

Classification of Road Surface Conditions Using Vehicle Positional Dynamics

¹Timothy Tzen Vun Yap, ¹Hu Ng, ²Vik Tor Goh and ¹Jeng Weng Seah

¹Faculty of Computing and Informatics,

²Faculty of Engineering, Multimedia University, Persiaran Multimedia,
63100 Cyberjaya, Malaysia

Abstract: The objective of this research is to collect and analyze road surface conditions in Malaysia and develop a classification model that can identify road surface conditions from the collected data. Data is collected through a mobile application that collects positional dynamics of vehicles on the road. Features considered include statistical measures such as minimum, maximum, standard deviation, median, average, skewness and kurtosis. Selection of the extracted features is performed using Ranker, Tabu search and Particle Swarm Optimization (PSO) followed by classification using k-Nearest Neighborhood (k-NN) Random Forest (RF) and Support Vector Machine (SVM) with linear, Radial Basis Function (RBF) and polynomial kernels. The classification model that gave the highest accuracy is SVM (RBF) with a Correct Classification Rate (CCR) of 91.71%. Trailing closely was RF at 91.17%. Although not as accurate as SVM, the difference was negligible and its computational time was much lower than the former. In the feature selection process, features which provide positive contribution to the classification process were chosen and the best performances were produced by PSO with an average CCR of 89.88%. Tabu selected 11 features while PSO selected 13 features where the extra two features made a difference in the results. Ranker selected every single feature but has the lowest average CCR. This is attributed to a subset of features that were selected were ineffectively impeding the classification. The features and classification model employed were able to effectively classify road surface conditions from vehicle positional dynamics. Using only 3D positional readings of the vehicle and standard statistical measures, road surface conditions can be effectively identified for the prioritisation and facilitation of road maintenance.

Key words: Road surface condition, feature selection, classification, vehicle positional dynamics, machine Learning

INTRODUCTION

Many cities spend large amounts of money to repair their roads (Anonymous, 2013). In 2014, the state government of Selangor in Malaysia spent more than half a billion ringgit in improving roads (Anonymous, 2013). Although, the amount spent on improving and maintaining the roads is substantial, deterioration of roads are inevitable due to prolonged use and weather. Bad roads damage vehicles and are hazardous to motorists especially motorbikes and they are one of the causes of traffic accidents. Potholes in specific can cause serious damage to vehicles (CNBC, 2015).

However, maintaining roads in good and safe condition is challenging due to heavy traffic as well as unpredictable weather, especially in Malaysia. Due to resource constraints, determining which roads need fixing or maintenance becomes important. Road surface condition monitoring cannot be easily solved by

deploying static sensors on the roads (Eriksson *et al.*, 2008). Thus, this research seeks to develop a platform that is able to collect data of the surface conditions of roads while vehicles traverse them and automatically classify them for the purpose of informing the authorities and assist in decision making and resource allocation. This is accomplished by the development of a mobile application which collects vehicle positional dynamics from the built-in accelerometer and a classification model that identifies road surface conditions from the collected data.

The data collected are recordings of 3D positional readings of the vehicle (x, y and z axes) while the features considered are standard statistical measures. The simplicity of the approaches underlines the ease for implementation and deployment.

Literature review: Chen *et al.* (2013) implemented a crowdsourcing-based road surface monitoring system by mounting hardware modules on distributed vehicles to

perform detection of road potholes and evaluation of levels of road roughness. Vibration pattern, location and vehicle velocity was collected with the use of cost effective accelerometers and GPS devices. During the study, a set of hardware devices which included a three-axis accelerometer, a Global Positioning System (GPS) Module, a Microcontroller (MCU) and a Global System for Mobile communications (GSM) module were placed on a vehicle (Chen *et al.*, 2013). While the vehicle was travelling, the accelerometer recorded positional data, while the GPS module recorded the time, location and the velocity. The MCU extracted the collected data using the improved Gaussian Mixture Model algorithm (i-GMM) developed by Chen *et al.* (2013) and the results were sent to the central server through the GSM module. i-GMM is able to learn background signal online without the need to train parameters for different road conditions beforehand (Chen *et al.*, 2013).

Mohan *et al.* (2008) presented Nericell which utilized the sensors on a mobile device to detect braking, bumps and honking in the surrounding area. Nericell made use of a microphone and an accelerometer to detect honks, braking and bumps. During the study, a canonical frame of reference was outlined with the x-axis facing the front of the vehicle, the y-axis facing the side and the z-axis vertically facing upwards. To detect the rate of braking, the mean of acceleration along the X-axis (aX) over the sliding window was observed together with indications of a braking event when the mean of aX of 0.12 g was over 4 sec long (Mohan *et al.*, 2008). It was found out that when the vehicle was traveling at low speeds, the value of acceleration along the z-axis (aZ) increased sharply when the wheel entered a pothole. However, the continuous dip in the values of aZ was also marked. Therefore, a new heuristic called z-sus was proposed which looked for a continuous dip in aZ to differentiate potholes from bumps (Mohan *et al.*, 2008).

Jokela *et al.* (2009) developed a system called IcOR that could detect slippery road conditions and warn the driver when the car is close to a slippery road. The IcOR system utilised a Monochrome stereo camera pair that produced a maximum resolution of 640×480 pixel, to identify light polarization changes and perform graininess analysis for ice detection on the surface of the road with a classification accuracy of >90%.

Eriksson *et al.* (2008) developed a pothole detection system that can differentiate pothole from other road anomalies using accelerometer data that were recorded by a group of taxi drivers. The taxi drivers were using a GPS device and an accelerometer to collect raw sensor data. The data was uploaded to the central server automatically whenever there was an available network connection. The

central server classified detections based on location. The data was labelled manually through constant driving on several streets in Boston, USA (Eriksson *et al.*, 2008). One of the concerns in this study was that the placement of the accelerometers in the vehicle might affect signal quality. They had placed accelerometers in three places inside a car such as the dashboard, windshield and an embedded PC not firmly attached to the vehicle. Signal accuracy was important because for the potholes to be properly traced, multiple signals have to be combined (Eriksson *et al.*, 2008). Processing filters were applied to detect the presence of pothole impact where every filter was designed as a control to eliminate every single non-pothole event such as filtering slow speed for door slams events and low-frequency components from the acceleration signal for braking, turning and veering (Eriksson *et al.*, 2008).

Perttunen *et al.* (2011) developed a pattern recognition system for identifying road surface anomalies, using data collected from a mobile device placed on a holder on the windshield of a vehicle. The holder had to be positioned carefully so that the accelerometer coordinates could be maintained approximately the same across data collection drives. The irregularities were classified into two classes according to severity where Type 1 represented small potholes and railroads and Type 2 represented speed bumps and irregularities that could cause accidents or damage the vehicle. The anomaly recognition system was based on accelerometer and GPS data, where the GPS data was used to estimate speed (Perttunen *et al.*, 2011). Fast Fourier Transform (FFT) features were used in the classification process. Filtering was also applied to remove signal artifacts from non-anomalies such as door slams and braking (Perttunen *et al.*, 2011).

Luo *et al.* (2012) proposed a multi-sensor integrated mobile surveying system which consisted of laser sensors (LADAR), GPS, Inertial Navigation System (INS) odometer and camera. INS was used as the reference navigation sensor because it could provide constant location data, velocity data and attitude data. Odometer was used to acquire the speed and mileage of the vehicle while GPS was used to obtain assured position and velocity measurement in open space as well as correct the errors of the INS and odometer. Navigation data was extracted from INS and odometer in order to derive the location and altitude for each surveyed road resource when the system was in areas without location signal or low location signal such as in a tunnel. This system could monitor white lines and road surface markers through remission, the capability of a material to redirect light

back. The system was able to perform road surface profiling, detecting surface inhomogeneity as well develop 3D models of roads (Luo *et al.*, 2012).

The techniques discussed in this Section utilized elaborate setups and multitudes of equipment. In view of this, this research intends to make data collection and classification simple. This is accomplished with the use of a single smart device such as a smartphone and it can be placed in an arbitrary position in the vehicle (but the position is fixed for the whole duration of the data collection). A smartphone is common and easily accessible; this can reduce cost in data collection as well as placing the task on everyday drivers without the need for training and complex instructions. Although a single smart device setup may not be sufficient to isolate non-anomalies such as door slam, the collection of the data as a time series may mitigate this effect through the use of conventional statistical features.

MATERIALS AND METHODS

Data collection: A Samsung Galaxy W. Android smartphone was used to collect data. A data collection application was developed to collect accelerometer and GPS readings. Accelerometer readings (x, y and z axes) readings relative to the phone were collected every 1 ms for the duration of the collection. GPS readings including latitude and longitude were also recorded for reference but not used in the classification.

Only one car was used to collect the data, a Proton Waja. The phone was attached to a mount that is attached to the windshield of the car as shown in Fig. 1. The accelerometer positional axes were aligned to the car's axes. When a surface condition was identified, the data collection was started before traversing and only stopped when the vehicle had gone passed. Road surface conditions considered include:

- Smooth surfaces
- Uneven surfaces
- Potholes
- Speed bumps
- Hazard lines

Turns and hard stops are used as control for the classification process. Data were collected on the roads in the vicinity of the townships of Cyberjaya and Rawang, at a constant speed of 20 kmh⁻¹. Examples of the surface conditions are shown in Fig. 2. For each condition, 50 sets of readings were recorded. In total, 1050 sets of data (time series of the x, y and z axes readings) were collected.



Fig. 1: Position of phone in the vehicle

Table 1: List of statistical parameters

| Statistical parameters | Description |
|------------------------|---|
| Minimum | Smallest number in a data set |
| Maximum | Largest number in a data set |
| Standard deviation | Measurement of the variation of a set of data from its mean |
| Median | The number in the middle of the set of the data |
| Average | The arithmetic mean of the arguments in a data set |
| Skewness | Characterization of the degree of asymmetry of the data set around its mean |
| Kurtosis | The peak value of the data according to the normal distribution of the data set |
| Absolute Skewness | Absolute value of the skewness |
| Absolute Kurtosis | Absolute value of the kurtosis |

Feature extraction: For each condition, nine new features were formed from major statistical parameters of its corresponding set of data. Table 1 lists the nine statistical features. The features were normalized between 0 and 1 so that the features were regulated and no bias is present. Without normalization more weight would have been allocated implicitly to features with larger values than those with smaller ones for distance measures such as Euclidean distances.

Feature selection: Feature selection was carried so that the dimensions of the extracted features can be reduced. Ranker, Tabu Search and PSO were applied to determine extracted features which could provide constructive contribution in the classification progression.

Ranker utilizes a correlation attribute evaluator to rank extracted features (Hall and Holmes, 2003). The worth of an attribute is measured by evaluating the correlation between the attribute and the subject where each nominal extracted feature is treated as the individual significance indicator on a merit basis and a weighted average



Fig. 2: Road surface conditions from left, smooth surface, uneven surface, pothole, speed bump and hazard lines

establishes the overall correlation for a nominal extracted gait feature (Hall and Holmes, 2003). Ranker utilizes correlation for selecting the most relevant attributes in a dataset. The correlation is calculated between each attribute and the output variable and selects only those attributes that have a moderate-to-high positive or negative correlation and drop those attributes with a low correlation.

Tabu search is used for mathematical optimization with a meta-heuristic search method that employs local search method (Glover, 1986). Local search is able to obtain good solutions by continually considering a better solution from the current solution's neighborhood. However, there is a tendency of being stuck in suboptimal regions where more than one solution is fitting similarly. By relaxing the basic rules of the local search, Tabu Search enhanced the performance of local search by using memory structures that describe visited solutions. If a potential solution violated a rule or is visited more than once within a short time it will be marked as "Tabu" which means forbidden in order to prevent the algorithm from considering the same possibility again (Glover, 1986).

PSO is a global search algorithm that optimizes the problem by improving a potential solution iteratively according to a given measure of the quality (Moraglio *et al.*, 2007). From a number of potential solutions which are called 'particles', these particles are moved around and inside the search-space using a mathematical formula to determine the particle's positions and velocities. Every particle movement is affected by its own local best position. The particles will eventually be guided to the

best position in the search-space (Moraglio *et al.*, 2007). The movements of the particles are guided by their own best known position in the search-space as well as the entire swarm's best known position. When improved positions are being discovered these will then come to guide the movements of the swarm. The process is repeated however with no guarantee, so that a satisfactory solution will eventually be discovered. These methods are used to identify unneeded, irrelevant and redundant features that do not contribute to the accuracy of the classification model.

Classification: Three classification techniques, k-NN with Euclidean distance metrics, RF and SVM were employed to assess the implementation of this approach. k-NN, a non-parametric classifier is used in the differentiation established different subjects in the training data through the exploitation of the overall collected data in the feature space by substantiating its memory (Fix and Hodges, 1989). The information of its k-nearest neighbors in that memory controls the classification of unknown classes-the majority vote of the nearest neighbors in the training data determines which class the subjects belong to. A class is assigned based on the number of neighbor, k (Fix and Hodges, 1989). The k-NN algorithm models a decision problem with instances or examples of training data that are deemed important or required to the model. Such methods typically build up a database of example data and compare new data to the database using a similarity measure in order to find the best match and make a prediction. RF is a concept of the general

technique of random decision forests that are an ensemble learning method where multiple algorithms are used to obtain better predictive results for classification and regression (Breiman, 2001). RF describes a margin function that is used to measure the amount of the average number of votes for the exact class go above the average vote for any other class that exist in the dependent variable. This measure provides a useful way of making predictions and also a way that associates a confidence measure with those predictions (Breiman, 2001). The parameters utilized include the number of attributes (K) number of execution Slots (S), depth (d) and number of Iterations (I). RF models are composed of multiple weaker models that are independently trained and whose predictions are combined in some way to make the overall prediction.

SVM is a learning machine for two-group classification problems (Cortes and Vapnik, 1995). In SVM, the input vectors are mapped to a very high dimension feature space in a non-linear manner. A linear decision surface is structured collectively based on the feature space. Three kernels were used, namely, linear, RBF and polynomial kernels. The parameters that were trained for these three kernels:

- Linear kernel: Cost (C)
- RBF kernel: Cost (C) and Gamma (G)
- Polynomial kernel: Cost (C) Gamma (G), Degree (D) and coefficient (R)

Ten folds cross validation was employed in the classification process. The collected road surface condition data were divided into ten disjoint subsections in a random manner, so that nine could be utilized for training and one for validation. The cross-validation process was repeated for ten turns. The feature vectors of each disjoint subsection were channeled into classifiers as the validation test. Then, the single mean correct classification rate was obtained by averaging the cross validation results.

Performance evaluation: Three quality measures were used in this experiment: Correct Classification Rate (CCR), True Positive Rate (TPR) and False Positive Rate (FPR).

CCR is represented as the percentage of the correctly classified number of subjects divided by the total number of subjects in the dataset, given as:

$$CCR = \frac{T_p}{T} \times 100\% \quad (1)$$

Where:

T_p = The number of road surface conditions recognized

T = The total number of road surface condition data

Table 2: Parameter values for each classifier

| Classifiers | Parameters |
|---|--------------------------------------|
| k-Nearest neighbourhood | k = 1 |
| Random forest | K = 2, S = 2, d = unlimited, I = 900 |
| Support vector machine with linear kernel | C = 48 |
| Support vector machine with polynomial kernel | C = 4096, G = 0.5, D = 3, R = 1.8 |
| Support vector machine with RBF kernel | C = 2048, G = 0.5 |

TPR is denoted as the percentage of the number of correctly classified subjects divided by the total number of the subjects in a class. It is given as:

$$TPR = \frac{Q_p}{P} \times 100\% \quad (2)$$

Where:

Q_p = The number of road conditions correctly recognized of a class

P = The total number of data in a class

FPR is denoted as the percentage of the subjects wrongly labeled belonging to a class but belonging to a different class among all the subjects which are not of that class. It is represented as:

$$FPR = \frac{Q_N}{T - P} \times 100\% \quad (3)$$

where, Q_N represents the number of road surface conditions wrongly assigned to a different class.

Designs of experiments: The experiments were performed in two phases: training and testing. All data collected were used during the training and testing phases. For training, the classification models were obtained through the optimization of the parameters for each classifier. The parameters for testing were set according to heuristic results obtained from training and the values are shown in Table 2. During testing, classification is performed through the utilization of the models obtained from training.

RESULTS AND DISCUSSION

Performance of classifiers and feature selectors: Three classifiers, k-NN, RF and SVM were applied to find the CCR, TPR and FPR and to verify the consistency of the results. Table 3 shows the average CCR, TPR and FPR obtained for each feature selector. For clarity, the chart of the CCR of the classifiers for each feature selector is shown in Fig. 3.

Based on the results in Table 3 the classifier that gave the highest accuracy is SVM (RBF) with CCR of 91.71%. In particular, SVM (Polynomial) and SVM (RBF) outperformed SVM (Linear). This is due to the non-linear

Table 3: CCR, TPR and FPR of the classification results

| Parameter | Ranker | | | Tabu | | | PSO | | |
|------------|--------|-------|------|-------|-------|------|-------|-------|------|
| | CCR | TPR | FPR | CCR | TPR | FPR | CCR | TPR | FPR |
| k-NN | 85.17 | 85.10 | 2.50 | 86.91 | 87.40 | 2.10 | 87.66 | 87.40 | 2.00 |
| RF | 91.17 | 91.40 | 1.40 | 89.89 | 90.90 | 1.50 | 90.06 | 91.70 | 2.20 |
| SVM (Ln) | 90.29 | 90.30 | 1.60 | 89.14 | 89.10 | 1.80 | 90.00 | 90.00 | 1.70 |
| SVM (RBF) | 91.71 | 91.70 | 1.40 | 91.14 | 91.10 | 1.50 | 91.40 | 91.40 | 1.40 |
| SVM (Poly) | 89.71 | 89.70 | 1.70 | 90.00 | 90.00 | 1.70 | 90.29 | 90.30 | 1.60 |
| Average | 89.61 | 89.64 | 1.72 | 89.42 | 89.70 | 1.72 | 89.88 | 90.16 | 1.78 |

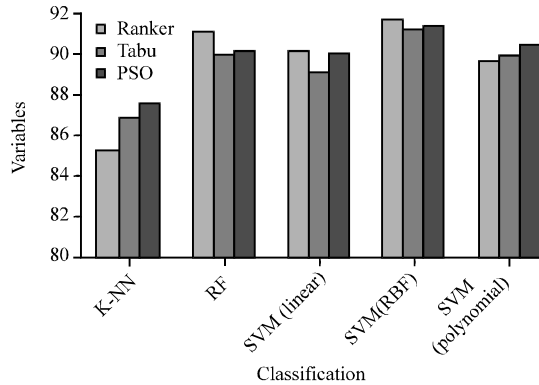


Fig. 3: Correct classification rate of classifiers

nature of the generated features which allows for the kernel trick in the nonlinear SVM (Polynomial and RBF) algorithms to adapt to the maximum-margin hyper-plane in the transformed feature space. SVM in nature computes optimal hyper-plane with respect to margin maximization, thus is able to perform well in high dimension feature space (Suykens *et al.*, 2002, Suykens and Vandewalle, 1999). Trailing behind was RF at 91.17%. Although not as accurate as SVM, the difference was negligible and its computational time was much lower than the former.

For the feature selectors, it is also observed that the best performances were produced by PSO with an average CCR of 89.88% where 13 of the 27 extracted features were selected for classification, compared to 11 (Tabu) and 27 (Ranker). Features in common selected by Ranker, Tabu and PSO are listed.

Common features for Ranker, Tabu and PSO Features:

- Minimum of x-axis measurements
- Standard deviation of x-axis measurements
- Median of x-axis measurements
- Average of x-axis measurements
- Kurtosis of x-axis measurements
- Minimum of y-axis measurements
- Standard deviation of y-axis measurements
- Skewness of y-axis measurements
- Minimum of z-axis measurements
- Maximum of z-axis measurements

Tabu selected 11 features while PSO selected 13 features, the two extra features being maximum of x-axis measurements and the median of z-axis measurement. PSO gave the highest average CCR with the extra two features making a difference in the results. PSO was also found to perform better than a well-designed Tabu search on certain applications, due to the nature of the PSO’s algorithm in obtaining the global optimal solution (Ho *et al.*, 2007; Allahverdi and Al-Anzi, 2006). On the other hand, Ranker selected every single feature but has the lowest average CCR, attributed to the features were selected but ineffectively impeding the classification.

From data along the x-axis are more evident. As the phone was vertically held by a mount when data was collected, data along the x-axis are more impactful as the data movement in the vertical plane is more apparent when the vehicle goes over speed bumps, potholes and uneven surfaces. However, this may also lead to misclassification of these three surface conditions (any one condition is classified as the other two) as they share certain similar characteristics in terms of elevation of the vehicle from its normal position. To overcome this, additional features such as vehicle linear acceleration and angular velocity calculated from the accelerometer readings using the Euclidean norm and frequency domain versions of the features can be included to mitigate the variations in the readings. These features will be considered in future research.

CONCLUSION

A classification model using statistical parameters is presented to classify road surface conditions. Three feature selectors and classifiers were employed to assess the performance of the proposed model. The model is found to be able to distinguish road surface conditions. It can potentially predict road damage, facilitate maintenance and resource management.

However, additional features such as vehicle linear acceleration and angular velocity as well as frequency domain features may improve classification as these features could be robust to variations in the readings. The model could also be improved by incorporating features from other forms of input for example video. A fusion classification model incorporating these features with statistical parameters and video will be considered in future with more road surface conditions introduced.

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REFERENCES

- Allahverdi, A. and F.S. Al-Anzi, 2006. A PSO and a tabu search heuristics for the assembly scheduling problem of the two-stage distributed database application. *Comput. Operat. Res.*, 33: 1056-1080.
- Anonymous, 2013. The high cost of America's bad roads and bridges. *CNN Money, USA*. <http://money.cnn.com/2013/02/12/news/economy/infrastructure-spending>
- Breiman, L., 2001. Random forests. *Mach. Learn.*, 45: 5-32.
- CNBC., 2015. Bad roads cost car owners billions: Report. *NBCUniversal, New York, USA*. <http://www.cnbc.com/2015/07/23/bad-roads-cost-car-owners-billions-report.html>
- Chen, K., M. Lu, G. Tan and J. Wu, 2013. CRSM: Crowdsourcing based road surface monitoring. *Proceedings of the 2013 IEEE Joint 10th International Conference on High Performance Computing and Communications and Embedded Ubiquitous Computing (HPCC/EUC), November 13-15, 2013, IEEE, New York, USA., ISBN:978-0-7695-5088-6*, pp: 2151-2158.
- Cortes, C. and V. Vapnik, 1995. Support-vector networks. *Mach. Learn.*, 20: 273-297.
- Eriksson, J., L. Girod, B. Hull, R. Newton, S. Madden and H. Balakrishnan, 2008. The pothole patrol: Using a mobile sensor network for road surface monitoring. *Proceedings of the 6th International Conference on Mobile Systems, Applications and Services, June 17-20, 2008, Breckenridge, Colorado*, pp: 29-39.
- Fix, E. and J.L. Hodges, 1989. Discriminatory analysis: Nonparametric discrimination: Consistency properties. *Int. Stat. Rev./Revue Int. Stat.*, 57: 238-247.
- Glover, F., 1986. Future paths for integer programming and links to artificial intelligence. *Comput. Operat. Res.*, 13: 533-549.
- Hall, M.A. and G. Holmes, 2003. Benchmarking attribute selection techniques for discrete class data mining. *IEEE Trans. Knowledge Data Eng.*, 15: 1437-1447.
- Ho, S.L., S.Y. Yang, G.Z. Ni and K.F. Wong, 2007. An improved PSO method with application to multimodal functions of inverse problems. *IEEE. Trans. Magn.*, 43: 1597-1600.
- Jokela, M., M. Kutila and L. Le, 2009. Road condition monitoring system based on a stereo camera. *Proceedings of the IEEE 5th International Conference on Intelligent Computer Communication and Processing (ICCP 2009), August 27-29, 2009, IEEE, Finland, ISBN:978-1-4244-5007-7*, pp: 423-428.
- Luo, X., X.T. Ren, Y. Li and J.J. Wang, 2012. Mobile surveying system for road assets monitoring and management. *Proceedings of the 2012 7th IEEE Conference on Industrial Electronics and Applications (ICIEA), July 18-20, 2012, IEEE, Sydney, New South Wales, Australia, ISBN:978-1-4577-2118-2*, pp: 1688-1693.
- Mohan, P., V.N. Padmanabhan and R. Ramjee, 2008. Nericell: Using mobile smartphones for rich monitoring of road and traffic conditions. *Proceedings of the 6th ACM Conference on Embedded Network Sensor Systems, November 5-7, 2008, ACM, Raleigh, North Carolina, USA., ISBN:978-1-59593-990-6*, pp: 357-358.
- Moraglio, A., D.C. Chio and R. Poli, 2007. Geometric particle swarm optimisation. *Proceedings of the European Conference on Genetic Programming, April 11-13, 2007, Springer, Berlin, Germany*, pp: 125-136.
- Perttunen, M., O. Mazhelis, F. Cong, M. Kauppila and T. Leppanen *et al.*, 2011. Distributed Road Surface Condition Monitoring Using Mobile Phones. In: *Ubiquitous Intelligence and Computing, Hsu, C.H., L.T. Yang, J. Ma and C. Zhu (Eds.)*. Springer-Verlag, Berlin, Germany, pp: 64-78.
- Suykens, J.A., D.J. Brabanter, L. Lukas and J. Vandewalle, 2002. Weighted least squares support vector machines: Robustness and sparse approximation. *Neurocomputing*, 48: 85-105.
- Suykens, J.A.K. and J. Vandewalle, 1999. Least squares support vector machine classifiers. *Neural Process. Lett.*, 9: 293-300.