyer, T., 1908. On two-dimensional motions in a gas flowing at a supersonic velocity. Inf. Res. Field Eng., 62: 31-67.

Molder, S., 1960. Head-on Interaction of Oblique Shock Waves. University of Toronto, Toronto, Ontario,.

Nicholl, C.I.H., 1951. The head-on collision of shock and rarefaction waves. Master Thesis, University of Toronto Institute of Aerophysics (UTIA), Ontario, Canada.

Omelchenko, A.V. and V.N. Uskov, 2002. Interference of nonstationary oblique shock waves. Tech. Phys. Lett., 28: 491-493.

Omelchenko, A.V., T. Gang and V.N. Uskov, 2002. On the Behavior of Gas-Dynamic Variable at Oblique Shock. In: Contemporary Problems of the Non-Equilibrium Gasdynamics and Thermodynamics: Collection of Articles, Uskov, V.N. (Ed.). Baltic State Technical University, Saint Petersburg, Russia, pp: 179-191.

Raad, B., N.S. Moghaddam and M. Elahinia, 2016. A numerical simulation of the effect of using porous superelastic Nitinol and stiff Titanium fixation hardware on the bone remodeling. Proceedings of the SPIE International Conference on Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring, May 13, 2016, SPIE, Bellingham, Washington, USA., pp: 98021T-98021T.

Rahmani, M., M.T. Ahmadi, N. Shayesteh, N.A. Amin and K. Rahmani et al., 2011. Current-voltage modeling of bilayer graphene nanoribbon schottky diode. Proceedings of the 2011 IEEE Regional Symposium on Micro and Nanoelectronics (RSM), September 28-30, 2011, IEEE, Kota Kinabalu, Malaysia, ISBN:978-1-61284-846-4, pp: 256-258.

Schardin, H., 1932. [Remarks on the pressure equalization process in a pipeline (In German)]. Phys. Zeits, 33: 60-64.

Seyedhosseini, S.M., M.J. Esfahani and M. Ghaffari, 2016. A novel hybrid algorithm based on a harmony search and artificial bee colony for solving a portfolio optimization problem using a mean-semi variance approach. J. Cent. S. Univ., 23: 181-188.

Silnikov, M.V., M.V. Chernyshov and V.N. Uskov, 2014. Analytical solutions for Prandtl-Meyer wave-oblique shock overtaking interaction. Acta Astronaut., 99: 175-183.

Tahmassebpour, M. and A.M. Otaghvari, 2016. Increase efficiency data processing with using an adaptable routing protocol on cloud in wireless sensor networks. J. Fundam. Appl. Sci., 8: 2434-2442.

Taub, A.H., 1947. Refraction of plane shock waves. Phys. Rev., 72: 51-60.

Uskov, V. and M. Chernyshov, 2014. The interaction of prandtl-meyer wave with the oblique shock of the same direction. J. Energy Power Eng., 8: 121-136.

Uskov, V.N. and M.V. Chernyshov, 2010. Analytical solutions for overtaking prandtl-meyer wave-oblique shock. Proceedings of the 19th International Shock Interaction Symposium, August 31-September 3, 2010, Publishers Moscow, Moscow, Russia, pp: 1-4.

Uskov, V.N. and M.V. Chernyshov, 2014. Extreme shockwave systems in problems of external supersonic aerodynamics. Thermophys. Aeromech., 21: 15-30.

Uskov, V.N. and P.S. Mostovykh, 2008. Triple configurations of traveling shock waves in inviscid gas flows. J. Appl. Mech. Tech. Phys., 49: 347-353.

Uskov, V.N., V.P. Meshkov and A.V. Omelchenko, 2002. Interaction between a shock wave and counterpropagating rarefaction wave. Her. Spbgu, 2: 99-106.

Uskov, V.N., V.P. Meshkov and A.V. Omelchenko, 2002. Interaction between a shock wave and counterpropagating rarefaction wave. Her. Spbgu, 2: 99-106.

Uskov, V.N., 1980. Shock Waves and their Interaction. Baltic State Technical University, Saint Petersburg, Russia.