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High Voltage Power Transformer Transfer Function Based on Vector Fitting and Experimental Analysis

Moneer A. Faraj, Azah Mohamed and Ramizi Mohamed
Department of Electrical, Electronic and Systems Engineering,
Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia (UKM),
43600 Bangi, Selangor Darul Ehsan, Malaysia

Abstract: One of the most important equipment elements in HV transmission systems are transformers. Therefore, improvement techniques for transformer life assessment have been developed in recent years. Frequency Response Analysis (FRA) is a useful method for the reliable detection of electrical and mechanical faults in transformers. The point to be further emphasised in FRA is interpreting deviations in order to identify the failures .In this research an algorithm for automated analysis is developed based on analytical model results by approximating measurement data to a rational function to obtain minimum complexity by using the vector fitting method. The Root-Mean-Square (RMS) between the measured data and proposed-fitted transfer function is determined by searching the number of complex and real starting poles as well as, number of iterations, linear or non-linear distribution of the proposed poles over the reference measured frequency range is determined as well besides finding the order of the approximation function that consequently represents the transformer's transfer function. A MATLAB software program has been implemented for the vector fitting algorithm with the measured data.

Key words: Frequency Response Analysis (FRA) of power transformer, transformer faults diagnosis, transfer function, victor fitting method, implemented, electrical

INTRODUCTION

In transmission systems, power transformers are the most expensive single elements with a crucial role in facilitating power transfer to users. Therefore, any fault in the transformer will cause reduced power system reliability. Monitoring of a transformer increases over time. In this regard, several research notions such as dissolved gas analysis, partial-discharge measurements, thermal measuring, frequency response analysis, etc., have been implemented. Each method is able to detect a corresponding fault. A comparison between TF measurements can reveal information about the mechanical condition of transformer windings (Rahimpour *et al*., 2010).

Many approaches have been used to fit rational function approximation to the main frequency response. The vector fitting method among the better methods employed to approximate measurement data (Alharbi, 2014). The methodology of fitting measured frequency response data has been described in (Gustavsen and Semlyen, 1999) which presents the ability to fit a high number of resonance peaks by allowing complex starting poles.

The research by Bigdeli *et al.* (2011) used vector fitting to compare the TFs and specify the fault type, level and location. The transfer function coefficients were determined with the required accuracy under intact and fault conditions. The ability of the FI index has been used (to recognize the fault location). Karimifard *et al.* (2008) used vector fitting to determine the extent of the axial winding displacement which is based on transfer function, the sum of absolute displacement of pole was calculated and used as an index. While by Karimifard *et al.* (2009) HV winding of transformer has been modeled by the detailed model. The TF of this model was estimated using VF method for normal and deformed cases. A new index was proposed for the determination of the deformation extent.

Yielded a good model that is simple and robust and that can serve as a theoretical methodology for insulation design of transformer windings. Optimal pade

Corresponding Author: Ramizi Mohamed, Department of Electrical, Electronic and Systems Engineering,
Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, 43600 UKM Bangi,
Selangor Darul Ehsan, Malaysia

approximation was used to obtain order-reduction of the rational transfer functions, subsequently resulting in synthesizing voltage transfer functions with passive elements. Therefore, the poles and residues help produce ideal transformers (Ping *et al*., 2008).

Approximation of the admittance matrix with rational functions using vector fitting was used by Gustavsen (2003) to produce an EMTP-type compatible model suitable for transient studies from measured terminal characteristics. Alvarez *et al*. (2013) it was mentioned that the resonances in transformer windings can be calculated using vector fitting algorithm through simulations and impedance measurement with Mombello circuits to achieve correlation coefficient above 99.9%. By Alvarez *et al*. (2013) the voltage distribution in transformer windings was calculated using equivalent circuits with constants elements that could model the frequency dependence. Vector fitting was used to approximate the behaviour of R and L to rational functions with the aim of determining the elements of the equivalent circuit. A frequency-dependent loss of power transformer for very fast transient simulation was proposed (Zheng and Wang, 2013). The attenuation factors and the corresponding natural frequencies of the transformer in a measurable range were extracted using measured power transformer network functions and VFM. The researcher concluded that the frequency-dependent attenuation factor formula can be applied directly to the simulated fast transient response to reflect the loss effect. A complete procedure characterized an admittance matrix over a wide frequency band based on a sweep frequency response analysis measurement tool. The admittance matrix was subjected to rational modelling using vector fitting (Holdyk *et al*., 2014).

MATERIALS AND METHODS

Frequency response analysis principle: Frequency Response Analysis (FRA) is used to identify internal faults of power transformers based on transfer function analysis. The power transformer component like windings, core and insulation are described as equivalent circuits of capacitors, resistors and self and mutual inductances as in Fig. 1. These parameters are changed by mechanical and/or electrical faults within a transformer. Therefore, the transfer function's shape is also altered. The FRA guide serves to compare one TF in complex form as reference with another TF that is being tested and experimentally measured (measurement is done mostly when the transformer is offline). It is found that none or only slight differences between the TFs indicates no electrical or mechanical change inside the transformer and evidently, the deviations advocate potentially significant variances in relation to the reference.

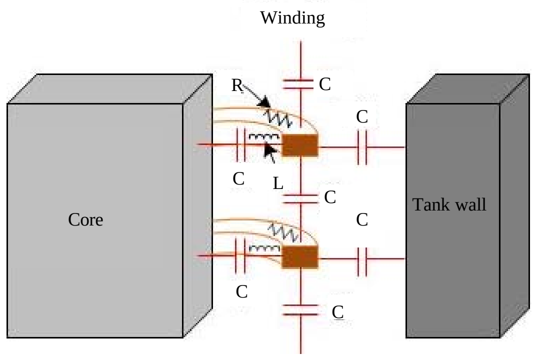

Fig. 1: Network behavior of a transformer's active part

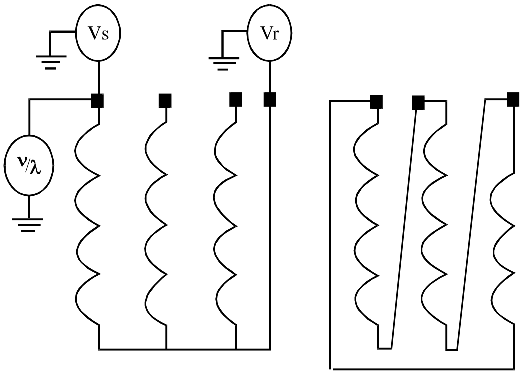

Fig. 2: Frequency response analysis end-to-end test configurations

There are different connection schemes while the end-to end connection shown in Fig. 2 is commonly used. Here, the FRA generates a signal in one end of a winding and measures the response at the other end of the same winding with the response in the frequency domain range of 20 HZ-20 MHZ (Jayasinghe *et al*., 2006).

Interpretation process: The majority of past investigations on the interpretation process of transformer FRA have focused mainly on the resonance characteristic of TFs. Some parameters such as new resonance peak locations, disappearance in the resonances and differences in damping frequencies with respect to the reference transfer function are considered evidence of mechanical and electrical changes in the power transformer. The experience is required to interpret the FRA correctly to identify the type and location of a fault. The FRA frequency range stated by Dick and Erven (1978) Pordanjani and Xu (2014) is indicated in Table 1.

The possibility of FRA measurement evaluation is applicable with algorithmic interpretation by comparing the deviations in the frequency resonances between the

Table 1: Frequency response analysis bands and their sensitivity to faults Abu *et al.* (2013)

Frequency range	Fault sensitivity
<20 (KHz)	This range is linked to core deformation, open circuit, shorted turns and residual magnetism
20-400 (KHz)	Here the transformer winding responses are dominated by the inductive components
>400 (KHz)	This range is associated with deformation in the main or tap winding. Here, the inductive and capacitive components result inmultiple resonances
	This range is linked to the movement of the main and tap windings and ground impedance variations. Here, the FRA signature is dominated by the capacitive components

FRA curves in analytical models. In the current study a rational approximation method which is the base of vector fitting, method is adopted to handle the analytical formulation of the transformer's TF.

Transformer frequency response approximation: The rational functions in polynomial form can be written as:

$$TF(s) = \frac{a_0+a_1s+,\ldots,a_ms^m}{b_0+b_1s+,\ldots,b_ms^m} \quad (1)$$

Where:

TF(s) = Estimated transfer function

a_m = Real coefficients denominator

b_m = Real coefficients denominator

m = Number of order

While in zeros and poles, the expression is written as:

$$TF(s) = A\frac{\prod_{m=1}^{m} s-z_m}{\prod_{n=1}^{n} s-p_n} \quad (2)$$

(zm, pn) with complex conjugate pairs of zeros and poles, respectively.

Vector fitting: As described in the introduction, Vector Fitting (VF) is a method of finding the best fit of a rational function based on the least squares sense for data measured in a complex frequency response. VF is an iterative technique based on pole-zero relocation which is explained in the following steps: The rational function approximation can be written as:

$$F(s) = \sum_{n=1}^{n} \frac{c_n}{s-a_n} + b_n + sh \quad (3)$$

Where:

a_n = Poles come in a real quantity or complex conjugate

c_n = Zeros come in a real quantity or complex conjugate

b, h = Real quantity

In estimating the coefficient in Eq. 3 it is obviously the optimization problem is of non-linear poles. Gustavsen and Semlyen (1999) proposed identifying the parameter indirectly. Two other transfer functions, σ(s) and H(s) are realized as follows:

$$\sigma(s) = \sum_{n=1}^{n} \frac{c_n}{s-a_n} + 1 \quad (4)$$

$$H(s) = \sigma(s)TF(s) = \sum_{n=1}^{n} \frac{c_n}{s-a_n} + d + se \quad (5)$$

Equation 4 and 5 have identical poles which are supposed to be known at the beginning of each iteration. Transfer functions H(s) and σ (s) can be described as:

$$H(s) = TF(s)\,\sigma(s) = \frac{\prod_{n=1}^{N+1} s-z_n}{\prod_{n=1}^{N} s-p^n} \quad (6)$$

$$\sigma(s) = \frac{\prod_{n=1}^{N} s-z^n}{\prod_{n=1}^{N} s-p^n} \quad (7)$$

Then f(s) can be written as follows:

$$TF(s) = \frac{TF(S)\sigma(s)}{\sigma(s)} = \frac{\prod_{n=1}^{N+1} s-z^n}{\prod_{n=1}^{N} s-z^n} \quad (8)$$

The poles of TF(s) become equal to the zeroes of σ(s). The starting poles are cancelled out because the same starting poles are used. The next step is to find the residues and the constant term. These parameters can be calculated from:

$$TF(s) = \sum_{n=1}^{n} \frac{c_n}{s-a_n} + b_n + sh$$

The flowchart in Fig. 3 represents the vector fitting algorithm (Pordanjani and Xu, 2014). The accuracy of the approximation is measured with the following Eq. 9:

$$RMS = \sqrt{\frac{1}{L}\sum_{V=1}^{L}\left(\frac{|TF_{orig}(V.\Delta f)|-|TF_{fitt}(V.\Delta f)|}{|TF_{orig}|}\right)^2} \quad (9)$$

Where:

L = The number of data points

v = The voltage magnitude and $|TF_{orig}|$ Calculated by using Eq. 10

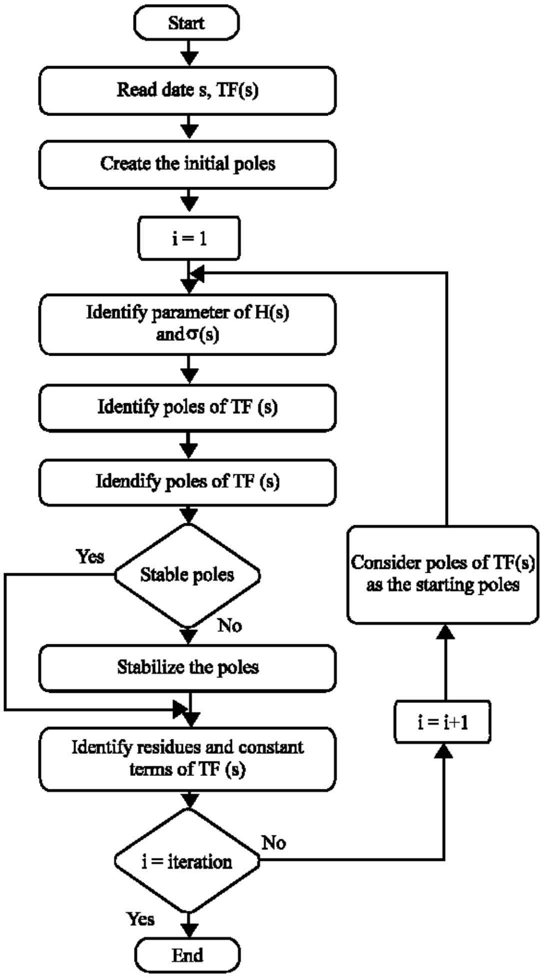

Fig. 3: The flowchart in of vector fitting algorithm

$$\left|TF_{orig}\right| = \frac{1}{L}\sum_{v=1}^{L}\left|TF_{orig}\ (V.\Delta f)\right| \qquad (10)$$

The variances of resonance peaks between the measured data points and the fitted ones should be smaller than the deviations of the respective data measurements. The fitting quality of the VF algorithm depends on the start pole locations over the frequency range.

The most suitable number of starting poles that represents the degree of the TF and iteration number should be chosen carefully to overcome complexity. It is found later that the higher numbers of iterations wouldn't improve the accuracy of fitting results.

RESULTS AND DISCUSSION

FRA measurements were performed on a transformer as shown in Fig. 4 to obtain a winding transfer function which has the following specifications: as show in Table 2. An FRA plot was drawn with the data points for the transfer function in MATLAM 2013a and the transformer was assembled as a single winding without neutral a point terminal as shown in Fig. 4. The measurement was done under 50 Ω impedance. The schematic diagram of FRA measurement is shown in Fig. 5.

Fig. 4: Site power transformer as a case study

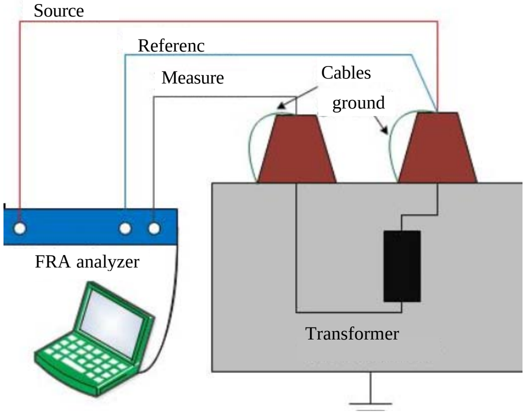

Fig. 5: Schematic diagram of FRA measurement

The transfer function of the transformer's frequency response for 1 Hz-2 MHz frequency range is estimated using the vector fitting technique as shown in Fig. 6. Good transfer function estimation for 24 starting complex conjugate poles that represent the order was obtained with root mean square of 0.979.

Table 2 and 3 displays the transfer function results. Note that the selected transfer function order, type of distribution, type of poles and number of iterations is based on the lowest rms value.

A slight shift between resonances at relatively high frequencies of about 10 Khz-1 MHz which as seen in

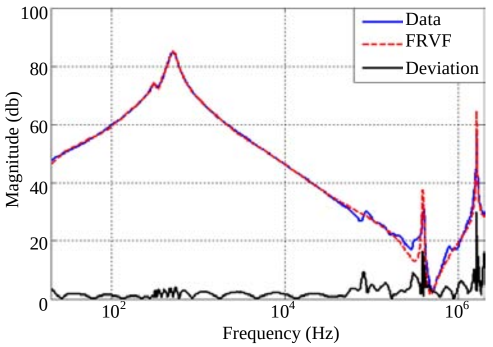

Fig. 6: FRA of a power transformer, experimental and fitted

Table 2: Site power transformer

Variables	Values
Serial number	951080
Manufacturer	LIAOYANG
Type code year	1998
MVA rating	7.5 MVA
High voltage	33 KV
Low voltage	11 KV
Tertiary voltage	0 KV
Phases	3
Vector group	Dyn11

Table 3: The coefficient of the transfer function of 24 order

Order of S	Coefficient of numerator	Coefficient of denominator
24	+29.63	1
23	-(2.98e07+6.773e-07i)	+2.935e06
22	+(6.107e15-2.671i)	+2.143e14
21	+(9.056e20-8.377e07i)	+4.266e20
20	+(3.475e29-3.062e14i)	+1.063e28
19	+(5.46e35-2.672e21i)	+1.615e34
18	+(2.997e42-3.525e27i)	-1.063e41
17	+(1.188e49-1.171e34i)	+4.013e47
16	+(2.649e51-3.301e40i)	-9.704e53
15	+(5.332e61+3.354e46i)	+1.722e60
14	-(1.664e67+3.416e52i)	+4.732e65
13	-(1.901e72-3.263e57i)	-3.485e70
12	+(5.357e76-6.472e61i)	-1.043e75
11	+(6.244e80-9.104e66i)	+7.765e78
10	-(1.148e83+1.298e71i)	-7.628e81
9	+(1.414e88+1.853e74i)	+1.818e86
8	-(1.029e91+3.555e78i)	+3.074e88
7	+(1.448e95+4.909e81i)	+1.808e93
6	-(1.995e97+4.039e85i)	-4.82e95
5	+6.393e101+3.419e88i)	+7.452e99
4	+(2.261e104-2.09e92i)	-6.533e102
3	+(9.629e107+6.047e94i)	+1.007e106
2	+(6.929e110-4.022e98i)	-1.627e109
1	-(3.023e113-4.882e99i)	-4.41e111
0	-(1.471e1154.409e103i)	+5.899e113

Fig. 7 under the magnifier. This slight difference was duo to the analysis of many frequency responses with respect to the reference, algorithm iterations, localization of the starting poles as well as their number and whether they were real and/or complex conjugate poles.

There are several reasons that could come from the reference measurements such as data that were recorded with equipment from different manufacturers and some measuring parameters such as number of measured frequency data points, the frequency range and the logarithmic or linear distribution of data points.

To show the effect of type of pole the transfer functions in a low frequency range of 1 Hz-10 KHz for real and complex conjugate poles are presented in Fig. 8 and 9. The rms values are 0.3030 and 0.7142, respectively.

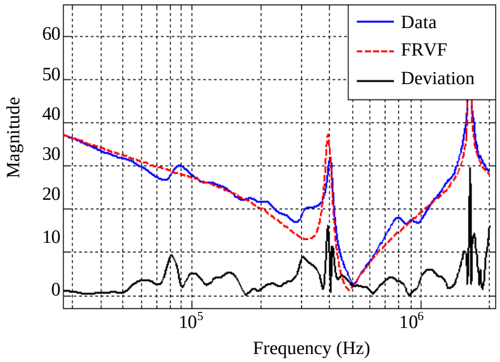

Fig. 7: Magnifier FRA of a power transformer (experimental and fitted data)

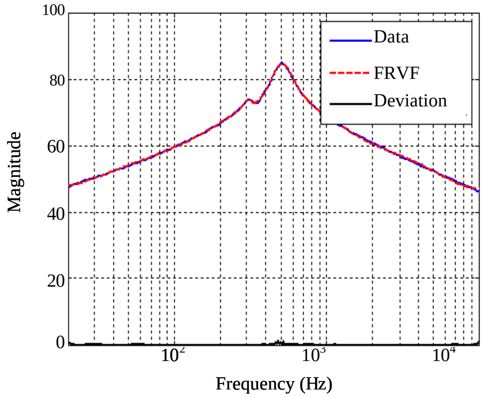

Fig. 8: Transfer functions at low frequency with real poles

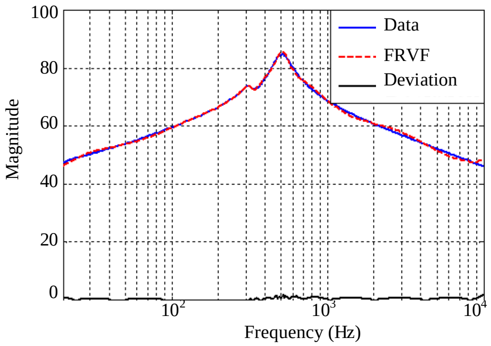

Fig. 9: Transfer functions at low frequency with complex conjugate poles

CONCLUSION

FRA still necessitates additional research to find an automatic parameter affective algorithm, meaning without having to estimate the number of starting poles or undergo a localization process of those poles. The importance of finding an analytical formula for a transformer transfer function from measured data points is to take it as a future frequency analysis reference and reduce the interpretation process complexity. The locations and prepositioning of the rational fitting function are highly affected by the pole estimation. An appropriate accuracy measure of the fitting results is an RMS which should be in small range to satisfy the requirements. The vector fitting approximation algorithm can be enhanced in a way such that accurate pre-estimation suitable for model complexity is achieved.

Future analysis using algorithms such as artificial neural networks may support The comparison of pole location and number for attaining more accurate analytical models.

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