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Quadratic Investment Portfolio Without a Risk-Free Asset Based on Value-at-Risk

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Abstract: This study will discuss the problems of quadratic investment portfolio without a risk-free asset based on value-at-risk. It is assumed that the risk of an investment portfolio measured by value-at-risk. The resolution of problems that do include: first, formulate models the trade-off problem. Secondly, formulate expectation maximization model of the problem. Third, formulate model minimization of value-at-risk problem. Based on the results of the discussions can be concluded that the trade-off between risk and expected return does not only depend on the type of investor but also on the size of the investment. In a realistic investment situation, it is likely that more constraints, e.g., restriction on short-selling, need to be considered.

Key words: Investment portfolio, value-at-risk, short-selling, maximization, minimization, discussions

INTRODUCTION

Investment in the capital markets is one way that can be done by an investor. The capital markets have provided many benefits to development the economy of a country. There are many investments that are cultivated through stocks and bonds (Strassberger, 2005). An investor who will invest in the capital market in particular stocks should be able to understand the risks involved because such investments generally investors will be faced with the forthcoming period containing uncertainty. This means that contain elements of risk for investors. The desire to make a profit is one of hope for all investors. The higher the risk faced by an investor, the higher the expectations of investors to profit (expected return) (Hult *et al*., 2012).

The need for a risk measure of reliability was further strengthened (Wang *et al*., 2016). The growth of trade activities and the market more erratic make market participants feel the need to develop risk measurement techniques are more accurate and reliable (Plunus *et al*., 2015). One method is to use risk measurement approach kuantil, better known as Value at Risk (VaR) (Alexander *et al*., 2006; Boudt *et al*., 2012).

Value at Risk (VaR) is a method of calculation of the market risk to determine the maximum risk of loss that can occur in a portfolio, either single-or multiple-instrument instruments on a certain level of trust, during the holding period and in normal market conditions (Goh *et al*., 2012; Ogryczak *et al*., 2015).

The advantage of this method is that VaR focus on downside risk does not depend on the assumption of the return distribution and this measurement can be applied to all financial trading products (Boudt *et al*., 2012).

Figures obtained from the measurements with this method are the result of the calculation in the aggregate or overall against the risk of products as a whole. VaR also provide estimates of the likelihood or probability as to the incidence of loss that the sum is greater than the number of losses that have been determined (Goh *et al*., 2012; Ogryczak and Sliwinski, 2010).

This indicates something that is not obtained from the methods of other risk measurement. VaR also noticed the price change of the existing assets and its influence on other assets. So, this allows doing measurements against the reduced risk due to the diversification of some instrument investment or portfolio.

The nature of the establishment of the portfolio is to allocate funds on a range of alternative investment, so that the overall investment risk can be minimized (Bansal *et al*., 2014; Cochrane, 2014). In investing, investors can choose to invest their funds in various assets, either assets that are at risk as well as risk-free assets or a combination of both of these assets.

Optimal portfolio is a portfolio of selected an investor from the many options that exist on the set of an efficient portfolio. Categorized portfolio efficiently if have the same risk level, capable of delivering higher profit level or is capable of producing the same level of profit but with lower risk (Ahmadi and Sitdhirasdr, 2016; Baweja and Saxena, 2015).

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Modern portfolio theory was first introduced by Harry Markowitz in 1952, until now many serve as reference in compiling a portfolio of stocks (Pinasthika and Surya, 2014). This gives a lot of Model Markowitz steps investors in putting together a portfolio and given also the weighting allocation of funds on such stocks often known as Markowitz Model is a model of mean-variance (Golafshani and Emamipoor, 2015). The mean-variance portfolio model is quadratic because variance is the shaped quadratic function (Qin, 2015; Shakouri and Lee, 2016).

Furthermore, based on the description above in this study was conducted research on "Quadratic Investment Portfolio without a risk-free asset based on value-at-risk". The goal is to specify the optimum weights combination formulations of a risk free asset without a portfolio based on the size of risk value-at-risk. As an illustration of numeric, analyzed some of the shares traded on the stock market in Indonesia.

MATERIALS AND METHODS

Mathematics models: In the descriptions of mathematical models are here discussed about the basic concepts to be used in the next discussion. Furthermore, the description followed by a discussion of modeling-based investment portfolio value-at-risk.

Basic concept: Suppose the length of time the investment is expressed by $t_0 = 0$ the initial investment and $t_1 = 1$ end investment. Investors form a portfolio with value expectations $E[V_1]$ at time $t_1 = 1$ is high. Because V_1 fluctuates, it is expected that the risk Var $[V_1]$ is minimum. Suppose V_0 initial investment and a simple asset market with $n \geq 2$ risky assets with spot prices S_t^k where t = 0, 1 and k = 1, ..., n. Note that S_0^k is known whereas S_1^k is not. We also allow positions in a risk-free zero-coupon bond that cost B_0 at time 0 and pays one unit of the chosen currency at time 1 (Hult *et al*., 2012).

A position in the risky assets is represented by a vector $h = (h_1, ..., h_n)^T \in \Re^n$, h_k is the number of units of asset number k held over the time period by the investor. We let h_0 denote the position in the risk-free bond. The prices or market values at time t = 0 and t = 1 of an affordable portfolio are (Hult *et al*., 2012):

$$h_0 B_0 + \sum_{k=1}^{n} h_k S_0^k \leq V_0 \quad \text{and} \quad V_1 = h_0 + \sum_{k=1}^{n} h_k S_1^k$$

If no risk-free bond is available, then we simply set $h_0 = 0$. The next, we take the initial monetary value of the position in the kth asset $w_k = h_k S_0^k$ and $w_k = h_0 B_0$. With monetary portfolio weights the current and future portfolio values can be expressed as (Hult *et al*., 2012; Pinasthika and Surya, 2014):

$$w_0 + \sum_{k=1}^{n} w_k \leq V_0 \quad \text{and} \quad w_0 \frac{1}{B_0} + \sum_{k=1}^{n} w_k \frac{S_1^k}{S_0^k} \qquad (1)$$

It is seen that determining the optimal allocation of initial capital V_0 requires the knowledge of the expected value μ and covariance matrix Σ of the vector R where (Hult *et al*., 2012):

$$R^T = \left(\frac{S_1^k}{S_0^k}, ..., \frac{S_1^n}{S_0^n}\right)$$

with $R_0 = 1/B_0$ and $w^T = (w_1, ..., w_n)^T$, we may write $V_1 = w_0 R_0 + w^T R$ and therefore (Hult *et al*., 2012; Pinasthika and Surya, 2014):

$$E[V_1] = w_0 R_0 + w^T \mu \qquad (2)$$

And:

$$Var[V_1] = w^T \Sigma w \qquad (3)$$

We assume that the covariance matrix Σ = Cov (R) = $E[(R-\mu)(R-\mu)^T]$ is positive definite: $w^T \Sigma w > 0$ for all $w \neq 0$. By definition, any covariance matrix is symmetric and also positive-semidefinite: for any $w \neq 0$, $w^T \Sigma w = Var(w^T R) \geq 0$. Therefore, assuming that Σ is positive definite is equivalent to assuming that Σ is invertible or equivalently that all the eigenvalues of $\Sigma > 0$ (Hult *et al*., 2012).

Modeling of portfolio investment based on value-at-risk item: In this study, are discussed about the investment portfolio optimization model Mean-VaR without the risk-free asset. It is assumed that the asset return has a certain distribution and the risk of the portfolio is measured using the Value-at-Risk (VaR). According to Alexander *et al*. (2006) and Boudt *et al*. (2012) risk measurement model of the value-at-risk for portfolios formulated as:

$$VaR_p = -V_0\{\mu_p + z_\alpha \sigma_p\}$$

Because it is a risk-free asset and is constant, then refer to Eq. 2 and 3, the value-at-risk for the portfolio can be expressed as:

$$VaR_p = -V_0\{w^T \mu + z_\alpha (w^T \Sigma w)^{1/2}\} \qquad (4)$$

Where:

- \- = Stated losses
- V_0 = The initial capital invested
- z_α = Percentile of the standard normal distribution when the given level of significance (1-α)%

So, the purpose of the function model of investment portfolio is maximize $\{\mu_p\text{-}\rho VaR_p\}$ or maximize $\{w^T\mu + c/2V_0V_0[w^T\mu + z_\alpha(w^T\Sigma w)^{1/2}]\}$. Thus the model of optimization of investment portfolio which need to be resolved are:

$$\text{Maximum } \left\{(1+\frac{c}{2})w^T\mu + \frac{c}{2}z_\alpha(w^T\Sigma w)^{1/2}\right\} \quad (5)$$

$$\text{Subject to, } w^T I \le V_0$$

Assume there are two investments with return R and $\tilde{R}$, investors who invested V_0 have expectations the same result, then the value of the constant c, can be determined through the equality that:

$$E[V_0R] - \frac{c}{2V_0}VaR[V_0R] = E[V_0\tilde{R}] - \frac{c}{2V_0}VaR[V_0\tilde{R}]$$

So, we get that:

$$c = \frac{2\{E[V_0R] - E[V_0\tilde{R}]\}}{\{VaR[V_0R] - VaR[V_0\tilde{R}]\}}$$

Next, to find the solution optimization Eq. 5 can be done by using the Lagrangian multiplier method and method of the Kuhn-Tucker. Lagrangian multiplier function from Eq. 5 is given as (Ghaemi *et al*., 2009; Mustafa *et al*., 2015):

$$L(w,\lambda) = (1+\frac{c}{2})w^T\mu + \frac{c}{2}z_\alpha(w^T\Sigma w)^{1/2} + \lambda(w^T I - V_0)$$

Based on the method of the Kuhn-Tucker, obtained the following system of equations:

$$\frac{\partial L}{\partial w} = (1+\frac{c}{2})\mu + \frac{cz_\alpha \Sigma w}{2(w^T\Sigma w)^{1/2}} + \lambda I = 0 \quad (6)$$

$$\frac{\partial L}{\partial \lambda} = w^T I - V_0 = 0 \text{ or } w^T I = V_0 \quad (7)$$

Equation 6 can be expressed as:

$$\frac{cz_\alpha \Sigma w}{2(w^T\Sigma w)^{1/2}} = -\left\{(1+\frac{c}{2})\mu + \lambda I\right\} \quad (8)$$

If the Eq. 8 multiplied by $2\Sigma^{-1}/cz_\alpha$ because $\Sigma^{-1}\Sigma = \Sigma\Sigma^{-1} = U$ and U the squared and matrix as a unit so that $Uw = w^TU = w$ thus can be obtained from the equation:

$$\frac{w}{(w^T\Sigma w)^{1/2}} = -\frac{(1+\frac{c}{2})\Sigma^{-1}\mu + \lambda\Sigma^{-1}I}{\frac{c}{2}z_\alpha} \quad (9)$$

Equation 9, if multiplied by I^T because $I^Tw = w^TI = V_0$ then the equation is obtained:

$$\frac{V_0}{(w^T\Sigma w)^{1/2}} = -\frac{\left\{(1+\frac{c}{2})e^T\Sigma^{-1}\mu + \lambda I^T\Sigma^{-1}I\right\}}{\frac{c}{2}z_\alpha} \quad (10)$$

Or:

$$(w^T\Sigma w)^{1/2} = -\frac{\frac{cV_0}{2}z_\alpha}{\left\{(1+\frac{c}{2})I^T\Sigma^{-1}\mu + \lambda I^T\Sigma^{-1}I\right\}} \quad (11)$$

When the Eq. 9 multiplied by Eq. 10 and 11, then the obtained optimal portfolio weights vector equations as:

$$w^* = \frac{V_0\left\{(1+\frac{c}{2})\Sigma^{-1}\mu + \lambda\Sigma^{-1}I\right\}}{(1+\frac{c}{2})I^T\Sigma^{-1}\mu + \lambda I^T\Sigma^{-1}I} \quad (12)$$

When the Eq. 9 multiplied by w^T and simplified, then retrieved the equation:

$$\frac{c}{2}z_\alpha(w^T\Sigma w)^{1/2} = -\left\{(1+\frac{c}{2})\mu^T w + \lambda\right\} \quad (13)$$

Furthermore, if the Eq. 11 and 12 of substituting into Eq. 13 and simplified being:

$$(I^T\Sigma^{-1}I)\lambda^2 + (1+\frac{c}{2})\{I^T\Sigma^{-1}\mu + \mu^T\Sigma^{-1}I\}\lambda + V_0\{(1+\frac{c}{2})^2\mu^T\Sigma^{-1}\mu - (\frac{c}{2}z_\alpha)^2\} = 0 \quad (14)$$

Equation 14 is a quadratic equation in λ so that it can be calculated using the equation ABC constants as follows:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \ \lambda > 0 \quad (14)$$

Where:
$a = I^T\Sigma^{-1}I$
$b = (1+c/2)\ \{I^T\Sigma^{-1}\mu + \mu^T\Sigma^{-1}I\}$
$c = V_0\{(1+c/2)^2\mu^T\Sigma^{-1}\mu - (c/2\cdot z_\alpha)^2\}$ with Σ^{-1} inverse from matrix Σ

Equation 15 and 11 are jointly used to determine the proportion of the weighting of asset allocation funds in some stocks in the formation of optimal investment portfolio.

RESULTS AND DISCUSSION

Numerical illustration: In the illustration of numeric is intended to demonstrate how the application of the model that has been formulated to analyze data shares traded on capital markets. Stock data were analyzed through the website http://www.finance.go.id//.

The data consists of 5 shares are selected for during the period from 2 January 2013 until 4 June 2016. The data include stocks: INDF, gods, AALI, LSIP and the ASII. The value of the return of the five rataan shares, respectively given in vector = (0.0154, 0.0390, 0.0033, 0.0088, 0.0003). While the value of the variansi return along with the value of the kovariansi between the return of the 5 stocks are given in the form of a matrix and its inverse kovariansi as follows:

$$\Sigma = \begin{bmatrix} 0.0026 & 0.0001 & -0.0002 & 0.0002 & 0.0001 \\ 0.0001 & 0.0028 & 0.0003 & 0.0000 & 0.0001 \\ -0.0002 & 0.0003 & 0.0013 & 0.0006 & 0.0004 \\ 0.0002 & 0.0000 & 0.0006 & 0.0019 & 0.0003 \\ 0.0001 & 0.0001 & 0.0004 & 0.0003 & 0.0002 \end{bmatrix}$$

$$\Sigma^{-1} = 10^4 \begin{bmatrix} 0.0450 & -0.0022 & 0.0365 & -0.0018 & -0.0916 \\ -0.0022 & 0.0370 & -0.0092 & 0.0039 & -0.0048 \\ 0.0365 & -0.0092 & 0.2310 & -0.0022 & -0.4723 \\ -0.0018 & 0.0039 & -0.0022 & 0.0694 & -0.1008 \\ -0.0916 & -0.0048 & -0.4723 & -0.1008 & 1.6441 \end{bmatrix}$$

Since, the number of shares that is analyzed is composed of five kinds, then the vector unit defined as a $I^T = (1, 1, 1, 1, 1)$. Suppose also that the initial capital invested is $V_0 = 1$ unit of currency. Furthermore, vector μ^T, vector I^T and matrices inverse, covariance Σ^{-1}, jointly used for the calculation of the investment portfolio optimization process. Here, it is assumed that in the traksaksi selling stock short sales are not allowed. The research was conducted using the optimization model Mean-VaR optimization in the process. These values are constants determined in simulated risk a version c>0 in sequence from the smallest to the largest value value. Investment portfolio optimization process conducted with the help of software Matlab 7.0.

For the process of optimization of investment portfolio of Mean-VaR done by referring to Eq. 14 and 11

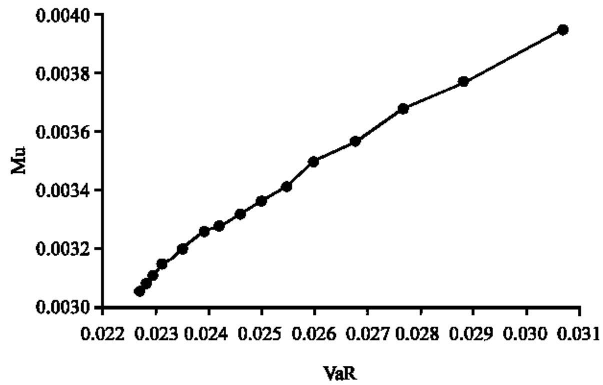

Fig. 1: Graphic of efficient portofolio Mean-VaR

as well as here defined value α = 5% so that the retrieved value $z_{5\%} = -1.645$. The value of the constant risk aversion c>0 that meet the assumption that short sales are not allowed is 0.25 $0.25 \leq c \leq 0.28$. Change the value c from 0.25-0.28 here is done with the use Δc of 0.001. Investment portfolio optimization process of Mean-VaR efficient portfolios such as the graph obtained (Fig. 1).

At Fig. 1, minimum portfolio is risk value $VaR_p = 0.0224$ and mean value $\hat{\mu}_p = 0.003$, occured when value of risk aversian constant c = 0.140. Minimum portfolio generated for the combination of weight portfolio as $w^{Min} = (0.1246, 0.0783, 0.0051, 0.0667, 0.7254)$. for a whilemaximum portfolio at risk value $VaR_p = 0.0307$ and mean value $\hat{\mu}_p = 0.0033$, occured when value of risk aversian constant c = 0.25. Maximum portfolio generated for the combination of weight portfolio is $w^{Max} = (0.2308, 0.1115, 0.2254, 0.1474, 0.2848)$. In optimization of investment portfolio process Mean-VaR that obtained value of global optimum portfolio $VaR_p = 0.0245$ and mean value $\hat{\mu}_p = 0.0033$ was occured when value of risk aversian constant c = 0.264. The most of global optimum portfolio have ratio $\hat{\mu}_p / VaR_p = 0.1346939$. The resulting of global optimum portfolio for the combination of weight portfolio is $w^{Glob} = (0.1538, 0.0874, 0.0657, 0.0889, 0.6041)$.

Based on that analysis at optimization of investment portfolio Mean-VaR, it seems that for a constant value of risk aversion which is greater, the investors will invest in a portfolio that has a size of value-at-risk which is smaller.

CONCLUSION

In this study has done research on optimization modeling portfolio without the risk-free asset based value-at-risk. So, that the conclusion of research, first the optimum solution of a model investment portfolio

Mean-VaR without risk-free assets expressed in terms of weight vector equation w* as given on Eq. 11 and 14. Second, based on the analysis of 5 stocks of assets for investment portfolio Mean-VaR obtained global optimum combination of weight portfolio is w^{Glob} = (0.1538, 0.0874, 0.0657, 0.0889, 0.6041) with expectation of return mean of portfolio and value at risk in the amount of 0.0033 and 0.0245 that achieved when value of the constant value of risk aversion is 0.264. Global optimum portfolio has a ratio between the average and the risk is the greatest.

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REFERENCES

Ahmadi, H. and D. Sitdhirasdr, 2016. Portfolio optimization is one multiplication, the rest is arithmetic. J. Appl. Finance Banking, 6: 81-93.

Alexander, S., T.F. Coleman and Y. Li, 2006. Minimizing CVaR and VaR for a portfolio of derivatives. J. Banking Finance, 30: 583-605.

Bansal, Y., S. Kumar and P. Verma, 2014. Commodity futures in portfolio diversification: Impact on investors utility. Global Bus. Manage. Res., 6: 112-121.

Baweja, M. and R.R. Saxena, 2015. Portfolio optimization with structured products under return constraint. Yugoslav J. Oper. Res., 25: 221-232.

Boudt, K., P. Carl and B.G. Peterson, 2012. Asset allocation with conditional value-at-risk budgets. J. Risk, 15: 39-68.

Cochrane, J.H., 2014. A mean variance benchmark for intertemporal portfolio theory. J. Finance, 69: 1-49.

Ghaemi, R., J. Sun and I.V. Kolmanovsky, 2009. An integrated perturbation analysis and sequential quadratic programming approach for model predictive control. Autom., 45: 2412-2418.

Goh, J.W., K.G. Lim, M. Sim and W. Zhang, 2012. Portfolio value-at-risk optimization for asymmetrically distributed asset returns. Eur. J. Operat. Res., 221: 397-406.

Golafshani, Z.Y. and S. Emamipoor, 2015. Portfolio optimization using two methods of mean-variance analysis and mean risk in Tehran Stock Exchange. Tech. J. Eng. Appl. Sci., 5: 128-133.

Hult, H., F. Lindskog, O. Hammarlid and C.J. Rehn, 2012. Risk and Portfolio Analysis: Principles and Methods. Springer, New York, USA., ISBN:978-1-4614-4102-1, Pages: 335.

Mustafa, S., S. Bano, M. Hanif and N. Jamal, 2015. Reducing portfolio quadratic programming problem into regression problem: Stepwise algorithm. J. Math., 47: 127-134.

Ogryczak, W. and T. Sliwinski, 2010. Efficient portfolio optimization with conditional value at risk. Proceedings of the 2010 International Multi Conference on Computer Science and Information Technology (IMCSIT), October 18-20, 2010, IEEE, Poland, Europe, ISBN:978-1-4244-6432-6, pp: 901-908.

Ogryczak, W., M. Przyluski and T. Sliwinski, 2015. Portfolio optimization with reward-risk ratio measure based on the conditional value-at-risk. Proceedings of the World Congress on Engineering and Computer Science (WCECS 2015), October 21-23, 2015, IAENG, San Francisco, USA., ISBN:978-988-14047-2-5, pp: 1-6.

Pinasthika, N. and B. Surya, 2014. Optimal portfolio analysis with risk-free assets using index-tracking and markowitz mean-variance portfolio optimization model. J. Bus. Manage., 3: 737-751.

Plunus1, S., R. Gillet and G. Hubner, 2015. Equivalent risky allocation: The new era of risk measurement for heterogeneous investors. Am. J. Ind. Bus. Manage., 5: 351-365.

Qin, Z., 2015. Mean-variance model forportfolio optimization problemin the simultaneous presence of random and uncertain returns. Eur. J. Oper. Res., 245: 480-488.

Shakouri, M. and H.W. Lee, 2016. Mean-variance portfolio analysis data for optimizing community-based photovoltaic investment. Data Brief, 6: 840-842.

Strassberger, M., 2005. Capital requirement, portfolio risk insurance and dynamic risk budgeting. Investment Manage. Financial Innovations, 3: 1-22.

Wang, Y., Y. Chen and Y. Liu, 2016. Modeling portfolio optimization problem by probability-credibility equilibrium risk criterion. Math. Prob. Eng., 2016: 1-13.