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Sliding Motion of the Yarn on the Package Surface During Yarn Unwinding

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Abstract: Tension in the yarn and its oscillations during the over-end unwinding of the yarn from stationary packages depend on the unwinding speed, the shape and the winding type of the package, the air drag coefficient and also the coefficient of friction between the yarn and the package. The yarn does not leave the surface package immediately at the unwinding point. Instead, it first slides on the surface and then lifts off to form the balloon. The problem of simulating the unwinding process can be split into two smaller sub-problems: the first task was to describe the motion of the yarn in the balloon, the second one to solve the sliding motion.

Key words: Dynamics of yarn, friction between yarn and package, balloon theory, boundary comditions, oscillations, simulating

INTRODUCTION

The problem of yarn motion on the package surface during the unwinding can be treated in analogy with the motion of the yarn forming the balloon between the lift-off point and the eyelet, through which the yarn is being pulled.

The yarn is being withdrawn with velocity V through an eyelet where we also fix the origin O of our coordinate system (Fig. 1). The yarn is rotating aroun the z axis with an angular velocity ω. At the lift-off point Lp the yarn lifts from the package and forms a balloon. At the unwinding point up the yarn starts to slide on the surface of the package. Angle ϕ is the winding angle of the yarn on the package.

The general equation of motion for the yarn was derived and justified in one of the previous researches (Pracek and Jaksic, 2002a, b; Pracek, 2004, 2005; Pracek and Jaksic, 2005; Pracek, 2006, 2007, 2010a, b; Pracek and Franci, 2010a, b, c; Pusnik and Pracek, 2016):

$$P\left(D^2 r + 2\omega\times Dr + \omega\times(\omega\times r) + \omega\times r\right) = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \quad (1)$$

Where:

- r = The points from the origin of the coordinate system to a chosen point along the yarn
- ρ = The linear density of the yarn mass
- ω = The angular velocity vector of the spinning coordinate system in which the yarn is being described and which points along the z-axis
- D = The operator of the total time derivative which follows the motion of the point inside the spinning coordinate system, $D = \partial/\partial t|_{r,\theta,z} - V\partial/\partial s$
- T = The mechanical tension
- f = The linear density of external forces

In the part of the yarn which forms the balloon:

$$f = -\frac{1}{2} c_u \rho d\,|v_n|\,v_n \quad (2)$$

Where:

- c_u = The effective air-drag coefficient
- d = The yarn diameter
- v_n = = v-(v•t)t is the normal component of the yarn velocity (t is the unit tangent vector to the yarn at the given point). When however, the yarn is sliding on the package surface the quantity
- f = The related to the friction between the yarn and the package surface

Friction between the yarn and the package surface: There is a friction between the package and the yarn which is sliding on its surface before it lifts off to form the balloon. The yarn is exerting a normal force on the package (i.e., a force perpendicular to the package surface, thus in radial direction). This force is not known a priori but must be

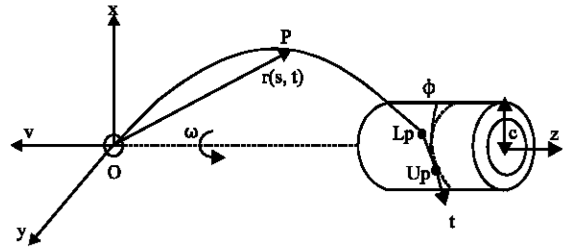

Fig. 1: Mechanical setup in overend yarn unvinding from cilimdrical package

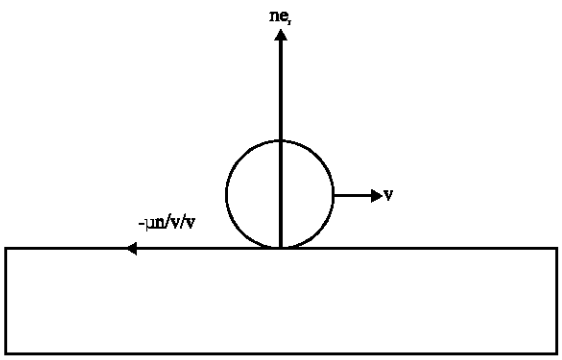

Fig 2: The force of friction between the package surface and the yarn

determined as part of the solution to the full problem. The simplest expression of the friction law states that the friction force is proportional to the normal component of the force. The coefficient of proportionality is known as the coefficient of friction μ. The friction force points in the direction opposite to the yarn motion.

The quantity f in Eq. 1 therefore has two components: the radial force of the package on the yarn (which is equal in magnitude to the force of the yarn on the package in accordance with the Newton's law of reciprocal action) and the friction force proper (Fig. 2):

$$f = ne_r - \mu n \frac{v}{|v|} \quad (3)$$

Where:

- n = The linear density of the normal component of the force between the yarn and the surface
- e_r = The unit vector in the radial direction
- v/|v| = The unit vector in the direction of the yarn

When the yarn slides on the surface, it thus experiences the normal force ne_r and the friction force $-\mu n\, v/|v|$. The friction law is at best a rough approximation to a more complex real behavior. In reality, the coefficient of friction depends in a complicated way on the sliding velocity (Pracek, 2004) and it is different at various points of the package surface since the package is seldom fully homogenous. We thus take μ to be some average coefficient of friction which one can determine empirically (Pracek and Pusnik, 2016).

Quasi-stationary approximation: Equation 1 is generally valid and it describes an arbitrary motion of the yarn, even in cases when the conditions are rapidly changing for example near the package edges. Near the package edge the winding angle suddenly changes, therefore the motion of the yarn on the package surface and in the part of the balloon near the lift-off point becomes very complex. Near the edges undesired events can occur; the yarn can fall off the package or a layer of the yarn collapses. The description of such transient effects is beyond the validity of our simplified model, since one should accurately model the behavior of the yarn also in the layers forming the package bulk. For example, the residual forces of the yarn in the package would also play a role (Pracek, 2010a, b; Pracek *et al*., 2011a, b, c).

Strictly speaking, the yarn undergoes sliding motion on the package surface only when the unwinding point is at a certain distance away from the packageedges. In such circumstances, the conditions are quasi-stationary: in the rotating coordinate frame the yarn only slowly changes its form. For this reason, in the first approximation the time dependence can be fully described by time-variable boundary conditions while the time-derivative terms in the equation of motion can be neglected (Pracek *et al*., 2016):

$$\rho\left(v^2\frac{\partial^2 r}{\partial s^2} - 2V\omega\times\frac{\partial r}{\partial s} + \omega\times(\omega\times r)\right) = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \quad (4)$$

The equation of motion for the yarn on the package, simplification to a two-dimensional problem: When the yarn slides on the package surface, its motion effectively occurs within a two-dimensional subspace. This fact can be taken into account in Eq. 4 in order to simplify the problem to a two-dimensional problem which can be handled more easily. It turns out that in the case of sliding motion on the cylindrical package the problem can be solved to a large extent using analytical techniques. Analytical solutions allow for a more direct understanding of the relation between the different quantities. For this reason we will henceforth assume that the package is cylindrical and we will determine the analytical solution. The radius vector to a point on the surface of a cylinder can be expressed as (compare with Eq. 2 by Pracek *et al*. (2016):

$$r(s) = ce_r(\theta(s)) + z(s)e_z \quad (5)$$

The quantity c is the constant distance of the point r from the package axis. It is equal to the radius of the layer which is being unwound. The unit vector e_z points along the direction of the package axis, the unit vector e_r points in the radial direction with the polar angle θ(s), Fig. 3. There are two unknowns in this expression, θ(s) and z(s) while the third [r(s)] drops out since it is constant on the surface. The motion of the yarn has this been translated to a two-dimensional problem. This ansatz will be used in Eq. 4 to find a simplified equation of motion.

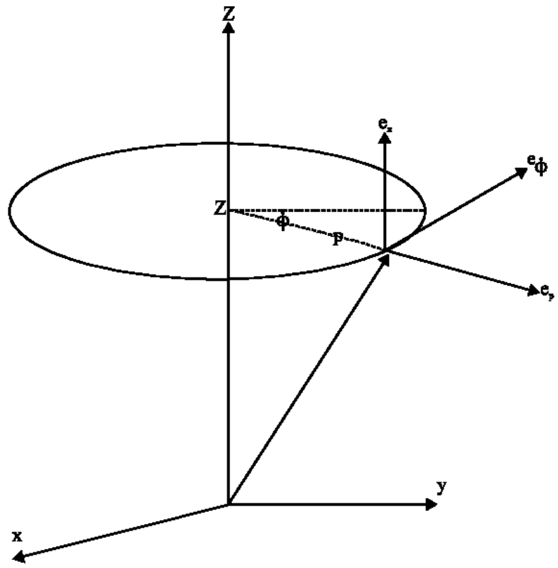

Fig. 3: The cylindrical coordinate system

The arc-length derivatives of the radius vector are computed using the relations 9 and 10 from Praeek *et al.* (2016) to obtain:

$$\begin{aligned} r'(s) &= c\theta(s)e_\theta + z'(s)e_z \\ r''(s) &= c\theta'(s)e_\theta - c\left[\theta'(s)\right]^2 e_r + z''(s)e_z \end{aligned} \qquad (6)$$

where, the dashes indicate the arc-length derivative. We then obtain:

$$\begin{aligned} &\frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) = \frac{\partial T}{\partial s}\frac{\partial r}{\partial s} + T\frac{\partial^2 z}{\partial s^2} = \\ &T'(c\theta' e_\theta + z' e_z) + T(c\,\theta'' e_\theta - c(\theta')^2 e_r + z'' e_z) = \\ &-cT(\theta')^2 e_r + c\left(T'\theta' + T\theta''\right)e_\theta + \left(T'z' + Tz''\right)e_z \end{aligned}$$

We also need the relations:

$$\omega \times p' = -c\omega\theta'(s)e_r$$

And:

$$\omega \times (\omega \times r) = -\omega^2 c e_r$$

which can be derived using a simple calculation of the vector products. Equation 4 may then be decomposed along its different components:

$$(r)\rho\left(-cv^2(\theta')^2 + 2Vc\omega\theta' - \omega^2 c\right) = -cT(\theta')^2 + f_r \qquad (7)$$

$$(\theta)\rho\left(cV^2\theta''\right) = cT\theta'' + cT\theta' + f_\theta \qquad (8)$$

$$(z)\rho\left(V^2 z''\right) = Tz'' + Tz' + f_z \qquad (9)$$

The quantities f_r, f_θ and f_z are the components of the linear density of the external force (Eq. 3). The first one in simply $f_r = n$ while the other two still need to be determined. The velocity of the yarn in the quasi-stationary approximation is (Eq. 15 by Praeek *et al.* (2016) where we substitute $v_{rel} = 0$):

$$v = -Vt + \omega \times r = c\left(\omega - V\theta'\right)e_\theta - z'Ve_z$$

This expression can then be used to derive the unit vector in the direction of the yarn velocity:

$$\frac{v}{|v|} = \frac{1}{\sqrt{c^2(\omega - V\theta')^2 + z'^2 V^2}}\left[c(\omega - V\theta')e_\theta - z'Ve_z\right]$$

From which then finally follow the two components of the linear density of the force:

$$\begin{aligned} f_\theta &= \frac{-\mu nc(\omega - V\theta')}{\sqrt{c^2(\omega - V\theta')^2 + z'^2 V^2}} \\ f_z &= \frac{\mu nz'V}{\sqrt{c^2(\omega - V\theta')^2 + z'^2 V^2}} \end{aligned} \qquad (10)$$

Equation 7-10 are the simplified equations of motions that we required. At first they appear more complex than the vector Expressions 3 and 4 since, they are expressed component by component. Nevertheless, they are indeed simpler: the unknown functions are θ, z, n_θ, n_z, T but we have managed to eliminate r and n_r. In the section part of this research we will show that the function T can equally be eliminated.

CONCLUSION

We have shown how the equation of motion on the package surface can be obtained from the general equation of yarn motion by considering the force of friction (instead of the air-drag as was the case for the part of the yarn which forms the balloon). The external force has two components: the normal force of the package surface and the force of friction. We have described the conditions for the validity of the quasi-stationary approximation which was then used to simplify the equation of motion to a two-dimensional problem. Analytical solutions allow a better understanding of the relation between the different quantities.

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