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QSAGE Iterative Method For The Numerical Solution of Two-Point Fuzzy Boundary Value Problem

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Abstract: In this study, system of linear equation is solved by using iterative method which is family of Alternating Group Explicit (AGE) generated from discretization of two point Fuzzy Boundary Value Problem (FBVPs). In addition, to that the fuzzy linear system has been solved iteratively by using Gauss-Seidel (GS), Full-Sweep AGE (FSAGE), Hall-Sweep AGE (HSAGE) and Quarter-Sweep AGE (QSAGE). Then numerical experiments are carried out onto two examples to verify the effectiveness of the method. Results show that the QSAGE method is superior than the other three methods in terms of execution time, number of iterations and Hausdorff distance.

Key words: Two-stage iteration, finite difference scheme, fuzzy boundary value problems, hausdorff distance, QSAGE, Alternating Group Explicit (AGE)

INTRODUCTION

Abdullah (1991) discovered the new concept known as half-sweep iteration via. Explicit Decoupled Group (EDG) iterative method in solving two-dimensional Poisson equations. Following to this concept, further investigations have been extensively conducted in (Sulaiman *et al*., 2004; 2008; Muthuvalu and Sulaiman 2008; 2012; 2011; Dahalan *et al*., 2014; 2013; 2015) for demonstrating the capability of the half-sweep iteration. In 2000, Othman and Abdullah (2000) expanded this approach to initiate the Modified Explicit Group (MEG) method based on the quarter-sweep approach. Later, many studies have been conducted to demonstrate the capability of the quarter-sweep iteration (Sulaiman *et al*., 2004; 2009; 2010; Dahalan and Sulaiman, 2015). The QSAGE method is used as linear solver to solve fuzzy linear systems generated from the discretization of the two-point Fuzzy Boundary Value Problems (FBVPs) whereas Gauss-Seidel (GS), Full-Sweep AGE (FSAGE) and Hall-Sweep AGE (HSAGE) methods are used as control solvers.

MATERIALS AND METHODS

Second-order finite difference approximation equations: Based on the general form, lets define two point linear FBVPs as follow:

$$\begin{aligned} & x''(t)+c(t)x'(t)+d(t)x(t)=f(t),\ t\in[a,b] \\ & x(a)=\sigma \\ & x(b)=\omega \end{aligned} \qquad (1)$$

Where:

x(t)	=	A fuzzy function
f(t)	=	Continuous functions
c(t) and d(t)	=	Continuous functions on [a, b]
σ and ω	=	Fuzzy numbers

To be clear, let $\tilde{x}$ be a fuzzy subset of real numbers. It is characterized by a membership function evaluated at t, written $\tilde{x}(t)$ a number in [0, 1]. Fuzzy numbers can be identified through the membership function. The α cut of $\tilde{x}$ which α is denote as a crisp number can be write as $\tilde{x}(\alpha)$ and define $\{x \mid \tilde{x}(t)\geq\alpha\}$ for $0<\alpha\leq 1$. The interval of the α cut of fuzzy numbers will be write a $\tilde{x}(\alpha)=[\underline{x}(\alpha),\overline{x}(\alpha)]$ for all α, since they are always closed and bounded (Allahviranloo, 2002). Suppose $(\underline{x},\overline{x})$ be parametric form of fuzzy function x, now for arbitrary positive integer n subdivide the interval t = [a, b] as $a = t_0<t_1<...<t_{n-1} = b$ defined by $t_i = a+ih$ (i = 0, 1, 2, ..., n) and h = b-a/n.

To simplify the formulation of the full, half and quarter-sweep second-order finite difference approximation equations the finite grid network will be used as shown in Fig. 1. Implementations of these point

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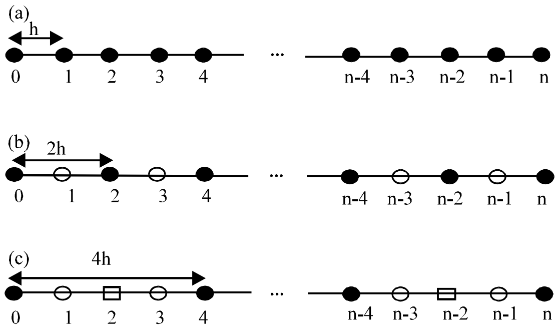

Fig. 1: a-c) Distribution of uniform solid node points for the full, half and quarter-sweep cases, respectively

iterative methods are executed onto the interior solid nodal points until convergence test is found. Meanwhile, the approximation solutions for the remaining points can be computed directly (Abdullah, 1991; Othman and Abdullah, 2000).

Denote the value of x and $(\underline{x},\overline{x})$ at the representative point t_i (i = 0, 1, 2, ..., n) by x_i and $(\underline{x}_i,\overline{x}_i)$, respectively. Thus by using the second-order full, half and quarter-sweep scheme central difference schemes, Eq. 1 can be developed as:

$$\underline{x}''_i \approx \frac{\underline{x_{i-p}}-2\underline{x_i}+\underline{x_{i+p}}}{(ph)^2} \quad (2)$$

$$\overline{x}''_i \approx \frac{\overline{x_{i-p}}-2\overline{x_i}+\overline{x_{i+p}}}{(ph)^2} \quad (3)$$

And:

$$\underline{x}'_i \approx \frac{\underline{x_{i+p}}-\underline{x_{i-p}}}{2ph} \quad (4)$$

$$\overline{x}'_i \approx \frac{\overline{x_{i+p}}-\overline{x_{i-p}}}{2ph} \quad (5)$$

Will gived:

$$x''_i = \left(\underline{x}''_i, \overline{x}''_i\right) \quad (6)$$

$$x'_i = \left(\underline{x}'_i, \overline{x}'_i\right) \quad (7)$$

respectively. By using parametric form of fuzzy functions, Eq. 1 can be written as:

$$\underline{x}''_i = \underline{f(t_i)-c(t_i)x'_i-d(t_i)x_i} \quad (8)$$

$$\overline{x}''_i = \overline{f(t_i)-c(t_i)x'_i-d(t_i)x_i} \quad (9)$$

Suppose that $c(t_i)>0$ and $d(t_i)>0$ for i = 0, 2, ..., n. Then:

$$\underline{x}''_i + c(t_i)\underline{x}'_i + d(t_i)\underline{x}_i = f(t_i) \quad (10)$$

$$\overline{x}''_i + c(t_i)\overline{x}'_i + d(t_i)\overline{x}_i = f(t_i) \quad (11)$$

By applying Eq. 2 and 4, 10 will be reduced to:

$$\frac{\underline{x_{i-p}}-2\underline{x_i}+\underline{x_{i+p}}}{(ph)^2} + c(t_i)\frac{\underline{x_{i+p}}-\underline{x_{i-p}}}{2ph} + d(t_i)\underline{x_i} = f(t_i) \quad (12)$$

For i = 1p, 2p, ..., n-2p, n-p. Meanwhile, by substituting Eq. 3 and 5 into Eq. 11 we have:

$$\frac{\overline{x_{i-p}}-2\overline{x_i}+\overline{x_{i+p}}}{(ph)^2} + c(t_i)\frac{\overline{x_{i+p}}-\overline{x_{i-p}}}{2ph} + d(t_i)\overline{x_i} = f(t_i) \quad (13)$$

Then, Eq. 12 and 13 can be rewritten as follows:

$$\left(2-phc(t_i)\right)\underline{x_{i-p}} + \left(2p^2h^2d(t_i)-4\right)\underline{x_i} + \left(2+phc(t_i)\right)\underline{x_{i+p}} = 2p^2h^2f(t_i) \quad (14)$$

$$\left(2-phc(t_i)\right)\overline{x_{i-p}} + \left(2p^2h^2d(t_i)-4\right)\overline{x_i} + \left(2+phc(t_i)\right)\overline{x_{i+p}} = 2p^2h^2f(t_i) \quad (15)$$

respectively, for i = 1p, 2p, ..., n-2p, n-p. As the value of p corresponds to 1, 2 and 4, it represets the full, half and quarter-sweep cases, respectively. Since, both of Eq. 14 have the same form in terms of the equation, except, based on the interval of the α cuts, the differences identified only in the upper and lower bound thus it can be rewritten as:

$$\rho_i x_{i-p} + \beta_i x_i + \phi_i x_{i+p} = F_i \quad (16)$$

where, ρ_i = 2-phc (t_i) and $\beta_i = 2p^2h^2$ d (t_i)-4, φ_i = 2+phc (t_i) and F_i $2p^2$ h^2 f (t_i). The approximation Eq. 16 leads a linear system in a matrix form as:

$$Ax = b \tag{17}$$

Family alternating group explicit iterative method: Let consider a class of methods mentioned by Evans (1987), Evans and Ahmad (1996) which is based on the splitting of the matrix A into the sum of two as follows:

$$A = G_1 + G_2 \tag{18}$$

Where:

$$G_1 = \left[\begin{array}{cc|cc|c|cc} g_1 & \varphi_1 & & & & & \\ \rho_2 & g_2 & & & & & \\ \hline & & g_3 & \varphi_3 & & & \\ & & \rho_4 & g_4 & & & \\ \hline & & & & \ddots & & \\ \hline & & & & & g_{n-2} & \varphi_{n-2} \\ & & & & & \rho_{n-1} & g_{n-1} \end{array}\right]$$

$$G_2 = \left[\begin{array}{c|cc|c|cc|c} g_1 & & & & & & \\ \hline & g_2 & \varphi_2 & & & & \\ & \rho_3 & g_3 & & & & \\ \hline & & & \ddots & & & \\ \hline & & & & g_{n-3} & \varphi_{n-3} & \\ & & & & \rho_{n-2} & g_{n-2} & \\ \hline & & & & & & g_{n-1} \end{array}\right]$$

If n is odd. Similarly we define the following matrices:

$$G_1 = \left[\begin{array}{cc|c|cc|c} g_1 & \varphi_1 & & & & \\ \rho_2 & g_2 & & & & \\ \hline & & \ddots & & & \\ \hline & & & g_{n-3} & \varphi_{n-3} & \\ & & & \rho_{n-2} & g_{n-2} & \\ \hline & & & & & g_{n-1} \end{array}\right]$$

$$G_2 = \left[\begin{array}{c|cc|c|cc} g_1 & & & & & \\ \hline & g_2 & \varphi_2 & & & \\ & \rho_3 & g_3 & & & \\ \hline & & & \ddots & & \\ \hline & & & & g_{n-2} & \varphi_{n-2} \\ & & & & \rho_{n-1} & g_{n-1} \end{array}\right]$$

if n is even with $g_i = \beta_i/2$ ($i = 1, 2, \ldots, n-1$). In this study, we only consider case n is even. Evidently, G_1 and G_2 satisfy the following conditions:

- $(rI+G_1)$ and $(rI+G_2)$ are non-singular for any $r>0$ where r is called the acceleration parameter
- For any vectors c and d and for any $r>0$ it is practical to solve the systems

$$(rI+G_1)y = c(rI+G_2)z = d \tag{19}$$

In explicit form since they consist of only the (2×2) subsystems. Significant here is the situation where G_1 and G_2 are either small (2×2) block systems or can be made so by a suitable permutation of their rows and corresponding columns. In terms of the work required, this procedure is more convenient because is much less than it is required to solve the original system (Eq. 10) directly. Then becomes:

$$(G_1+G_2)x = b \tag{20}$$

And by applying the AGE method, $x^{(k+1)}$ can be determined in two sweeps, i.e.:

$$(rI+G_1)x^{\left(k+\frac{1}{2}\right)} = b+(rI-G_2)x^{(k)} \tag{21}$$

And:

$$(rI+G_2)x^{(k+1)} = b+(rI-G_1)x^{\left(k+\frac{1}{2}\right)} \tag{22}$$

Thus, the explicit form of AGE method can be written as:

$$x^{\left(k+\frac{1}{2}\right)} = (rI+G_1)^{-1}\left[b+(rI-G_2)x^{(k)}\right] \tag{23}$$

And:

$$x^{(k+1)} = (rI+G_2)^{-1}\left[b+(rI-G_1)x^{\left(k+\frac{1}{2}\right)}\right] \tag{24}$$

From the iteration matrix for AGE method is:

$$T_{AGE} = (rI+G_2)^{-1}(rI-G_1)(rI+G_2)^{-1}(rI-G_2) \tag{25}$$

Also, note that the spectral radius of the AGE method T_{AGE} is <1. From therefore, the implementation of the AGE method is presented in Algorithm 1.

Algorithm 1 (FSAGE, HSAGE and QSAGE methods):

Initialize $x^{(0)} \leftarrow 0$ and $\varepsilon \leftarrow 10^{-10}$.
For i = 1p, 2p, ..., n-p, calculate
$\rho_i = 2\text{-}phc(t_i)$, $\beta_i = 2p^2h^2 d(t_i)\text{-}4$,
$\varphi_i = 2+phc(t_i)$, $F_i = 2p^2h^2 f(t_i)$

First sweep for i = 1P, 2p, ..., n-p, compute
$x^{(k+1/2)} = (rI+G_1)^{-1}[b+(rI-G_2)x^{(k)}]$
Second sweep i = 1p, 2p, ..., n-p,, for , compute
$x^{(k+1/2)} = (rI+G_2)^{-1}[b+(rI-G_1)x^{(k+1/2)}]$
Convergence test. If the convergence criterion i.e. is
$\|x^{(k+1)}-x^{(k)}\|_\infty \leq \varepsilon$ satisfied go to step 6
Otherwise go back to step (ii)
Display approximate solution

RESULTS AND DISCUSSION

Numerical experiments: The effectiveness of GS, FSAGE, HSAGE and QSAGE methods can be verify by two examples of FBVPs. Three parameters each were observed namely as number of iteration, execution time (in second) and Hausdorff distance. The value of tolerance error proposed as $\varepsilon = 10^{-10}$ was considered during the implementation (Mohsen and El-Gamel, 2008).

Problem 1:

$$x''(t) = \tilde{k}(-6t),\ t \in [0,1] \quad (26)$$

where, $\tilde{k}[\alpha] = [\underline{k}(\alpha), \overline{k}(\alpha)] = [0.75+0.25\alpha, 1.25-0.25\alpha]$ with the boundary conditions $\tilde{x}(0) = 0$ and $\tilde{x}(1) = 0$. The exact solution for:

$$\underline{x}''(t;\alpha) = \underline{k}(\alpha)(-6t) \quad (27)$$

And:

$$\overline{x}''(t;\alpha) = \overline{k}(\alpha)(-6t) \quad (28)$$

Are:

$$\underline{x}(t;a) = \underline{k}(a)\left[-t^3+2t\right] \quad (29)$$

And:

$$\overline{x}(t;a) = \overline{k}(a)\left[-t^3+2t\right] \quad (30)$$

respectively.

Problem 2 (Farajzadeh *et al.*, 2010):

$$x''(t)-4x(t) = \tilde{k}(4\cosh(1))\ t \in [0,1] \quad (31)$$

where, $\tilde{k}[\alpha] = [\underline{k}(\alpha), \overline{k}(\alpha)] = [0.75+0.25\alpha, 1.25-0.25\alpha]$ with the boundary conditions $\tilde{x}(0) = 0$ and $\tilde{x}(1) = 0$. The exact solution for:

$$\underline{x}''(t;\alpha)-4\underline{x}(t;\alpha) = \underline{k}(\alpha)(4\cosh(1)) \quad (32)$$

And:

$$\overline{x}''(t;\alpha)-4\overline{x}(t;\alpha) = \overline{k}(\alpha)(4\cosh(1)) \quad (33)$$

Are:

$$\underline{x}(t;\alpha) = \underline{k}(\alpha)\left[\cosh(2t-1)-\cosh(1)\right] \quad (34)$$

And

$$\overline{x}(t;\alpha) = \overline{k}(\alpha)\left[\cosh(2t-1)-\cosh(1)\right] \quad (35)$$

respectively. Based on these two problems all numerical results from the implementation of GS, FSAGE, HSAGE and QSAGE methods have been recorded in Table 1-5.

Table 1: Comparison of three parameters between FSGS, FSAGE, HSAGE and QSAGE methods for $\alpha = 0.00$

Number of iterations								
Problem 1				Problem 2				
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
512	681711	96747	26028	6899	475487	67638	18213	4831
1024	2431928	354438	96747	26028	1692329	247434	67638	18213
2048	8548735	1279808	354438	96747	5930853	891667	247434	67638
4096	29480437	4549671	1279808	354438	20369573	3161503	891667	247434
8192	99066551	15883620	4549671	1279808	68062962	10997813	3161503	891667
Execution time								
512	48.94	8.00	2.00	1.00	35.27	6.00	2.00	0.00
1024	211.19	39.00	9.00	2.00	155.77	27.00	6.00	1.00
2048	989.91	202.00	40.00	9.00	764.09	141.00	27.00	6.00
4096	5719.20	1310.00	213.00	40.00	4457.31	912.00	144.00	33.00
8192	32465.10	8125.00	1327.00	215.00	26063.40	5676.00	908.00	152.00
Hausdorff distance								
512	2.6560e-06	3.2355e-07	7. 9382e-08	1.9308e-08	2.4952e-06	8.3545e-07	2.4775e-06	9.6975e-06
1024	1.0624e-05	1. 3084e-06	3.2355e-07	7.9382e-08	7.7115e-06	1.0823e-06	8.3545e-07	2.4775e-06
2048	4.2497e-05	5. 2674e-06	1.3084e-06	3.2355e-07	3.0279e-05	3.7861e-06	1.0823e-06	8.3545e-07
4096	1.6999e-04	2. 1148e-05	5.2674e-06	1.3084e-06	1.2097e-04	1.5058e-05	3.7861e-06	1.0823e-06
8192	6.7995e-04	8.4787e-05	2.1148e-05	5.2674e-06	4.8386e-04	6.0327e-05	1.5058e-05	3.7861e-06

Table 2: Comparison of three parameters between FSGS, FSAGE, HSAGE and QSAGE methods for $\alpha = 0.25$

Number of iterations								
Problem 1				Problem 2				
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
512	682475	96840	26050	6905	476030	67704	18230	4835
1024	2434982	354815	96840	26050	1694502	247701	67704	18230

Table 2: Continue

Number of iterations								
Problem 1				Problem 2				
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
2048	8560953	1281323	354815	96840	5939547	892745	247701	67704
4096	29529307	4555751	1281323	354815	20404350	3165828	892745	247701
8192	99262033	15908020	4555751	1281323	68202066	11015151	3165828	892745
Execution time								
512	49.07	9.00	2.00	1.00	35.26	6.00	2.00	0.00
1024	211.36	39.00	8.00	3.00	155.79	27.00	6.00	1.00
2048	991.23	202.00	40.00	9.00	756.06	142.00	27.00	6.00
4096	5874.81	1301.00	209.00	40.00	4465.35	903.00	163.00	27.00
8192	32551.12	8164.00	1420.00	215.00	25999.98	5652.00	911.00	156.00
Hausdorff distance								
512	2.6560e-06	3.2355e-07	7.9430e-08	1.9308e-08	2.4650e-06	8.0517e-07	2.3564e-06	9.2133e-06
1024	1.0624e-05	1.3083e-06	3.2355e-07	7.9430e-08	7.7039e-06	1.0748e-06	8.0517e-07	2.3564e-06
2048	4.2497e-05	5.2675e-06	1.3083e-06	3.2355e-07	3.0277e-05	3.7841e-06	1.0748e-06	8.0517e-07
4096	1.6999e-04	2.1148e-05	5.2675e-06	1.3083e-06	1.2097e-04	1.5058e-05	3.7841e-06	1.0748e-06
8192	6.7995e-04	8.4786e-05	2.1148e-05	5.2675e-06	4.8386e-04	6.0327e-05	1.5058e-05	3.7841e-06

Table 3: Comparison of three parameters between FSGS, FSAGE, HSAGE and QSAGE methods for $\alpha = 0.50$

Number of iterations								
Problem 1				Problem 2				
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
512	683007	96905	26066	6909	476410	67751	18241	4838
1024	2437112	355076	96905	26066	1696018	247888	67751	18241
2048	8569470	1282378	355076	96905	5945607	893496	247888	67751
4096	29563373	4559989	1282378	355076	20428592	3168843	893496	247888
8192	99398298	15925021	4559989	1282378	68299033	11027246	3168843	893496
Execution time								
512	49.25	9.00	3.00	1.00	35.40	6.00	2.00	0.00
1024	210.43	39.00	9.00	2.00	155.80	27.00	6.00	1.00
2048	988.93	203.00	40.00	9.00	757.38	141.00	28.00	6.00
4096	5784.36	1311.00	212.00	39.00	4585.51	912.00	146.00	28.00
8192	32665.34	8152.00	1329.00	215.00	26078.03	5696.00	932.00	152.00
Hausdorff distance								
512	2.6560e-06	3.2355e-07	7.9423e-08	1.9294e-08	2.4346e-06	7.7486e-07	2.2353e-06	8.7292e-06
1024	1.0624e-05	1.3084e-06	3.2355e-07	7.9423e-08	7.6963e-06	1.0672e-06	7.7486e-07	2.2353e-06
2048	4.2497e-05	5.2675e-06	1.3084e-06	3.2355e-07	3.0275e-05	3.7823e-06	1.0672e-06	7.7486e-07
4096	1.6999e-04	2.1148e-05	5.2675e-06	1.3084e-06	1.2097e-04	1.5057e-05	3.7823e-06	1.0672e-06
8192	6.7995e-04	8.4785e-05	2.1148e-05	5.2675e-06	4.8386e-04	6.0327e-05	1.5057e-05	3.7823e-06

Table 4: Comparison of three parameters between FSGS, FSAGE, HSAGE and QSAGE methods for $\alpha = 0.75$

Number of iterations								
Problem 1				Problem 2				
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
512	683321	96944	26075	6912	476633	67778	18247	4839
1024	2438369	355232	96944	26075	1696912	247998	67778	18247
2048	8574499	1283001	355232	96944	5949186	893940	247998	67778
4096	29583490	4562489	1283001	355232	20442908	3170624	893940	247998
8192	99478766	15935054	4562489	1283001	68356295	11034378	3170624	893940
Execution time								
512	49.22	8.00	2.00	1.00	35.42	6.00	2.00	1.00
1024	210.33	39.00	9.00	3.00	155.72	27.00	6.00	2.00
2048	1026.58	203.00	40.00	8.00	757.27	141.00	28.00	6.00
4096	5771.53	1298.00	211.00	40.00	4364.75	914.00	145.00	28.00
8192	32617.94	8186.00	1378.00	232.00	26127.43	5706.00	912.00	151.00
Hausdorff distance								
512	2.6560e-06	3.2355e-07	7.9460e-08	1.9254e-08	2.4044e-06	7.4463e-07	2.1143e-06	8.2450e-06
1024	1.0624e-05	1.3083e-06	3.2355e-07	7.9460e-08	7.6888e-06	1.0596e-06	7.4463e-07	2.1143e-06
2048	4.2497e-05	5.2675e-06	1.3083e-06	3.2355e-07	3.0273e-05	3.7803e-06	1.0596e-06	7.4463e-07
4096	1.6999e-04	2.1148e-05	5.2675e-06	1.3083e-06	1.2097e-04	1.5057e-05	3.7803e-06	1.0596e-06
8192	6.7995e-04	8.4786e-05	2.1148e-05	5.2675e-06	4.8386e-04	6.0327e-05	1.5057e-05	3.7803e-06

Table 5: Comparison of three parameters between FSGS, FSAGE, HSAGE and QSAGE methods for $\alpha = 1.00$

	Problem 1				Problem 2			
n	FSGS	FSAGE	HSAGE	QSAGE	FSGS	FSAGE	HSAGE	QSAGE
Number of iterations								
512	683426	96956	26078	6912	476706	67786	18250	4840
1024	2438784	355282	96956	26078	1697208	248034	67786	18250
2048	8576162	1283208	355282	96956	5950370	894086	248034	67786
4096	29590144	4563320	1283208	355282	20447642	3171216	894086	248034
8192	99505380	15938400	4563320	1283208	68375230	11036748	3171216	894086
Execution time								
512	49.45	9.00	2.00	0.00	35.43	6.00	1.00	0.00
1024	210.66	39.00	9.00	2.00	155.72	27.00	6.00	2.00
2048	809.53	202.00	39.00	9.00	755.20	141.00	28.00	6.00
4096	5758.67	1313.00	209.00	40.00	4615.31	915.00	145.00	32.00
8192	32519.13	8221.00	1478.00	221.00	25815.45	5662.00	914.00	155.00
Hausdorff distance								
512	2.6559e-06	3.2354e-07	7.9440e-08	1.9280e-08	2.3742e-06	7.1441e-07	1.9933e-06	7.7608e-06
1024	1.0624e-05	1.3084e-06	3.2354e-07	7.9440e-08	7.6812e-06	1.0521e-06	7.1441e-07	1.9933e-06
2048	4.2497e-05	5.2674e-06	1.3084e-06	3.2354e-07	3.0271e-05	3.7785e-06	1.0521e-06	7.1441e-07
4096	1.6999e-04	2.1148e-05	5.2674e-06	1.3084e-06	1.2097e-04	1.5056e-05	3.7785e-06	1.0521e-06
8192	6.7995e-04	8.4783e-05	2.1148e-05	5.2674e-06	4.8386e-04	6.0326e-05	1.5056e-05	3.7785e-06

CONCLUSION

Here, the family of AGE iterative method was used to solve linear systems arise from the discretization of two-point FBVPs using the second-order finite difference scheme. The results show that QSAGE method is more superio in terms of execution time, the number of iterations and Hausdorff distance compared to the GS, FSAGE and HSAGE methods. Since, AGE is well suited for parallel computation it can be considered as a main advantage because this method has groups of independent task which can be implemented simultaneously. It is hoped that the capability of the proposed method will be helpful for the further investigation in solving any multi-dimensional fuzzy partial differential equations (Farajzadeh *et al*., 2010). Also the family of AGE methods such as as Modified Alternating Group Explicit (MAGE) (Evans and Yousif, 1988; Yousif and Evans, 1987) and Two Parameter Alternating Group Explicit (TAGE) (Sukon, 1996; Mohanty *et al*., 2003) methods can be used as linear solvers in solving the same problem.

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