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Age Iterative Method Applied to 2D Fuzzy Poisson Equation (Application Method of Age Placement Against Poisson 2D Blur Equation)

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Abstract: In this study, iterative methods particularly the Alternating Group Explicit (AGE) iterative method is used to solve system of linear equations generated from the discretization of Two-Dimensional Fuzzy Poisson problems (2DFP). The formulation and implementation of the AGE method is also presented. Then numerical experiments are carried out on to two problems to verify the effectiveness of the methods. The results show that the AGE method is superior compared to GS method in terms of number of iterations, execution time and Hausdorff distance.

Key words: Two-stage iteration, finite difference scheme, fuzzy boundary value problems, complexity reduction approach, execution time formulation, AGE

INTRODUCTION

Fuzzy Boundary Value Problems (FBVPs) can be approached by two types. The first approach, addresses problems in which the boundary values are fuzzy where the solution is still in fuzzy function. Then, the second approach is based on generating the fuzzy solution from the crisp solution (Gasilov *et al*., 2011). Numerical methods well be used in obtain their approximate solution to solve these problems. For clarity, consider the two Dimensional Fuzzy Poisson Equation (2DFPE) as:

$$\begin{aligned}&\frac{\partial^2\tilde{U}}{\partial x^2}+\frac{\partial^2\tilde{U}}{\partial y^2}=\tilde{f}(x,y)\\&\left.\begin{aligned}\tilde{U}(x,0)=\tilde{g}_1(x)\\\tilde{U}(x,a)=\tilde{g}_2(x)\end{aligned}\right\}\quad 0\le x\le a,\\&\left.\begin{aligned}\tilde{U}(0,y)=\tilde{g}_3(x)\\\tilde{U}(a,y)=\tilde{g}_4(x)\end{aligned}\right\}\quad 0\le y\le a,\end{aligned}\qquad(1)$$

where $\tilde{f}(x,y)$, $\tilde{g}_1(x)$, $\tilde{g}_2(x)$, $\tilde{g}_3(y)$ and $\tilde{g}_4(y)$ were fuzzy numbers or fuzzy functions.

Second-order, central finite difference scheme will be applied to discretize the 2DFPE (Eq. 1) into linear systems, numerically based on the Seikkala derivative (Seikkala, 1987). In this study, the generated linear systems will be solved iteratively by using AGE method (Evans, 1987; Evans and Ahmad, 1996). Indeed, AGE method is also analogous to Alternate Direction Implicit (ADI) scheme which has been used extensively in solving large scale computations. From previous studies, findings of the studys related to the AGE iterative method and its variants (Evans and Yousif, 1988; (Golbabai and Arabshahi, 2010; Mohanty *et al*., 2003; Mohanty and Talwar, 2012; Sukon, 1996; Yousif and Evans, 1987) have shown that the efficiency of the family of AGE methods has been widely used to solve the non-fuzzy problems. Due to the efficiency of the methods, this study extends the application of AGE iterative method in solving fuzzy problems. Since, the fuzzy linear systems will be constructed, the iterative method becomes the natural option to get a fuzzy numerical solution of the problem.

Finite difference approximation equations: Consequently, let $\tilde{x}$ and $\tilde{y}$ be two fuzzy subset of real number. They are characterized by a membership function evaluated at t, written $\tilde{x}(t)$ and $\tilde{y}(t)$, respectively as a number in (0, 1). Fuzzy numbers can be identified via. the membership function. Th α- cut of $\tilde{x}$ and $\tilde{y}$ which α is denote as a crisp number can be written as $\tilde{x}(\alpha)$ and $\tilde{y}(\alpha)$ in $\{x\,|\,\tilde{x}(t)\ge\alpha\}$ and $\{y\,|\,\tilde{y}(t)\ge\alpha\}$ for $0<\alpha\le 1$. Since, they are always closed and bounded interval, the α cut of fuzzy numbers can be written as $\tilde{x}(\alpha)=[\underline{x}(\alpha),\overline{x}(\alpha)]$ and $\tilde{y}(\alpha)=[\underline{y}(\alpha),\overline{y}(\alpha)]$ for all α [Eq. 2]. Suppose $(\underline{x},\overline{x})$ and $(\underline{y},\overline{y})$ be parametric

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form of fuzzy function x and y, respectively, now for arbitrary positive integer n and m subdivided the interval $a \le t \le b$ whereas $x_i = a+ih$ (i = 0, 1, 2, ..., n) and $y_j = a+jl$ (i = 0, 1, 2, ..., m) for i and j, respectively and define the step size h and l by h = b-a/n and h = b-a/m.

Denote the value of x and y as $(\underline{x},\overline{x})$ and $(\underline{y},\overline{y})$ at the representative point t_i(i = 0, 1, 2, ..., n) and t_j (i = 0, 1, 2, ..., m) by x_i and y_j at $(\underline{x},\overline{x})$ and $(\underline{y},\overline{y})$, respectively. Then by using second-order central finite difference scheme Eq. 1, can be developed as:

$$\left.\frac{\partial^2 \underline{U}}{\partial x^2}\right|_{i,j} = \frac{\underline{U}_{i-1,j} - 2\underline{U}_{i,j} + \underline{U}_{i+1,j}}{h^2} \tag{2}$$

$$\left.\frac{\partial^2 \overline{U}}{\partial x^2}\right|_{i,j} = \frac{\overline{U}_{i-1,j} - 2\overline{U}_{i,j} + \overline{U}_{i+1,j}}{h^2} \tag{3}$$

And:

$$\left.\frac{\partial^2 \underline{U}}{\partial y^2}\right|_{i,j} = \frac{\underline{U}_{i,j-1} - 2\underline{U}_{i,j} + \underline{U}_{i,j+1}}{h^2} \tag{4}$$

$$\left.\frac{\partial^2 \overline{U}}{\partial y^2}\right|_{i,j} = \frac{\overline{U}_{i,j-1} - 2\overline{U}_{i,j} + \overline{U}_{i,j+1}}{h^2} \tag{5}$$

By using Eq. 2 and 3 lower boundary for Eq. 1 will be reduced to:

$$\frac{\underline{U}_{i-1,j} - 2\underline{U}_{i,j} + \underline{U}_{i+1,j}}{h^2} + \frac{\underline{U}_{i,j-1} - 2\underline{U}_{i,j} + \underline{U}_{i,j+1}}{h^2} = \underline{f}_{i,j} \tag{6}$$

Then, applying Eq. 4 and 5 into upper boundary for Eq. 1, it can be shown:

$$\frac{\overline{U}_{i-1,j} - 2\overline{U}_{i,j} + \overline{U}_{i+1,j}}{h^2} + \frac{\overline{U}_{i,j-1} - 2\overline{U}_{i,j} + \overline{U}_{i,j+1}}{h^2} = \overline{f}_{i,j} \tag{7}$$

For i = 1, 2, ..., n-1 and j = 1, 2, ..., m-1. Since, both Eq. 6 and 7 have the same form in terms of equation, except, based on the interval of the α-cuts, the differences identified only in the upper and lower boundary thus, it can be rewritten as:

$$\frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{h^2} + \frac{U_{i,j-1} - 2U_{i,j} + U_{i,j-1}}{h^2} = f_{i,j} \tag{8}$$

Equation 8 can be represented in matrix form as follows:

$$A\tilde{U} = \tilde{F} \tag{9}$$

MATERIALS AND METHODS

Alternating group explicit iterative method: Consider class of methods mentioned by Evans (1997) which is based on the splitting of the matrix A into the sum of its constituent symmetric and positive definite matrices as follows:

$$A = G_1 + G_2 + G_3 + G_4 \tag{10}$$

Where:

G_1 and G_2 = The forward and backward differences in the x-plane
G_3 and G_4 = The similar difference in y-plane

In which $\text{diag}(G_1) = \text{diag}(G_2) = 1/4\ \text{diag}(A)$. By reordering the points column-wise along y-direction, $G_3$and G_4 literally have the same structure as G_1 and G_2, respectively. Then, Eq. 10 becomes:

$$\left(G_1 + G_2 + G_3 + G_4\right)\underset{\sim}{U} = \underset{\sim}{F} \tag{11}$$

Thus, the explicit form of AGE method can be written as:

$$\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)} = \left(r_1 I + G_1\right)^{-1}\left[2\underset{\sim}{f} + \left(r_1 I + G_1 - 2A\right)\right] \tag{12}$$

$$\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)} = \left(r_1 I + G_2\right)^{-1}\left[G_2\,\underset{\sim}{U}^{(k)} + r_1\,\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}\right] \tag{13}$$

$$\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)} = \left(r_2 I + G_3\right)^{-1}\left[G_3\,\underset{\sim}{U}^{(k)} + r_2\,\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}\right] \tag{14}$$

And:

$$\underset{\sim}{U}^{(k+1)} = \left(r_2 I + G_4\right)^{-1}\left[G_4\,\underset{\sim}{U}^{(k)} + r_2\,\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}\right] \tag{15}$$

From Eq. 12-15, therefore, the implementation of the families of AGE methods is presented in Algorithm 1.

Algorithm 1 (Families of AGE methods):

i. Initialize $U^{(0)} \leftarrow 0$ and and $\varepsilon \leftarrow 10^{-10}$
ii. First sweep compute

$$\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)} = \left(r_1 I + G_1\right)^{-1}\left[2\underset{\sim}{f} + \left(r_1 I + G_1 - 2A\right)\right]$$

iii. Second sweep compute

$$\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)} = \left(r_1 I + G_2\right)^{-1}\left[G_2\,\underset{\sim}{U}^{(k)} + r_1\,\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}\right]$$

iv. Third sweep computer

$$\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)} = \left(r_2 I + G_3\right)^{-1}\left[G_3\,\underset{\sim}{U}^{(k)} + r_2\,\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}\right]$$

v. Fourth sweep compute

$$\underset{\sim}{U}^{(k+1)} = (r_2 I + G_4)^{-1}\left[G_4 \underset{\sim}{U}^{(k)} + r_2 \underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}\right]$$

vi. Convergence test. If the convergence criterion, i.e., $\left\|\underset{\sim}{U}^{(k+1)} - \underset{\sim}{U}^{(k)}\right\|_\infty \le \varepsilon$ is satisfied go to Step (vii). Otherwise go back to Step (ii).
Vii. Display approximate solutions.

RESULTS AND DISCUSSION

Numerical experiments: Two problems of 2DFPE are considered to verify the effectiveness of AGE iterative method via. the corresponding second-order central finite difference approximation equation. During implementation the proposed iterative methods, the value of the tolerance error, considered, $\varepsilon = 10^{-10}$.

Problem 1 (Allahviranloo, 2002):

$$\frac{\partial^2 \tilde{U}}{\partial x^2}(x,y) + \frac{\partial^2 \tilde{U}}{\partial y^2}(x,y) = \tilde{k} x e^y, \qquad 0 \le x \le 2,\ 0 \le y \le 1 \tag{16}$$

where, $\tilde{k}[\alpha] = \left[\underline{k}(\alpha), \overline{k}(\alpha)\right] = [0.75+0.25\alpha, 1.25-0.25\alpha]$ with the boundary conditions $\tilde{U}(0,y) = 0,\ \tilde{U}(2,y) = 2\tilde{k}e^y,\ 0 \le y \le 1$ and $\tilde{U}(x,0) = \tilde{k}x,\ \tilde{U}(x,1) = \tilde{k}ex,\ 0 \le x \le 2$. The exact solution for:

$$\frac{\partial^2 \underline{U}}{\partial x^2}(x,y;\alpha) + \frac{\partial^2 \underline{U}}{\partial y^2}(x,y;\alpha) = \underline{k}(\alpha) x e^y \tag{17}$$

And:

$$\frac{\partial^2 \overline{U}}{\partial x^2}(x,y;\alpha) + \frac{\partial^2 \overline{U}}{\partial y^2}(x,y;\alpha) = \overline{k}(\alpha) x e^y \tag{18}$$

Are:

$$\underline{U}(x,y;\alpha) = \underline{k}(\alpha) x e^y \tag{19}$$

And:

$$\overline{U}(x,y;\alpha) = \overline{k}(\alpha) x e^y \tag{20}$$

respectively.

Problem 2 (Abdullah, 1991):

$$\frac{\partial^2 \tilde{U}}{\partial x^2}(x,y) + \frac{\partial^2 \tilde{U}}{\partial y^2}(x,y) = \tilde{k}(x^2+y^2)e^{(xy)}, \quad 0 < x < 2,\ 0 < y < 1 \tag{21}$$

where, $\tilde{k}[\alpha] = \left[\underline{k}(\alpha), \overline{k}(\alpha)\right] = [0.75+0.25\alpha, 1.25-0.25\alpha]$ with the boundary conditions $\tilde{U}(0,y) = 0,\ \tilde{U}(2,y) = 2\tilde{k}e^y,\ 0 \le y \le 1$ and $\tilde{U}(x,0) = \tilde{k}x,\ \tilde{U}(x,1) = \tilde{k}ex,\ 0 \le x \le 2$. The exact solution for:

$$\frac{\partial^2 \underline{U}}{\partial x^2}(x,y;\alpha) + \frac{\partial^2 \underline{U}}{\partial y^2}(x,y;\alpha) = \underline{k}(\alpha)(x^2+y^2)e^{(xy)} \tag{22}$$

And:

$$\frac{\partial^2 \overline{U}}{\partial x^2}(x,y;\alpha) + \frac{\partial^2 \overline{U}}{\partial y^2}(x,y;\alpha) = \overline{k}(\alpha)(x^2+y^2)e^{(xy)} \tag{23}$$

Are:

$$\underline{U}(x,y;\alpha) = \underline{k}(\alpha) e^{(xy)} \tag{24}$$

And:

$$\overline{U}(x,y;\alpha) = \overline{k}(\alpha) e^{(xy)} \tag{25}$$

respectively based on these two problems, numerical results for GS and AGE methods have been recorded in Table 1-5. For the purpose of observations on the feasibility of the proposed methods, three parameters were observed such as number of iterations, execution time (in seconds) and Hausdorff distance (as mention in definition 1).

Table 1: Comparison of three parameters between GS and AGE methods at $\alpha = 0.00$

	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
Number of iterations				
32	3883	1108	3965	1129
64	14366	4100	14708	4190
128	52818	15150	54220	15521
256	192760	55682	198438	57183
512	697178	203164	720036	209210
Execution time				
32	0.49000	0.29000	0.52000	0.31000
64	2.77000	1.70000	2.79000	1.74000
128	29.4000	20.1400	30.4900	20.6700
256	370.330	285.280	381.060	292.670
512	5686.37	5462.46	6430.78	5682.34
Hausdorf distance				
32	6.8381e-06	6.8446e-06	3.8277e-06	3.8320e-06
64	1.6773e-06	1.7042e-06	9.3734e-07	9.5464e-07
128	2.9278e-07	3.9027e-07	1.6698e-07	2.1685e-07
256	6.0545e-07	1.1845e-07	6.3925e-07	1.5156e-07
512	2.6413e-06	6.8798e-07	2.6498e-06	6.9641e-07

Table 2: Comparison of three parameters between GS and AGE methods at $\alpha = 0.25$

	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
Number of iterations				
32	3886	1108	3967	1130
64	14379	4104	14720	4194
128	52866	15163	54268	15533
256	192951	55733	198629	57234
512	697942	203367	720800	209411
Execution time				
32	0.53000	0.31000	0.50000	0.30000
64	3.76000	1.68000	3.66000	1.72000
128	30.2900	20.1900	30.0700	20.6100
256	371.680	285.040	381.730	293.580
512	5642.22	5468.61	6733.05	5700.13
Hausdorf distance				
32	6.4958e-06	6.5023e-06	3.6360e-06	3.6403e-06
64	1.5916e-06	1.6185e-06	8.8939e-07	9.0663e-07
128	2.7217e-07	3.6893e-07	1.5594e-07	2.0491e-07
256	6.0056e-07	1.1383e-07	6.3716e-07	1.4960e-07
512	2.6401e-06	6.8671e-07	2.6492e-06	6.9595e-07

Table 3: Comparison of three parameters between GS and AGE methods at $\alpha = 0.50$

No. of iterations				
	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
32	3888	1109	3970	1131
64	14387	4106	14728	4196
128	52899	15172	54301	15542
256	193084	55768	198762	57269
512	698475	203507	721333	209552
Execution time				
32	0.49000	0.29000	0.56000	0.30000
64	2.64000	1.66000	2.86000	1.71000
128	29.9800	20.0900	30.1600	20.6700
256	370.700	286.240	382.090	293.380
512	5540.28	5467.58	6310.45	5695.10
Hausdorf distance				
32	6.1534e-06	6.1599e-06	3.4444e-06	3.4486e-06
64	1.5059e-06	1.5328e-06	8.4140e-07	8.5858e-07
128	2.5162e-07	3.4751e-07	1.4490e-07	1.9298e-07
256	5.9573e-07	1.0932e-07	6.3513e-07	1.4771e-07
512	2.6388e-06	6.8557e-07	2.6486e-06	6.9541e-07

Table 4: Comparison of three parameters between GS and AGE methods at $\alpha = 0.75$

No. of iterations				
	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
32	3889	1109	3971	1131
64	14392	4107	14734	4197
128	52920	15177	54320	15547
256	193163	55788	198841	57290
512	698789	203590	721646	209635
Execution time				
32	0.54000	0.29000	0.56000	0.34000
64	2.86000	1.65000	2.76000	1.73000
128	29.9800	20.0300	30.4100	20.6800
256	370.910	285.900	381.160	293.400
512	5577.81	5463.76	6600.82	5699.42
Hausdorf distance				
32	5.8111e-06	5.8176e-06	3.2527e-06	3.2569e-06
64	1.4202e-06	1.4471e-06	7.9347e-07	8.1049e-07
128	2.3132e-07	3.2613e-07	1.3402e-07	1.8107e-07
256	5.9092e-07	1.0483e-07	6.3310e-07	1.4583e-07
512	2.6376e-06	6.8433e-07	2.6482e-06	6.9488e-07

Table 5: Comparison of three parameters between GS and AGE methods at $\alpha = 1.00$

No. of iterations				
	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
32	3890	1110	3972	1130
64	14394	4108	14736	4198
128	52926	15178	54328	15550
256	193188	55796	198866	57296
512	698892	203618	721750	209662
Execution time				
32	0.53000	0.29000	0.56000	0.310000
64	2.79000	1.68000	2.95000	1.75000
128	30.1000	20.0700	30.7400	20.7400
256	370.350	285.630	380.580	293.320
512	5568.18	5472.62	5818.36	5694.80

Table 5: Continue

No. of iterations				
	Problem 1		Problem 2	
n	GS	AGE	GS	AGE
Hausdorf distance				
32	5.4688e-06	5.4753e-06	3.0611e-06	3.0653e-06
64	1.3345e-06	1.3615e-06	7.4552e-07	7.6248e-07
128	2.1107e-07	3.0471e-07	1.2322e-07	1.6919e-07
256	5.8616e-07	1.0031e-07	6.3112e-07	1.4399e-07
512	2.6364e-06	6.8306e-07	2.6477e-06	6.9442e-07

Definition 1 (Nutanong *et al.*, 2011): Given two minimum bounding rectangles P and Q a lower bound of the Hausdorff distance from the elements confined by P to the elements confined by Q is defined as:

$$\text{HausDistLB}(P,Q) = \text{Max}\left\{\text{MinDist}(f_\alpha, Q) : f_\alpha \in \text{FacesOf}(P)\right\}$$

CONCLUSION

In this study, the AGE method is used to solve linear systems arises from the discretization of two-point FBVPs using the second-order central finite difference scheme. The results showed that AGE method is more superior in terms of the number of iterations, execution time and Hausdorff distance compared to the GS method. Since, AGE is also known as the two stage iterative method which is suitable for parallel computation in solving the associated matrix equation it can be considered as a main advantage because this method has groups of independent task which can be implemented simultaneously. It is hoped, that the capability of the proposed method will be helpful for the further investigation in solving any multi-dimensional fuzzy partial differential equations (Farajzadeh *et al*., 2010). Also, other family of AGE methods can be used as linear solvers in solving the same problem. Basically the results of this study, can be classified as one of full-sweep iteration. Apart from the concept of the full-sweep iteration, further investigation of half-sweep (Abdullah, 1991; Dahalan *et al*., 2013; Dahalan *et al*., 2014; Dahalan *et al*., 2015; Muthuvalu and Sulaiman, 2008; Sulaiman *et al*., 2004) and quarter-sweep (Dahalan and Sulaiman, 2015) (Mohanthy *et al*., 2013) (Muthuvalu and Sulaiman, 2011; Othman and Abdullah, 2000; Sulaiman *et al*., 2009) iterations can also be considered in order to speed up the convergence rate of the standard proposed iterative methods.

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