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The Evaluation of the Confidence Intervals for the State Parameters of a DC Power System

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Abstract: State estimation in power engineering is used as a tool to find the unknown parameter values from the hypothesized model by utilizing the specified information available about the system. Due to random noises that are added from different sources, the exact value of the state vector cannot be found. This study is an effort to describe the simultaneous and individual confidence intervals for the state parameters in view of the heteroscedastic structure of the error terms. The performance of the constructed intervals in terms of coverage probability has been evaluated by using the Monte Carlo simulation study. The results of the study demonstrate that it is an effective method for practical implementation in the state estimation a power system.

Key words: DC load flow model, F-distribution, interval linear estimation, Student t-distribution, state estimation, weighted least squares

INTRODUCTION

The theoretical soundness of the model used in the power system state estimation is based on statistical grounds. Measurements that are used in the state estimation are polluted by noises which are added from different sources. Even with the measuring instrument, regardless of how carefully designed and well maintained it is, noise always exits in its measurements (Montgomery, 2007). In general, the model used in the power system estimation is based on deterministic and stochastic parts.In the statistical aspect, these estimated values of the state vector are called the point estimates which usually come from the maximum likelihood method of estimation. Point estimation provides a single value as an estimate of the true parameter whereby interval estimation means, the interval which is likely to cover the true parameter (Hahn and Meeker, 2011). The values of the estimated vector can be found by other method of estimation like the ordinary Least Squares (short, LS) method without making any normality assumption of the error terms because this method does not require any assumption about the probability distribution of the error terms. In short, the point estimation of state parameters can be achieved without a normality assumption. But for interval estimation, we have to make an assumption of normality for random noises in order to draw the inference about the unknown parameter. In addition to this, assumptions regarding the parameters of the assumed distribution have also played a significant role in the estimation procedure. The reliability of interval and point estimates is affected by these parametric assumptions. For example, relaxing the assumption of the same common variance or autocorrelation for all disturbance terms will affect the desirable statistical properties of the least square estimator (Montgomery *et al*., 2006).

In the presence of the random errors only in the measurements, the state estimator provides a reliable estimate for the unknown state vector while the inaccuracy can be resulted in the presence of other types of gross or topological errors. That is why, bad data detection and identification are also very important areas of the interest in most of the research (Lin and Pan, 2007; Dnguez *et al*., 2008).

In this effort, we have been extended the application of statistical inference to power system engineering. Before a similar attempt was made by Kyriakides and Heydt (2006) but the validity of the found results relied on parametric assumptions that were made about the error terms of the model.

For the implementation of the derived results in this article, an application has been taken from the power system state estimation problem. The power system state estimation is a fundamental method for on-line power

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system monitoring, analysis, control functions and has been an integral part of the energy management system since the innovative research of Schweppe and Wildes (1970), Schweppe and Rom (1970) and Schweppe (1970). It is a technique of reading field measurements and developing the best estimate of the state of a power system (Baalbergen *et al*., 2009). In the statistical context, the state estimation of a power system is like the multiple regression problem where the parameters to be estimated are the bus voltage magnitudes and the phase angles (Caro *et al*., 2013). The forgoing work is therefore in conjunction with the effort carried out by Kyriakides and Heydt (2006) under the generalized error structure model. General expressions for the confidence interval are described and applied in this study where the results in the referred research can be deduced as a particular case.

The major contribution of this research is an implementation of the inference procedure in the conventionally static state estimation procedures for a power system. The state of a power system is actually not static and is changed at every instant due to a change in the random load and other disturbances. The statistical estimation theory can, therefore, be implemented to characterize the random behavior of the system parameters. This research also highlights the importance of the assumptions that have been taken about the error terms in the hypothesized model.

MATERIALS AND METHODS

Model and estimation method: In the model form, the state estimation problem can be written as:

$$Z = h(X)+\varepsilon = HX+\varepsilon \quad (1)$$

Where:

$H = \partial h(X)/\partial X$ = The Jacobian matrix of the order $m\times n$
$Z = [z_1, z_2,..., z_m]^T$ = The vector of measurements
$h(X) = [h_1(X), h_2(X),..., h_3(X)]'$ = Measurements function vector
$X = [x_1, x_2,..., x_n]^T$ = A state vector
$\varepsilon = [\varepsilon_1, \varepsilon_2,..., \varepsilon_m]^T$ = The vector of random errors
m = Number of measurements
n = Number of the parameters that are to be estimated assumptions
$E(\varepsilon) = 0$ = Mean vector
$\Omega = E(\varepsilon\varepsilon^T)$ = Diag var-cov matrix of random errors with:

$$E(\varepsilon_i\varepsilon_j) = \begin{cases} \sigma_i^2; & \text{for all } \quad i = j \\ 0\ ; & \text{for all } \quad i \neq j, i, j = 1,2, ..., m \end{cases}$$

where, σ_i^2; i = 1, 2, ..., m is associated with the accuracy of the ith measuring device. The distribution is multivariate normal, i.e., $\varepsilon \sim N(0, \Omega)$. Confidence intervals for the unknown parameters in the nonlinear regression model are approximated and based on the results in the asymptotic theory (Gadzhiev, 1995; Bukac, 2008). In this case, the estimated parameters do not have the desirable properties (Huang *et al*., 2010). The emphasis here is not to elaborate between linear and nonlinear estimation procedures at this stage. But for the interest readers, details on the nonlinear estimation procedures and their applications in the context of probabilistic models can be found by Ross (1990), Bates and Watts (2007).

The assumption of equal variances which has been taken by Kyriakides and Heydt (2006), seems impractical in the interval estimation of the state parameters of a power system. Therefore, we have used the Model (Eq. 1) in the construction of individual and simultaneous confidence intervals with the assumptions as listed above. Under these assumptions, the most recommended method for the estimation is the use of the Weighted Least Square (WLS) which is a sub class of the Generalized Least Square method (GLS) (Hayes and Cai, 2007). By assuming the model given in Eq. 1, the solution is obtained by minimizing the following objective function:

$$\Phi(X) = \min\left(Z-\hat{Z}\right)^T \times W\left(Z-\hat{Z}\right) \quad (2)$$

Where:

$$W = \begin{bmatrix} \frac{1}{\sigma_1^2} & \cdots & 0 \\ \vdots & \frac{1}{\sigma_i^2} & \vdots \\ 0 & \cdots & \frac{1}{\sigma_m^2} \end{bmatrix} = \Omega^{-1} \text{ is the weighted matrix.}$$

Construction of confidence interval: In this study, we have developed the individual Confidence Intervals (CI) for the state unknown parameters and also extended the same to the simultaneous construction of the confidence region with the probability coverage 100 (1-α)%.

Estimation of the individual interval: In correspondence to Eq. 1, consider the following the transformed form of Eq. 3:

$$Z_t = H_t X + \varepsilon_t \quad (3)$$

Where:

$$Z_t = \Omega^{-\frac{1}{2}} Z,\ H_t = \Omega^{-\frac{1}{2}} H,\ \varepsilon_t = \Omega^{-\frac{1}{2}}\varepsilon,\ \varepsilon_t = N(0,I)$$

Now, the error variances are the same in Eq. 3. Hence, by simply using the LS method, the unknown state vector can be obtained from Eq. 3 and is given by:

$$\hat{X}_t = \left(H_t^T H\right)^{-1} H_t^T Z_t \quad (4)$$

Now, Eq. 5 can further be written as:

$$\hat{X}_t\text{-}X=\left(H_t^T H_t\right)^{-1} H_t^T \varepsilon_t \quad (5)$$

this yield:

$$E\left(\hat{X}\right)=X \text{ and } \Sigma\left(\hat{X}\right)=\left(H_t^T H_t\right)^{-1} \quad (6)$$

from Eq. 5 and 6, it is deduced that:

$$\hat{X}_t = N\left(X, \Sigma\left(\hat{X}\right)\right)$$

Since, $E(\hat{Z}_t) \neq Z$ so let us define a new estimator unbiased $\hat{Z}_w = H\hat{X}_t$ such that $E(\hat{Z}_w) = Z$. Now, let $\hat{\varepsilon}_t = Z_t - \hat{Z}_t$ be the weighted residuals from the model given in Eq. 4, then:

$$\hat{\varepsilon}_t = W^{1/2}\varepsilon_a \quad (7)$$

Where:

$$\varepsilon_a = Z - \hat{Z}_w \text{ or } \varepsilon_a = \left(I - H^*\right)\varepsilon \text{ with } H^* = H\left(H^T W H\right)^{-1} H^T W$$

From Eq. 7, it is implied that:

$$E\left(\hat{\varepsilon}_t\right)=0,\ V\left(\hat{\varepsilon}_t\right)=W^{1/2}\left(I-H^*\right)W^{-1/2}$$

Hence,

$$\hat{\varepsilon}_t : N\left(0, V\left(\hat{\varepsilon}_t\right)\right)$$

The sum of squares of the weighted residuals is now given by:

$$SSE_t = \hat{\varepsilon}_t^T \hat{\varepsilon}_t$$

This follows the χ^2 distribution with v = m-n degree of freedom. Hence, the ratio: $t_i = \frac{\hat{x}_{ti} - x_i}{\hat{\sigma}_t\sqrt{\omega_{ii}}}, i = 1, 2, ..., m$ follows t-distribution with v d.f. Where, $\hat{\sigma}_t = \sqrt{\frac{SSE_t}{v}}$ and w_{ii} is the ith diagonal element of the variance covariance matrix given in Eq. 7. Therefore, the individual 100 (1-α)% confidence interval for the state parameter is x_i given by:

$$P\left[-t_{\frac{\alpha}{2}} < \frac{\hat{x}_{ti} - x_i}{\hat{\sigma}_t\sqrt{\omega_{ii}}} < t_{\frac{\alpha}{2}}\right] = 1 - \alpha \quad (8)$$

Or:

$$\hat{x}_i \pm t_{\frac{\alpha}{2}, v}\hat{\sigma}_t\sqrt{\omega_{ii}}$$

The expression that is described by Kyriakides and Heydt (2006) for one at a time confidence intervals is the same as given in Eq. 8 with the exception that the standard error for each estimator here is based on the weighted residuals.

Estimation of the joint interval: The individual interval estimation for each parameter in the multivariate model may be a poor estimate. This is due to the fact that variability is measured by the interval estimate for a particular parameter. This is inferred under the assumption of the known optimized values for the all other parameters in the model (Lane and Mouchel, 1994). This is not clearly a realistic assumption. This leads the construction of the simultaneous confidence region with an overall confidence level probability of (1-α).

Under the known weights structure of the error terms, the most common strategy for the correctness of heteroscasdicity is weighting the measurement which in turn gives estimates of the desired statistical properties (Long and Ervin, 2000). Therefore, the simultaneous confidence interval can be approximated by using Eq. 3 as follows the ratio:

$$F = \frac{\left(\hat{X}_t - X\right)^T \left(\hat{H}_t^T H_t\right)^{-1} \left(\hat{X}_t - X\right)}{SSE_t}$$

follows the F distribution with n and m-n degrees of freedom. (For proof see Appendix). Therefore, the required 100(1-α)% simultaneous confidence region of all the state vector parameters is:

$$P\left[\frac{\left(\hat{X}_t\text{-}X\right)^T \left(\hat{H}_t^T H_t\right)^{-1} \left(\hat{X}_t\text{-}X\right)}{SSE_t} \leq F_{m,\ n-m}\right] = 1\text{-}\alpha \quad (9)$$

Equation 9 represents an m dimension elliptical region where the shape of the region is determined by the eigenvalue decomposition of the variance covariance matrix $(\hat{x}_t^T x)^{-1}$. This interval estimate has a resemblance to the interval which is described by Kyriakides and Heydt (2006) except that it is based on the weighted errors which have the advantage to deal with the more general structure of the error variances. Its interpretation becomes considerably tedious to compute when many parameters are considered in the estimated model. It may not be necessarily elliptically bounded in the case that the model is nonlinear in the unknown vector X. Because of the complexity of the region bounded by the simultaneous confidence interval, there also exists many other straightforward procedures for the general linear model. A joint interval may be found by considering the general expression (Stapleton, 2009):

$$\hat{x}_i \pm \gamma SE(\hat{x}_i) \quad (10)$$

where, $\hat{x}_i$ is a value of the ith estimator with the standard error $SE(\hat{x}_i)$ and γ is a multiplier whose value is assigned by the different methods in such a way that the desired probability of all intervals being correct is obtained. Among them, Bonferroni and Scheff e* are the common methods due to their easier applicability in many applied problems (Meng *et al*., 2010). In the (Bonferroni, 1935) method, the value γ is set as follows:

$$\gamma = t_{\alpha/2m, m-n}$$

This yield:

$$\hat{x}_i \pm t_{\alpha/2m, m-n} SE(\hat{x}_i) \quad (11)$$

which is based on the student t-distribution. In order to have a simultaneous interval of the probability coverage (1-α) for an unknown vector of the dimension (m×1), each interval is evaluated with a 1/mα level of significance. Whereas for (Scheffe, 1953) the Scheffe method, the γ is based on the F-distribution and is as given below:

$$\gamma = \left(2F_{\alpha,m,m-n}\right)^{1/2}$$

For more than two degree of freedom for the error, the Bonferroni Confidence Interval (BCI) is better than Scheffe's method at the standard level of the confidence (Mi and Sampson, 1993). There are also many other methods in the joint estimation that are basically classified in two categories ones that give the combined intervals or corresponding hypotheses and those which are only implemented essentially in the hypotheses testing procedures and are commonly used in the statistical literature. Details of these procedures can be found by Miller (1981).

Application to the dc power system state estimation: Our described method provides (1-α)100% individual and simultaneous interval estimates in the presence of weighted observations. Its performance has been evaluated in terms of the probability coverage. Coverage probability is the proportion of samples in repeated sampling that contains the true value of the corresponding parameter. The calculation of the coverage probability through the Monte Carlo experiment is a usually procedure for the evaluation of CI. The coverage probability has been calculated in the presence of heteroscedastic error terms through the Monte Carlo experiment. In order to validate our method for both individual and simultaneous interval estimation, we have

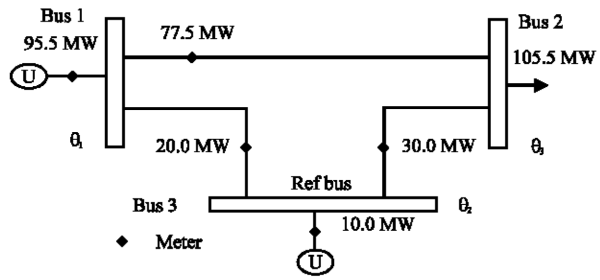

Fig. 1: DC load flow model for three bus system

Table 1: Available information for the given system

From bus	To bus	Power flow (pu*)	Reactance (pu)	Bus angles (rad)
1	2	0.775	0.4	0.00
1	3	0.200	0.8	0.16
2	3	0.300	0.5	-0.15

Pu: Per unit system representation of value in 100 base; (σ_i): Standard deviation related to accuracy of measurements of ith meter

considered simple the DC load flow network model. These types of models are very useful for rapidly computing the actual flow of power and in the security studies analysis (Wood and Wollenberg, 2012). The purpose of using this model is a non-iterative state estimation solution for the unknown state vector and it's resemblance to the model that has been used by Kyriakides and Heydt (2006) in terms of linearity. Consider the 3-bus DC flow model as shown in Fig. 1. Here, power flow meters of the different accuracy standards are installed at the different locations to measure the power in MW. Generally in a power system problem, a state vector is consisted of voltage magnitudes and the corresponding phase angles for all network nodes except the reference bus (Monticelli, 2000).Therefore, our state vector here, also consists of unknown phase angles for Buses 2 and 1. Whereas, Bus 3 is taken as a swing bus hence, its phase angle is assumed to be zero. Before running the simulation experiment on our described results for the individual and the simultaneous confidence intervals, we have to know the actual values of our parameters. Henceforth, it has been further assumed, so far, that the measuring devices are subject to provide measurements with no random disturbances. Therefore, for the given information in Table 1, the actual values of the phase angles for Buses 1 and 2 have been obtained as 0.16 and -0.15 in radians, respectively.

In the real situation, available measurements in the state estimation of a power system are not actual but are polluted with the disturbances from different sources that are assumed to follow the Guassian density function.

For implementation purposes, it has been assumed that the source of this uncertainty is contributed in the actual measurements by the accuracy parameter of each measuring device. The weighted least squares method has

Table 2: Performance of the confidence intervals for the measurements of equal accuracies ($\sigma_1 = 0.0001$, $\sigma_2 = 0.0001$, $\sigma_3 = 0.0001$)

Parameters to be estimated	Actual values of parameter	Point estimates	Individual confidence interval	Bonferroni confidence interval	Coverage probability ICI	BCI
θ_1	0.1600	0.1600	(0.0716, 0.3916)	(0.3042, 0.6242)	0.9466	0.9740
θ_2	-0.1500	-0.1499	(0.3837, 0.0838)	(0.6185, 0.3185)	0.9457	0.9734

Table 3: Performance of the confidence intervals for the measurements of unequal accuracies parameters to be estimated ($\sigma_1 = 0.01$, $\sigma_2 = 0.001$, $\sigma_3 = 0.0001$)

Parameters to be estimated	Actual values of parameter	Point estimates	Individual confidence interval	Bonferroni confidence interval	Coverage probability ICI	BCI
θ_1	0.1600	0.1600	(-0.0716, 0.3916)	(-0.3042, 0.6242)	0.9524	0.9759
θ_2	-0.1500	-0.1499	(-0.3837, 0.0838)	(-0.6185, 0.3185)	0.9520	0.9766

been used to estimate the unknown parameters θ_1 and θ_2. We have simulated the 10000 sets of measuring values z_1, z_2 and z_3 by adding errors from the assumed normal density models; all having zero means and variances corresponding to the accuracy parameter attached with each meter as shown in Table 2. In this way, for each set of simulated measurements, the interval estimates are calculated by expressions given in Eq. 8 and 11 at a nominal level of α = 0.05%. The performance of the interval estimates is then measured by the actual coverage probability values for θ_1 and θ_2 that are close to the chosen nominal value as shown in Table 2.

RESULTS AND DISCUSSION

The R language (28) has been used for the performance evaluation of these interval estimates and to report the necessary results in Table 2 and 3. We have also illustrated our procedure graphically up to 400 runs that can be seen in Fig. 2 and 3. Here, each vertical line is in correspondence to the width of an interval and the desired parameters values are represented by the central horizontal lines. The vertical lines that are above and below the parametric lines are in correspondence to intervals that have not been captured the true values of the parameters.

The nominal coverage regions for the individual and the simultaneous confidence intervals are 0.950 and 0.975, respectively, at the chosen significance level of 0.05. It has been seen from the simulated results, that the coverage probabilities for the desired parameters in Table 2 and 3 are not exactly equal to the nominal confidence regions for the individual and the simultaneous confidence intervals.

The reason behind this is a hortage of redundant ratio. In our case, it is 1.5. For a good interval estimate, enough numbers of the redundant measurements are required. This approximation can be achieved closer to the chosen level of significance provided that enough redundancy is involved. The confidence intervals that are listed in Table 2 and 3 are the average intervals over the

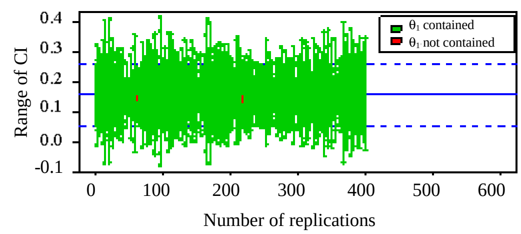

Fig. 2: Simulation of individual C1 for θ_1

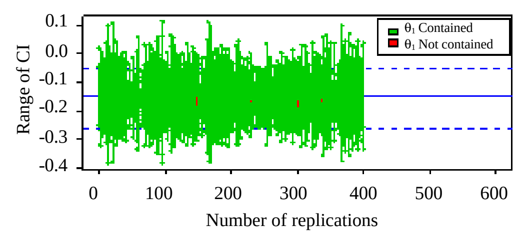

Fig. 3: Simulation of individual C1 for θ_2

10000 simulations. The width of the BIC is larger than the corresponding individual confidence which is not surprising but rather a fact that is supported by the common theoretical result. The coverage probabilities in case of unequal weights as shown in Table 3 still remained stable. The intervals that are described in the referred to study cannot be implemented here because of inclusion of the weighted measurements. But in our described method, once all the weights that are used for the different errors are assumed to be one, both will provide identical results. So, one of the important advantages of the confidence interval as described in Eq. 8 and 11 is its generalization and practical implementation in a power system state estimation problem that commonly involves measurements with different weight structures.

CONCLUSION

Kyriakides and Heydt (2006) have implemented, the described method for the individual and the simultaneous interval estimates for the state parameters of a power system under the assumption that error variances are constant. In this scenario, the least square method is one of the appropriate techniques for finding the optimum estimate of the system state parameters. This parametric assumption is questionable in a power system state estimation problem and looks impractical in the sense that uncertainties are assumed to be added in the measurements from the different sources. In our opinion, it is a very significant assumption of the state estimation problem from the application point of view and should be kept in consideration. In view of its importance, we have described individual confidence and simultaneous confidence intervals for an unknown state vector under the weighted least square scenario which is more commonly applicable to the power system state estimation problem.

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