

Traffic Lights Extension Schedules Control System with Classic Fuzzy Logic and Petri Net Time

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Abstract: This study describes the traffic lights control system for a signalized intersection with railway at the Southern arm. On regular state, the traffic lights schedule has been fitted with a fixed time strategy based on empirical data. At the reality, the intersection needs the traffic lights extra schedule that can adapt to the emergencies that caused by the railway doorstop closure. We construct the extension schedules that modeled by petri net time using classic fuzzy logic inference. The analysis and verification of the model using invariant, occurrence graphs and petri net simulator 2.0 and Delphi 7 for simulation. This research purposes the traffic lights model with appropriate allocation of travel time in order to reduce the number of queues of vehicles. Especially while the train is passing and certain time intervals after the railway doorstop is opened. The traffic lights using extension schedules have better performance despite using the open-loop control system only.

Key words: Traffic lights control system, signalized intersection, classic fuzzy logic, petri net time, Southern, Indonesia

INTRODUCTION

Fuzzy logic was proposed by Lotfi A. Zadeh from the University of California at his study in 1965. It has introduced the linguistic variables (Zimmerman, 1985). The main aspect in a fuzzy logic is the membership degree of uncertainty like probability but it is not similar. The input of a fuzzy control always in the crisp value and in “fuzzy sets” it must be converted by the membership function to its proximity degree to an attribute (Kusumadewi *et al.*, 2006).

This study aim proposes the fuzzy logic as the control systems of traffic lights for traffic management, safety and efficiency planning at the urban networks while the train is passing (Mamdani and Pappis, 1977). The traffic light systems have connection with railway doorstop system. The traffic lights must apply the extension schedule to solve the congestion that always arises because the railway doorstop closure exceeds than a cycle of traffic lights duration.

The signal systems based on time strategies has two classifications (Papageorgiou *et al.*, 2003): fixed time signal strategy and. Traffic responsive signal strategy. Fixed time signal strategy that used in this study topic is based on historical data. The setting assumes that the demand is constant and it couldn't be influenced by special events. Of course, the traffic lights can not accommodate the obstacles caused by the the railway doorstop closure. We construct the traffic responsive signal strategy using real time of the railway doorstop closure measurements. We uses the

train detectors. We simulate using petri net because it can present the discrete events of the traffic lights states. Petri Net (PN) has proven to be a good technique for the problem solving of difficult various kinds that related with modeling, formal analysis, design and the coordination of the discrete event systems control (Soares, 2010; Murata, 1989). Classic petri net consists of a four-tuple elements: places, transitions, arcs and tokens (Cassandras and Lafortune, 1999; Subiono, 2008). In this study, each places equipped with duration. This is called a place of a petri net time (Silva and Foyo, 2012; Ramchandani, 1973; Huang and Chung, 2010; Khansa *et al.*, 1996). The analysis and the verification using invariant, occurrence graph also petri net simulator 2.0 and Delphi 7 for simulation.

Many researchers have studied the behavior of adaptive traffic lights to construct efficient scheduling, i.e. (Papageorgiou *et al.*, 2003; Mamdani and Pappis, 1977; Soares, 2010; Subiono, 2008; Huang and Chung, 2010; Barzegar *et al.*, 2010). They used petri net time and Fuzzy logic also but no one of them had connected to the railways doorstop.

MATERIALS AND METHODS

Regular traffic signal system design: The signalized intersection as the main topic is at the urban traffic network consists of four arms (Fig. 1). The traffic light has two phases only. Phase North-South (NS) and Phase East-West (EW). The trip schedule of traffic lights systems are installed on the fixed time strategy

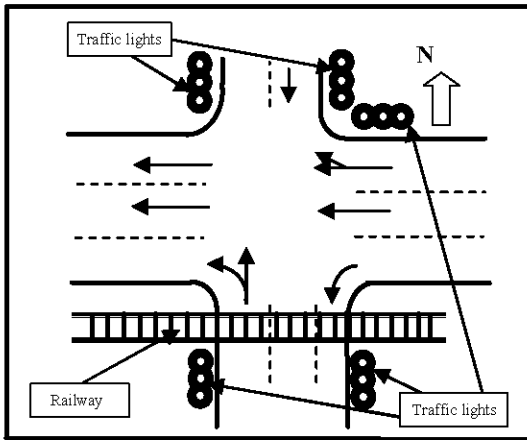


Fig. 1: The signalized intersection with railway on the Southern arm (Tristono, 2013)

(Tristono, 2013). The traffic flow from the North and South move in two directions, i.e., entering and away from the intersection. Traffic flow from the East arm moving in one direction which is entering the intersection only. Traffic flow in the West arm moving on one direction, being away from the intersection. The traffic streams approach from East arm and go to South arm is Left Turn on Red (LTOR). The traffic from North is allowed to go straight to South arms only and they are prohibited to turn right although NS traffic light turn on the green light (Tristono, 2013).

It bases on the left-hand traffic. It means that the traffic movement keeps on the left side of the road (Utomo *et al.*, 2014). We do not include the pedestrian signals in this model. This does not mean that their flow do not exist but in which they must refer to vehicle traffic light indications.

A railway crossing in the Southern arm. The presence of train will interrupt the traffic trip (Utomo *et al.*, 2014). The traffic from North-South and East that going to turn left must stop. This event often cause severe congestion. Due the safety of the road users and efficiency then we construct the traffic lights extension schedule.

Extension schedule design control: The extension schedule is constructed to deal a state of emergency caused by doorstop closure. There are two types of extension schedule, i.e., “train extension” and “NS/North-South green extension”. The first extension schedule is written by *tr*. This extension is applied during the train crossing time interval. At this state, the traffic lights regular schedule is interrupted for a while.

The design of “train extension” schedule is the South-North and East that turn left should light up red.

Table 1: The rules during NS turn green/yellow

| Traffic light turn green/Yellow | Membership of closing of train doorstop | | |
|---------------------------------|---|----------|----------|
| | Short | Middle | Long |
| Beginning_1 | Medium_1 | High_1 | High_1 |
| Center_1 | Low_1 | Medium_1 | High_1 |
| Ending_1 | Low_1 | Low_1 | Medium_1 |

Table 2: The rules when ns turn red

| Traffic light turn red | Membership of closing of train doorstop | | |
|------------------------|---|----------|--------|
| | Short | Middle | Long |
| Beginning_2 | High_2 | High_2 | High_2 |
| Center_2 | Medium_2 | High_2 | High_2 |
| Ending_2 | Low_2 | Medium_2 | High_2 |

The East turn left traffic signal is a virtual traffic lights, it is exist in the model only. The traffic signal at the East arm is provided for those who will go straight to West arm and turn right. This signal turns green. The green light has duration “train extension”. This is ended at *tr*, i.e., the time train has completed passing the crossing (Fig. 2).

The real time measurements that gathered by train detectors collect the information about the presence of the train on the track that ready to pass the crossing to controllers (Arduino). While it has completed, controllers send messages to traffic lights to give extra NS green interval and to open doorstop. This sec extension schedule is named “NS green extension”. It is written with the abbreviation *ext* only. The controller hasn’t been attached with vehicular traffic detectors.

Figure 3 is flowchart of traffic lights design. It defines the term midnight that the time interval between 10.00 p.m. until 04.00 a.m. in the morning which includes midnight. The traffic volume in this time interval is very low. It is the reason why extension schedule is invalid and it uses the traffic light regular schedule only.

Figure 4 is the flowchart of classic fuzzy logic inference engine. This function has two inputs. The first input is time interval of doorstop closure and the second input is the state of traffic lights. The two choices of state of traffic light, i.e., during green or yellow light is on and when red light is on. They have different rules and their inference engine separated each other. Its memberships at Fig. 5-7.

There are 18 rules in fuzzy inference engine based on knowledge from the empirical data. They both at Table 1 and 2. The rules at Table 1 is used while doorstop closing starts at NS light turns green or yellow. Table 2 is used when latch train closing starts at NS light turns red.

The output of fuzzy inference engine has extra time the green light duration on phase NS. The red extra interval of extension schedule on phase EW adjusted with phase NS. The red light on phase EW has been

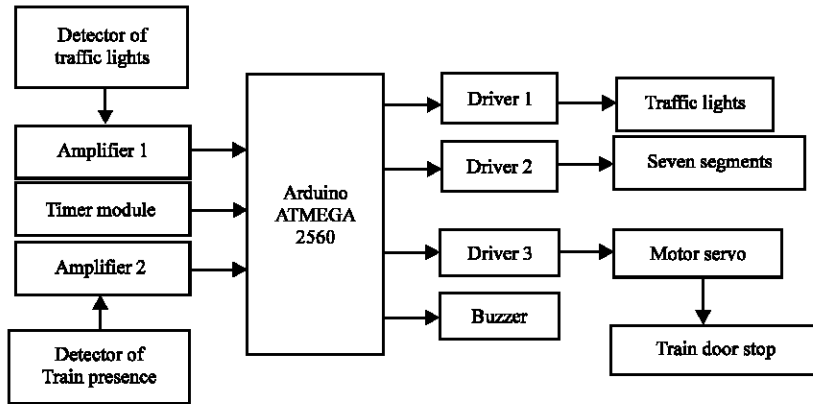


Fig. 2: Traffic lights block diagram

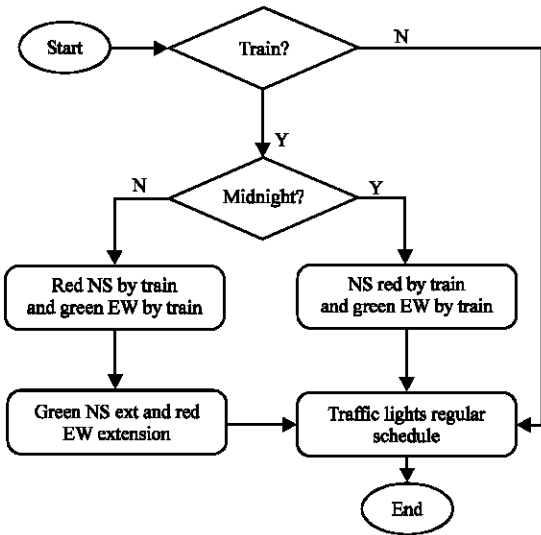


Fig. 3: Flowchart of traffic lights design

started 2 sec earlier than green light on phase NS and the red light on phase NS also has been turning up 2 sec earlier than green light on phase EW. The yellow light always turn on for 2 sec before the red light turn up. The yellow light is the transition color in sequence of time order after the green light has turned off and before the red turn up. All of these algorithm have been programmed in controller core (Putra, 2010).

Traffic lights petri net time presentation: It defines:

$$P = \{GNS, YNS, RNS, GEW, YEW, REW\}$$

Where:

P = A set of places representing traffic light colors coming from and towards a certain direction

GNS = Means as Green South-North and North-South

YNS = Yellow South-North and North-South

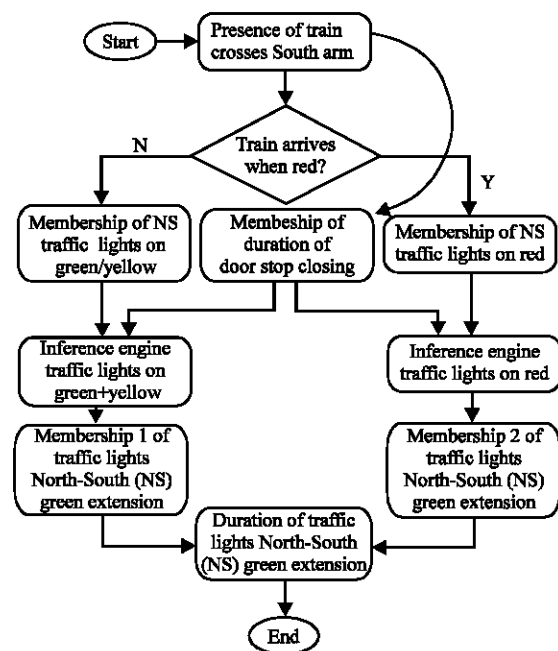


Fig. 4: Flowchart of fuzzy logic inference engine

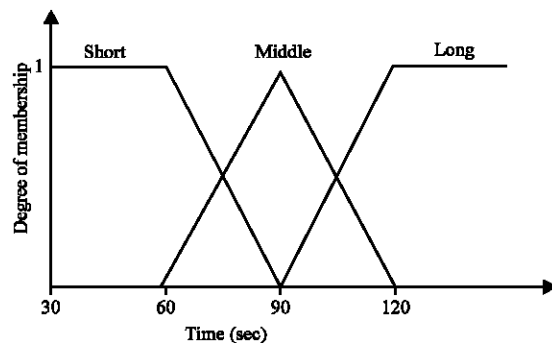


Fig. 5: Membership of antecedent railway doorstop closure

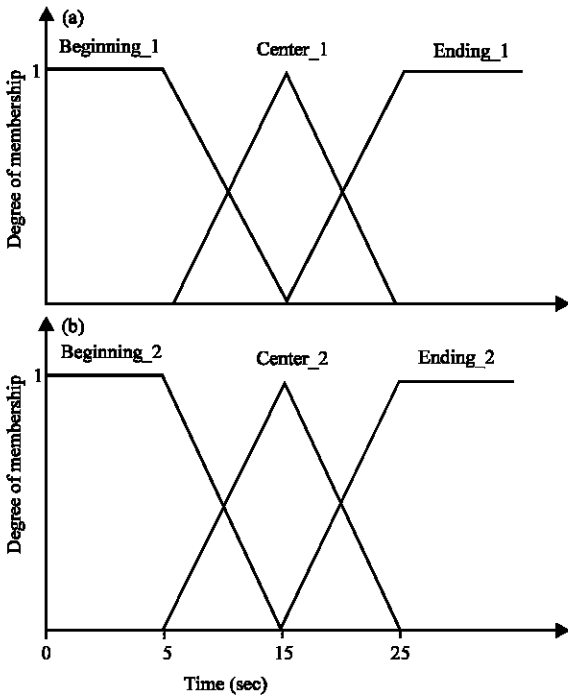


Fig. 6: Membership of antecedent state of traffic light: a) Cycle = 75 sec), NS turn green or yellow and b) NS turn red

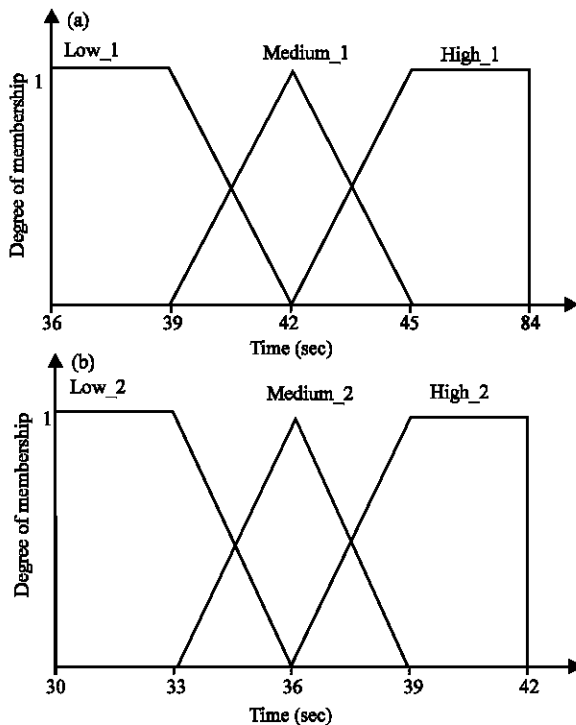


Fig. 7: Membership NS green extension as output of fuzzy logic: a) Doorstop closing start at green/yellow and b) doorstop closing start at red

RNS = Red South-North and North-South
 GEW = Green East-West
 YEW = Yellow East-West and
 REW = Red East-West

$$P_v = \{GES, YES, RES\}$$

Where:

P_v = A set of virtual places traffic lights. It is provided for the traffic coming from the East to turn left toward the Southern arm. It means that it does not exist at real world
 GES = Green East-South
 YES = Yellow East-South
 RES = Red East-South

$S = \{S_1, S_2\}$ is a set of places of intermediate to connect between the phase North-South and the phase East-West. It defines $T = \{t_1, t_2, t_3, t_4, t_5, t_6\}$. T is a set of transitions and $T_v = \{t_7, t_8, t_9\}$. T_v is a set of virtual transitions. Interval $[\tau_i, \tau_j]$ represents the duration of each place. τ_6 is the duration of one cycle of the traffic lights. Arcs connect the transition to a place and a place to transition. A is a set of arcs describe the direction of movement of the tokens:

$$t_0 < t_1 < t_2 < t_3 < t_4 < t_5 < t_6, t_0 = 0, t_1 \in \mathbb{N}_+$$

Figure 8, the top down model petri net time of traffic lights with sequence of time order. There is a token in each place GNS, REW and GES on a regular schedule. It means that the traffic light is green on the North-South, red on the East-West and green lit on the East turn left to head South.

Figure 9, during there is a train passing a token available in each place RNS, GEW and RES. It on “train extension” means that the traffic light is red on the North-South and South-North, green on the East-West and red lit to the East turn left to head South. The intent of $[\tau_2, \tau_r]$ at RNS is the time interval from τ_2 up to τ_r . τ_2 , initial of doorstop closing is the beginning of “train extension”. At τ_r the train finishes passing.

Place GNS at $[\tau_r, \tau_{ext}]$ means that “North-South green extension” is the time interval starting from the railway crossing ended up to a time extension. Its time interval is the result of the fuzzy logic inference engine. $\tau_0 < \tau_1 < \tau_2 < \tau_r < \tau_{ext}$; $\tau_i \in \mathbb{N}_+$ (positive natural numbers). $\tau_2 = \tau_1 + 2$; $\tau_r - \tau_2 > \tau$; $\tau_{ext} + 4 = \tau_3$; $\tau_{ext} - \tau_r > \tau_1 - \tau_0$; the τ is constant that the minimum of doorstop closure.

Invariant: The marking of a petri net time could be represented by the firing of enable transitions. All of

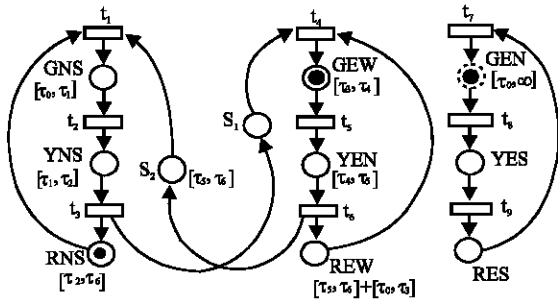


Fig. 8: Model petri net of traffic lights with sequence of time order (Genter, 2015)

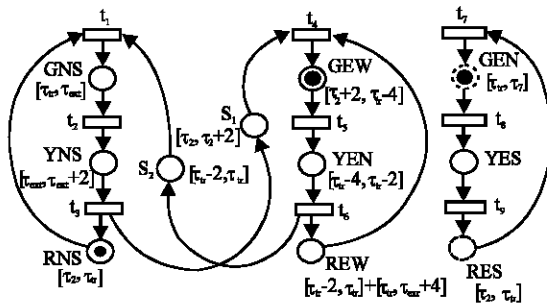


Fig. 9: Petri net model during the train is passing and the extension

markings that could be reached have some common properties. The properties guarantee that they would not differ while the transitions are fired. This equation is said invariant:

$$GEW + YEW + REW = 1 \quad (1)$$

The invariant at Eq. 1 asserts that could be only a token available in one of three places GEW, YEW and REW. It is on regular schedule. It also indicates that the sequence of event to be in time order at East-West traffic lights:

$$GNS + YNS + RNS = 1 \quad (2)$$

$$GES + YES + RES = 1 \quad (3)$$

Equation 4 indicates that if there is a token in place RNS, then there must be a token in either place GEW, YEW or REW. It means that if the red signal lights up in the North-South traffic light then the East-West traffic lights should turn on green, yellow or red:

$$GEW + YEW + REW = RNS \text{ while } RNS = 1 \quad (4)$$

$$GNS + YNS + RNS = REW \text{ while } REW = 1 \quad (5)$$

Table 3: Schedule of duration of regular traffic lights

| Phases | Inter green | | | Red | Cycle |
|----------|--------------------|--------------------|--------------------|---------------------------------------|--------------------|
| | Green | Yellow | All red | | |
| Phase NS | $[\tau_0, \tau_1]$ | $[\tau_1, \tau_2]$ | $[\tau_2, \tau_3]$ | $[\tau_3, \tau_6]$ | $[\tau_0, \tau_6]$ |
| Phase EW | $[\tau_3, \tau_4]$ | $[\tau_4, \tau_5]$ | $[\tau_5, \tau_6]$ | $[\tau_5, \tau_6] + [\tau_0, \tau_3]$ | $[\tau_3, \tau_6]$ |

The invariant Eq. 6 is equipped with intermediation places S_1 and S_2 . This invariant means that the direction of the vehicles movement. This invariant hints that the models system guarantee the safety of traffic flow:

$$S_2 + GEW + YEW + S_1 + GNS + YNS = 1 \quad (6)$$

All equations represent the key invariant in petri net model system (Table 3). The invariant while extension schedule as following: $GEW [\tau_2, \tau_r - 4] + YEW [\tau_r - 4, \tau_r - 2] + REW [[\tau_r - 2, \tau_{ext}] + [\tau_r, \tau_{ext} + 4]] = 1$ at Fig. 9 satisfies Eq. 1. This invariant asserts that it could be only a token available in one of three places GEW, YEW and REW. The duration $[\tau_2, \tau_r - 4]$ is attached on GEW means that would start from τ_2 up to 4 sec before train have finished passing. $REW [[\tau_r - 2, \tau_{ext}] + [\tau_r, \tau_{ext} + 4]]$ means that REW begins at 2 sec before the train has completed passing up to its completed passing $[\tau_{tr}, \tau_{ext} + 4]$ is the interval from the ending of the train passing up to 4 sec after the green extension has completed. First and sec duration of REW goes without time delay:

$$RNS [\tau_2, \tau_r] + GNS [\tau_r, \tau_{ext}] + YNS [\tau_{ext}, \tau_{ext} + 2] = 1 \text{ satisfies} \quad (7)$$

$$YES [\tau_2 - 2, \tau_2] + RES [\tau_2, \tau_r] + GES [\tau_r, \tau_7] = 1; \tau_7 = \infty \text{ satisfies} \quad (8)$$

$$GEW[\tau_2 + 2, \tau_r - 4] + YEW [\tau_r - 4, \tau_r - 2] + REW [\tau_r - 2, \tau_{ext} + 4] = RNS [\tau_2, \tau_r] \text{ where } RNS [\tau_2, \tau_r] = 1 \text{ satisfies} \quad (9)$$

$$RNS [\tau_2, \tau_r] + GNS [\tau_r, \tau_{ext}] + YNS [\tau_{ext}, \tau_{ext} + 2] = REW [\tau_r - 2, \tau_{ext} + 4] \text{ where } REW [\tau_r - 2, \tau_{ext} + 4] = 1 \text{ satisfies}$$

$$RES [\tau_2, \tau_r] = RNS [[\tau_2, \tau_r]] \quad (10)$$

$$S_1[\tau_2, \tau_2 + 2] + GEW[\tau_2 + 2, \tau_r - 4] + YEW [\tau_r - 4, \tau_r - 2, t_{ext} + 2, \tau_r] + GNS[\tau_r, \tau_{ext}] + YNS[\tau_{ext}, \tau_{ext} + 2] = 1 \text{ satisfies} \quad (11)$$

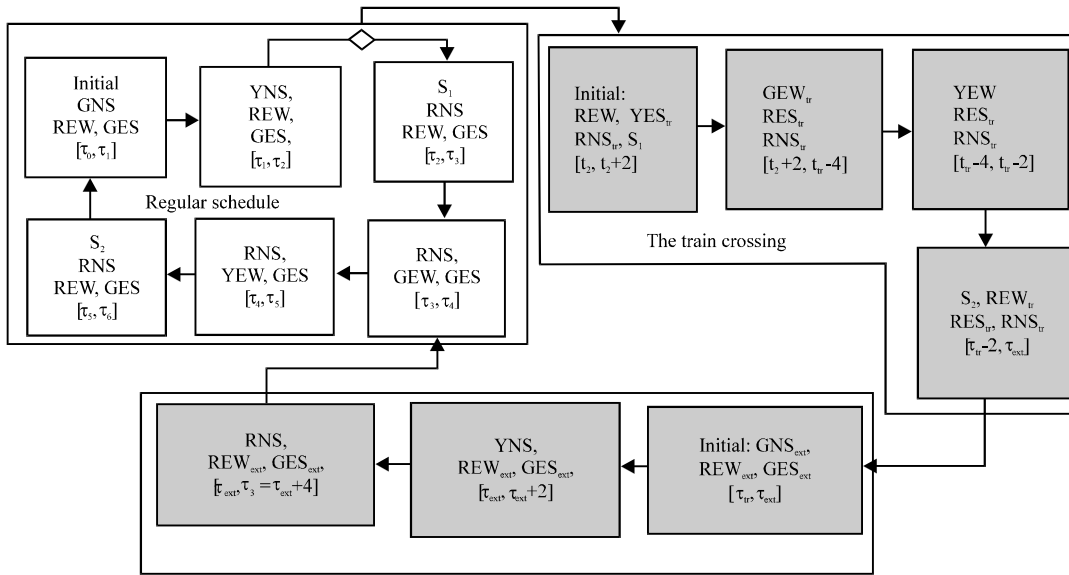


Fig. 10: Occurrence Graph (OG)

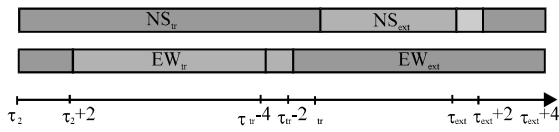


Fig. 11: Simulation result of traffic lights during the train crossing and NS green extension

RESULTS AND DISCUSSION

Occurrence Graph (OG) and simulation result: OG at Fig. 10 contains nodes for each marking that are reached and arcs for each element binding that are occurred. It could be seen from this approach that there is no possibility of deadlock in the model and it is reversibled to initial state. Figure 11 is the simulation result of traffic lights during “train crossing” at $[\tau_2, \tau_u]$ and “NS green extension” at $[\tau_{ext}, \tau_{ext}]$.

It uses standart metric Passanger Car Unit (PCU). This standart uses in transportation engineering to assess traffic rate on a highway. This is the effect of traffic variables that is compared to a single passanger car. The typical values of PCU are Low Vehicle/LV, Heavy (UM) (UM) (IHMC, 1997). The arrival of vehicles is assumed has uniform distribution.

The Degree of Saturation (DS) of East arm with extension schedule while interval doorstep closure 65 sec is 0.52 while interval doorstep closure 99 sec is 0.46 and while interval doorstep closure 120 sec is 0.4. All of these are lower than the normal degree of saturation of Eastern arm, i.e., 0.58 (At normal state the unsaturated degree is

Table 4: The simulation of traffic lights regular schedule

| Phase | Inter green | | | | Cycle |
|----------|-------------|--------|---------------|-----|-------|
| | Green | Yellow | All red (sec) | Red | |
| Phase NS | 29 | 2 | 2 | 44 | 75 |
| Phase EW | 38 | 2 | 2 | 35 | 75 |

Table 5: The duration of NS green extension on three kinds of doorstep closure

| Initial doorstep closure at NS state | NS green extension while doorstep closing for | NS green extension while doorstep closing for | | | | |
|--------------------------------------|---|---|-----------|--------|--------|---------|
| | | Green/Yellow (sec) | Red (sec) | 65 sec | 99 sec | 120 sec |
| 0 | | 43 | 46 | 46 | | |
| 10th | | 41 | 44 | 46 | | |
| 20th | | 39 | 42 | 44 | | |
| | 0 | 36 | 40 | 40 | | |
| | 10th | 36 | 40 | 40 | | |
| | 20th | 34 | 37 | 40 | | |
| | 30th | 33 | 36 | 37 | | |
| | 40th | 32 | 34 | 36 | | |

$0 < DS \leq 1$). The regular schedule at Table 4 using fix time strategy can not be influenced by special events except the train doorstep closure. The fuzzy inference logic will yield green extension time interval of North-South arms. It use consideration that on the yellow/red state of traffic lights the vehicles movement must stop, otherwise on the green state. It is presented at Table 5.

Table 6 conclude that the North-South queuing of vehicles stop that caused by train with extension schedule of traffic lights is same or lower than queuing of vehicles stop that caused by train using reguler schedule of traffic lights. It means that the performance of extension schedule of traffic lights is better than reguler schedule for long or short train doorstep closure.

Table 6: Simulation result of the train arrived during NS traffic light turn on green at 20th sec

| Interval of door stop closure (sec) | Door stop closing with regular schedule of traffic lights | | The queuing of vehicles stop (PCU) | | | | |
|-------------------------------------|---|------------------------|--|-------|--|-------|--------------------------|
| | Stop for (sec) | Final stop of state NS | Caused by train and regular schedule of traffic lights | | Caused by train with extension schedule of traffic lights train only | | Caused by East turn left |
| | | | South | North | South | North | |
| 65 | 67 | Green | 11.7 | 13.2 | 11.7 | 13.2 | 2.8 |
| 99 | 130 | Red | 22.8 | 20.7 | 17.8 | 16.4 | 4.2 |
| 120 | 130 | Red | 22.8 | 20.7 | 21.5 | 18.9 | 5.1 |

Table 7: The simulation result of the performance of traffic lights and the train arrived during NS turn on green at 20th sec

| North interval of door stop closing (sec) | Green duration of fuzzy inference engine result (sec) | Total vehicles that successfully crossed the intersection at the first green after a train door stop (PCU) | | | | Total vehicles that left yet crossed the intersection on the first green after a train door stop without including the arrival of the vehicles at a later stage (PCU) | | | |
|---|---|--|-------|-----------|-------|---|-------|-----------|-------|
| | | Regular | | Extension | | Regular | | Extension | |
| | | South | North | South | North | South | North | South | North |
| 65 | 39 | 3.2 | 3.4 | 7.4 | 7.9 | 8.5 | 9.8 | 4.3 | 5.3 |
| 99 | 42 | 5.5 | 5.9 | 8.0 | 8.5 | 17.3 | 14.8 | 9.8 | 7.9 |
| 120 | 44 | 5.5 | 5.9 | 8.4 | 8.9 | 17.3 | 14.8 | 13.1 | 10.0 |

Table 7 presents that total vehicles on North-South arms that successfully crossed the intersection at the first green after a train doorstop using extension schedule is higher than we use regular schedule only. The total vehicles that left yet crossed the intersection on the first green after a train doorstop without including the arrival of the vehicles at a later stage using extension schedule is lower than we use regular schedule.

CONCLUSION

The traffic lights extension schedule that has better performance is a solution to solve the rising queue of vehicles after train has passed through the intersection. The new schedule is very suitable to tackle congestion in urban areas with obstacles that come periodically although, this work is an open loop control systems only. The simulation by computer software represent that there is better time sharing and the turning down of the queuing of vehicles. The result could be checked from total vehicles that successfully crossed the intersection at the first green after a train door stop and the remaining vehicles that have not passed. The queuing of vehicles would decrease step by step at each cycle and the traffic flow would return to normal condition.

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