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The Effect of Surface Pressure and Elasticity to the Surface Minimum Energy with Fractional Order

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Abstract: Minimization and saving are two key words that currently often related to the solution of energy problem. In this study, we discuss the fact that when an elastic flat surface is pressed from the bottom at some points then a potential energy is formed at any points on the surface towards the bottom of the surface which is called the surface energy. The problem addressed in this study is to determine the functional relationship between the minimum surface energy with the value of the surface elasticity and the value of the pressure. In the previous research, the surface can be represented as a function of two variables in the form of double sine series. In this case, the energy is defined as the integral of the square of the Laplace operator with the order of the derivative generalized into fractional value. The pressure is the value of function at the pressed point coordinates while elasticity is the value of the fractional order. Some values of the minimum energy which depend on the surface elasticity, pressure and pressure point coordinates are used as the data to obtain a regression model of the functional relationship. Knowing the relationship between the involved varaiables, the resulting regression model can be used to determine the minimum energy easily. The model reveals that the coordinates of the pressure point does not significantly affect the surface energy. The surface energy is only depend on the pressure and surface elasticity.

Key words: Minimization, surface energy, fractional order, elasticity, regression models, operator

INTRODUCTION

This study discusses an issue on how to make a functional relationship model between the minimum surface energy with the surface pressure and surface elasticity. The surface is a function of two variables as a result of interpolation that minimizes energy as discussed by (Gunawan *et al.*, 2011). Their research discussed the problem on how to build a surface function as interpolation results that expressed in a double sine series:

$$u(x,y) := \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[a_{mn} \sin m\pi x \sin n\pi y] \quad (1)$$

with, $0 \leq y \leq 1$ and $0 \leq x \leq 1$ as well as minimizing energy of the form:

$$E_\beta(u) := \int_0^1\int_0^1 \left|(-\Delta)^{\beta/2} u\right|^2 dx\, dy \quad (2)$$

Where, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The last expression is called the Laplace operator, β is a fractional-order derivative of u (x, y). From u as Eq. 1, then Eq. 2 becomes:

$$E_\beta(u) = \frac{1}{4}\pi^{2\beta}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}{}^2 \left(m^2 + n^2\right)^\beta \quad (3)$$

These two studies ensure the solution of this problem to guarantee the existence of a solution, obtained through hilbert space:

$$W_\beta := \left\{u(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin m\pi x \sin n\pi y \,\middle|\, \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(m^2+n^2)^\beta a_{mn}{}^2 < \infty\right\}$$

with, $\beta > 1$ and the inner product:

$$< u, v >= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(m^2+n^2\right)^\beta a_{mn} b_{mn}$$

Following the existence of the function u (x, y) they constructed this function by using an interpolation method to find the solutions iteratively by considering the minimum energy value.

Teorema-1: Let V subspace of Hilbert space W_β where:

$$V = \left\{u(x,y) \in W_\beta \,\middle|\, u\left(x_i, y_j\right) = 0, i = 1,2,\ldots,M, j = 1,2,\ldots,N\right\}$$

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and U subset of Hilbert space W_β where:

$$U = \left\{u(x,y) \in W_\beta \middle| u(x_i, y_j) = c_{ij}, i = 1,2,...,M, j = 1,2,...,N\right\}$$

Then V is close d and U is nonempty, closed and convex. Furthermore, based on the above theorem and based the best approximation theory by Atkinson and Han (2001) obtained the following theorem of existence of a solution.

Theorem 2: Energy minimization (Eq. 2) has a single solution in W_β and the solution is $u = u_0 - \text{proj } v\ (u_0)$ with u_0 is an arbitrary element of U and proj v (u_0) states orthogonal projection of u_0 to V.

Proof: Suppose u_0 is an element in U. Then for some $v \in V$, u_0-v is also in U. Since, U is a convex subsets of W_β, then there should be a single element such that $v_0 \in V$ is the smallest norm. So, $u = u_0 - v_0$ is the solution in W_β in the minimization (Eq. 2).

Based on the theory of best approximation in Hilbert space, the element $v_0 \in V$ which makes the minimum value is the orthogonal projection of u_0 in V, i.e., v_0 = proj v (u_0) (Gunawan *et al*., 2008). In practice, this was done with the help of a tedious computation using known mathematical softwares. The steps to obtain the solutions is briefly outlined below.

MATERIALS AND METHODS

In this study develop a technique to construct the solution iteratively by considering the minimum energy value. The detail steps taken to get a solution is described by Bretscher (1997) with the outline is as follows. The 1st is determining the basis of:

$$V = \left\{u(x,y) \in W \middle| u(x_i, y_i) = 0\right\} \text{ i.e.,} \left\{v_1, v_2, v_3, ...\right\}$$

The 2nd is orthogonalizing the basis $\{v_1, v_2, v_3, ...\}$ into a basis $\{w_1, w_1, w_3...\}$ using the Gram-Schmidt Method where:

$$w_i = v_i - \text{proj}_{v_{i-1}} v_i = v_i - \frac{<w_1, v_i>}{<w_1, w_1>} w_1 - \dots - \frac{<w_{i-1}, v_i>}{<w_{i-1}, w_{i-1}>} w_{i-1}$$

with $<w_j, v_i>$ expresses the inner product of vector w_j and vector v_i. In a more detail, the vectors can be broken down into:

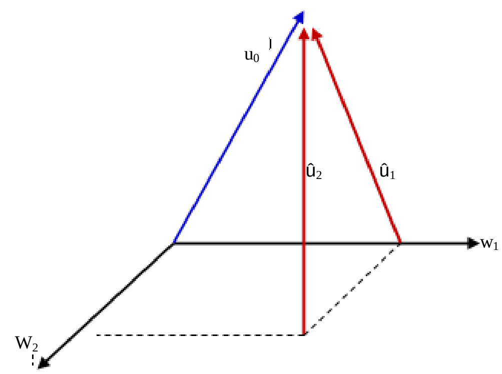

Fig. 1: Process to finding solutions functions $\hat{u}_n$

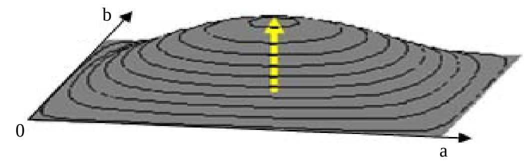

Fig. 2: Pressed elastic surface

$$w_1 = v_1$$

$$w_2 = v_2 - \frac{<v_2, w_1>}{<w_1, w_1>} w_1$$

$$w_3 = v_3 - \frac{<v_3, w_1>}{<w_1, w_1>} w_1 - \frac{<v_3, w_2>}{<w_2, w_2>} w_2, ...\text{etc}$$

The 3nd is determining the solutions function: $\hat{u}_n$, i.e.:

$$\hat{u}_1 = u_0 - \frac{<u_0, w_1>}{<w_1, w_1>} w_1$$

$$\hat{u}_2 = \hat{u}_1 - \frac{<u_0, w_2>}{<w_2, w_2>} w_2, ..., \text{etc.}$$

The last step is to calculate the energy by Eq. 3 to obtain the value of En for each solution function $\hat{u}_n$. The following Fig. 1 shows the process of finding the solution functions performed iteratively through orthogonalization and projection.

Implementation: The implementation of the theoretical problems described above can be explained as follows . If a flat surface of an elastic object is pressed from below at a point, then at each point on the surface will form potential energy directed towards the bottom of the surface. This potential energy is called the surface energy (Fig. 2).

According to Langhaar (1962), the surface equation which occurs after it has been pressed can be represented by a double sine series (Hogg and Craig, 1995):

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (4)$$

which satisfy the boundary condition w = 0, $w_{xx} = 0$ and $w_{yy} = 0$ for x = 0 or x = a and for y = 0 or y = b. Voltage energy on the surface is directed downward and given by:

$$E = \frac{1}{2}D\int_0^a\int_0^b \left(w_{xx}+w_{yy}\right)^2 dx\,dy = \frac{1}{2}D\int_0^a\int_0^b (\Delta w)^2 dx\,dy \qquad (5)$$

where, D is the elasticity of the surface. By substituting Eq. 4 into Eq. 5 then it is obtained:

$$E = \frac{1}{8}\pi^4 abD\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 {a_{mn}}^2 \qquad (6)$$

It is seen that the function of two variables in Eq. 1 is equal to the surface Eq. 4 for a = b = 1 whereas, the energy in Eq. 2 and 3 is identical to the energy in Eq. 5 and 6 that differ only in the constant. By calculating the energy value in Eq. 3, the minimum energy will depend on the value of the fractional order β which can be identified with the value of the elasticity of D in Eq. 6. Thus, the surface energy in general will depend on three things:

- Elasticity surface denoted by
- The coordinates of the pressure point on the surface which denoted by (x_i, y_i)
- The pressure at the pressure point which denoted by $C_i = u(x_i, y_i)$

The calculation of the minimum surface energy requires steps of hundreds or even thousands of iterations which is regarded as inefficient, eventhough it is implemented using some commercial software like maple or matlab. Given this situation in this study we propose a regression model with the aim in addition to simplify the calculation is to determine the functional relationship between variables. Independent variables are elasticity, the point coordinates where the pressing occurs and the compressive stress value at the pressing point while the dependent variable is the surface energy. The discussion of the problem is divided into three cases regarding on how the surface of the elasticity β is pressed:

- Pressed at the arbitrary point (x, y) with a pressure of one unit
- Pressed at any two points with a pressure of C_1 unit at (x_1, y_1) and of C_2 units at the point (x_2, y_2)
- Pressed at any one point with a pressure of C units

Table 1: Surface energy data distribution with n = 40

Min	1Q	Median	3Q	Max
0.3172	0.5453	0.64028	0.6997	0.88525

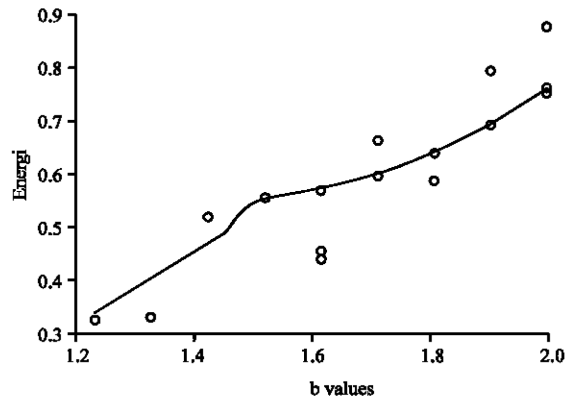

Fig. 3: Scatter Plot β-En

RESULTS AND DISCUSSION

Surface energy model with one point of pressure: In the first study, a rectangular elastic surface that is placed on a coordinate axis with 0≤ x ≤1 and 0≤ y ≤1 is pressed in one arbitrary point (x, y) with a pressure of one unit. Using the software that has been made with the input value of x and y as the coordinates of the point pressand the elasticity of the surface that the range of values in the interval $1 \leq \beta \leq 2$. By taking the number of iterations 1225, the data is obtained with the distribution as shown in Table 1.

Values in the table is the surface energy (E_n) whose magnitude depends on the elasticity of the surface (β) and the coordinates of the point press (x, y). 1Q stated first quartile and 3Q stated third quartile. Based on the data above will be created regression models in which En as the dependent variable and β, x, y as independent variable. Furthermore, to see the linearity elasticity of the surface energy, scatter plots are used in addition to this. It is intended as an early detection is a linear regression fit when applied.

From Fig. 3, it is clear that there is a linear relationship between the surface energy with the elasticity. Thus, the linear regression model is suitable for the functional form of the surface energy which depends on the elasticity. The resulting surface energy model with a single press point is given by:

$$En = -0.35938+0.55412\beta+0.02399x+0.01299y \qquad (7)$$

The p-value, F-count of this model is 1.615e-15 which is samller then 0.05, showing that the regression model is fit for use. To see how far the influence of all independent variables on the dependent variable, simultaneously, we use adjusted r-squared value and not the value of R^2. This is because this model has more than two independent variables (Hogg and Craig, 1995).

With adjusted R^2 the value is of 0.8499, meaning that β variables, x and y affect the surface energy simultaneously by 85%. However, the analysis of this model shows that the variables x and y do not significantly affect the surface energy because of its p-value (0.534 and 0.761 which >0.05).

Surface energy model with two point of pressure: On the second issue which is the pressure on the elastic surface coordinate plane with $0\le x\le 1$ and $0\le y\le 1$ is located at any two points, namely at the point (x_1, y_1) with a pressure of C_1 units and at the point (x_2, y_2) with pressure of C_2 units.

Using the software that has been made with the input values x_1, y_1, x_2, y_2, C_1, C_2 and β with the range of values $1\le\beta\le 2$ and the number of iterations is 1225, the resulting surface energy is obtained with the distribution as shown in Table 2. Surface energy model with a double press point is:

$$En = 0.709 + 1.104\beta - 0.993x_1 - 0.925y_1 + 1.847C_1 - 2.553x_2 - 0.675y_2 + 0.51C_2 \quad (8)$$

The analysis of the regression model gives the value of 0.003 significance, <0.05 which means that the regression model is fit for use. However, we find the fact that the only variable which have a significant influence on the surface Energy En is only β and C_1 with the significance value 0.05 while the value of another variable of significance is >0.05. Similarly, the value of adjusted R^2 is only 0.451, meaning that the effect of six independent variables on the surface energy simultaneously is only 45% while the rest is an effect that cannot be explained.

General surface energy model: As noted before that the analysis of the surface energy model with a single tap point presented in Eq. 7 turns the variables x and y do not significantly affect the energy En . Likewise, based on the model of the surface energy with two hit points as shown in Eq. 8, it turns out the variables x_1, y_1, x_2, y_2 and C_1 are also not significantly affect the energy En. Based on the above results, the 3rd retrospective case series was deemed necessary to create a regression model by eliminating variables that are not much to influence the surface energy.

Here are the results of the research in which the independent variables x_1, y_1, x_2, y_2 and C_1 whose influence on surface energy is not significant are eliminated. Thus, we need only to show the relationship between the magnitude of the surface energy En as the dependent variable with the surface elasticity β as the first

Table 2: Surface energy data distribution with n = 30

Min	1Q	Median	3Q	Max
0.02637	0.27929	0.47429	1.2341	2.39068

Table 3: Surface energy data distribution with n = 20

Min	1Q	Median	3Q	Max
0.01338	0.17934	0.2415965	0.527108	0.7656

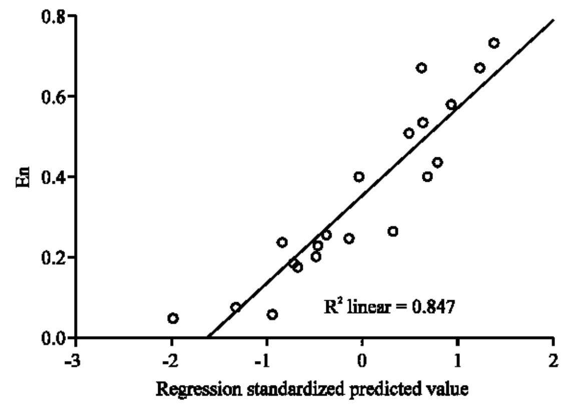

Fig.4: Scatterplot β and C to En; scattarpiot department variable: En

independent variable and the magnitude of the pressure at point C as the second independent variables in the form of:

$$E_n = K_0 + K_1\beta + K_2C$$

where, K_0, K_1 and K_2 are constants as parameter. For these needs by using the software, we calculated the surface energy which depends on β and C only. The results are obtained for n = 20 with the distribution is shown in Table 3. Furthermore, to see the linearity relationship between elasticity and surface energy, scatter plots are used looklike in Fig. 4. The results showed that the spearman correlation coefficient between β to En is 0.576 which regarded as a considerable influence with value of significance is 0.008 smaller than 0.05. Likewise for the relationship between C and En, we found with the correlation coefficient of 0.797 and 0.000 significance meaning the relationship is pretty good. The resulting model for the surface energy is given by:

$$E_n = -0.649 + 0.360\beta + 0.614C$$

With the value of "adjusted r^2" of 0.829, this shows that the influence of β and C together to En is 82.9% while the rest is influenced by other factors that have not been analysed. Therefore, based on the above analysis, regression models of surface energy obtained was sufficient to present a very significant influence of the elasticity of the surface and mmagnitude of pressure on the surface energy.

CONCLUSION

In this study, we have presented the discussion of the functional relationship between the surface energy with several independent variables like the elastisity of the surface, the magnitude of the pressure given to the surface and other related variables. The results showed that the coordinates of the location where the pressure is given and the magnitude of the pressure in the second pressure point do not have a significant effect on the magnitude of the surface energy. After this insignificant independent variables are omitted and taken just the elasticity and the magnitude of the tension at the first point, namely β and C, then the resulting model has a good level of significance. Thus, the surface energy is generally only depends on the elasticity of the surface and the magnitude of the pressure on a point where the pressure occur.

Another conclusion from what has been described in this study is after the existence of the solution of energy minimization problem has been guaranteed analytically, then we have two options for calculating the surface energy, namely through assisted numerical calculation software and through the regression model has been generated.

RECOMMENDATIONS

For the future, the research could be developed to the stage of implementation is to create a prototype medical devices are usually inserted into the human body so that the surface is smooth has the energy or minimum frictional forces that do not hurt the patient.

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