

Optimization of Wavelet Weighted Fuzzy Model for Time Series Data and its Application to Forecast Jakarta Composite Index

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Abstract: Jakarta Composite Index (JCI) is an indicator for monitoring the movement of the prices of all shares listed on the Jakarta stock exchange. Most studies of JCI prediction are conducted using conventional statistical methods. In this study, we come up with a new procedure to construct wavelet weighted fuzzy model and apply it to predict JCI. Wavelet weighted fuzzy modeling procedure is begun with wavelet transformation using a Discrete Wavelet Transform (DWT) mother Haar for time series data. The DWT results are used as an input of Mamdani Fuzzy Model. Furthermore, the weight of fuzzy rules is determined based on the training data. Finally, defuzzification process is performed to obtain the output of Wavelet Weighted Fuzzy Model. This procedure was applied to predict the value of JCI. The results indicate that the Wavelet Weighted Fuzzy Model has high accuracy for training and testing data. In addition, the prediction of JCI value is also performed with other models such as Weighted Fuzzy Model and Wavelet Fuzzy Model. Compared to the other models, Wavelet Weighted Fuzzy Model gives better results than that of the other models.

Keywords: Jakarta composite index, time series, prediction, Wavelet Weighted Fuzzy Model, Stock, DWT

INTRODUCTION

Composite index or stock price is a value used to measure the combined performance of all stocks listed on the stock exchange. It is used to take a hard look at the overall changes in stock prices in the market. The rise and fall of stock prices in the stock can be seen from the decrease and increase in the composite index. The rising trend signifies market excitement while the falling trend indicates market sluggishness. Since, it can show the general situation, it is also utilized as a basis in determining investments. Moreover, composite index values may affect some economic sectors in countries.

There has been many studies on the stock price using different methods. The predictions of stock price in some countries have been done by using classification (Bordel *et al.*, 2016), Support Vector Machine (SVM) (Heo and Yang, 2016) and fuzzy time series Markov chain methods (Rachmawati *et al.*, 2015). Some analytical methods to predict the stock market from classical to the latest methods can be found by Rusu and Rusu (2015). The randomness of the fluctuations of the stock market in China using a Markov process model has been investigated by Zhang and Zhang (2009). A new algorithm based on time series data for predicting the stock index in Shanghai China has been developed

by Xu (2010). The prediction of the stock data using neural network showed that the predicted results corresponded with the actual stock price (Liu and Wang, 2011). Furthermore, as by Abdullah and Fan (2011), Abdullah and Ling (2011), fuzzy method has been applied in predicting stock price. According to Sopipan *et al.* (2012), multiple regression is applied to predict the SET50 index (Stock Exchange of Thailand). Khadka *et al.* (2012), the concordance models and genetic programming for predicting the stock market (S and P 500 and NASDAQ indices) have been compared. According to Wang *et al.* (2013) researchers predicted the Shanghai stock price index using wavelet neural network based on ARIMA models. Moreover, artificial neural network models have been implemented to predict the stock Bursa Istanbul 100 (BIST 100) (Paksoy and Kilic, 2015; Sakarya *et al.*, 2015). Another research looked at the performance of ARIMA and GARCH models to predict Malaysia market properties and shares (Miswan *et al.*, 2014) and several methods of ANN, NAR and NARX to predict China stock index have been compared (Wang, 2015).

There has been several studies on application of fuzzy model especially in economics. Adzic and Sedlak (1998) researchers build the model of macro economics using fuzzy set with the transition process. According to Marcek (2003), fuzzy relations have been

used to determine the financial chaotic process and application of fuzzy model to calculate the probability of winning or losing has been investigated by Magno *et al.* (2005). Moreover, a mathematical model with fuzzy sets to describe the economic system has been introduced by Stojakovic (2005) and fuzzy model to determine the investment project decisions has been investigated by Sheen (2009). Furthermore, a model of zero-order Sugeno fuzzy weighted prediction for the stock price has been constructed (Nurhayadi *et al.*, 2014). Recently we constructed a fuzzy model of translation in the extent of the median prediction error for stock price (Abadi, 2014). We also formed the optimal fuzzy model with translation in the extents of the mean error (Abadi, 2014).

Moreover, studies on wavelet fuzzy model have been done. A new method of combination of discrete wavelet transform and fuzzy system can be found by Karatepe and Alci (2005). According to Popoola, researchers analyzed the modeling of non-stationary time series data with wavelet fuzzy model. Research on wavelet models combined with neural network models is carried out by Thuillard (2000). In researchers predicted the price of oil using wavelet neural network and by Bodyanskiy *et al.* (2008), researchers used wavelet neuro fuzzy prediction to complete the process of non-stationary. Furthermore, researchers compared the wavelet models, fuzzy logic and artificial neural network in predicting stock prices (Homayouni and Amiri, 2011). The model of wavelet neural network in predicting finance with time series data can be seen by Ortega (2012) and the application of linear wavelet neural network to predict the exchange rate of the Indian Rupee against the US Dollar can be found by Mohapatra *et al.* (2013).

The research in optimization of wavelet fuzzy models is still developed. In this study, we will provide a new method to construct wavelet weighted Mamdani fuzzy model. Furthermore, the resulting model is applied to forecast the Jakarta Composite Index (JCI).

MATERIALS AND METHODS

First, The data is taken from www.finance.yahoo.com. This consists of 1300 data from 1st January, 2010 until 27th April, 2015. Afterwards, the data is grouped into training data and testing data which are 700 data and 600 data for training data and testing data, respectively. The method used in this research is shown in Fig. 1.

Wavelet transformation: In general, wavelet is a wave function that is built with the full calculation so that this function has the mathematical properties. Wavelet provides information about the scale and frequency

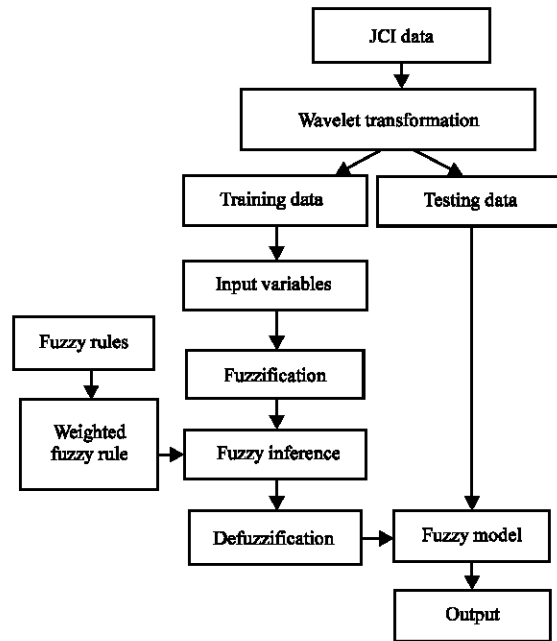


Fig. 1: Research diagram

combinations. Wavelet transformation is used to sort through the data, function or operator into components of different frequencies. Each frequency component is examined separately and then the results are reunited into a single unit.

Wavelet can be used to decompose the time series data into several sub time series data. Each sub time series data are processed separately and the results can be reunited. Wavelet with coarse resolution can easily capture the global behavior while wavelet with fine resolution can capture the local behavior of a function accurately (Bruce and Gao, 1996). Yu *et al.* (2001) have shown that the wavelet can improve the predictive ability of a method of modeling. Wavelet is a new base that can be used to represent functions with consideration of techniques for the analysis of time to frequency (Chui, 1992). Some examples of the wavelet family are the Haar, Daubechies, Symlets, Coiflets, Bior Splines, Reverse Bior, Meyer, D Meyer, Gaussian, Mexican hat, Morlet, Complex, Shannon, Frequency B-Spline, Complex Morlet, Riyad, etc.

Wavelet transformation converts the signal into different wavelet bases with a variety of shifting and scaling. Therefore, wavelet coefficients of some scale or resolution can be calculated from wavelet coefficients at the next high resolution. This makes it possible to implement wavelet transformation using a tree structure known as pyramid algorithms (pyramid algorithm). There are some characteristics of wavelet system as by Toufik and Mokhtar (2012):

- Wavelet consists of common functions used to represent the signal
- Wavelet has places to put the data frequency
- Wavelet is able to help the transforms algorithm quickly and efficiently

The wavelet transformation is a process of converting data into another form that is more easily analyzed. Wavelet transformation uses two important components in transforming the scaling function and the wavelet function. Scale function is also called low pass filter whereas wavelet function is also known as high pass filter. Wavelet transformation process is carried out by convolving the signal with a filter or process the data averaging and reduction repeatedly, often called the filter bank method. The original signal can be restored back to the reconstruction of a signal that has been decomposed by applying Inverse Discrete Wavelet Transform (IDWT). Wavelets transformation is divided in to 2 types (Chui, 1992).

Continue Wavelet Transform (CWT): Continue Wavelet Transform (CWT) is used for a function whose domain is real numbers on the X-axis. Continue Wavelet Transform (CWT) is working by calculating the convolution of a signal with a wavelet function at any time with any desired scale. This function is commonly used in more analytical scientific research.

Discrete Wavelet Transform (DWT): DWT is used for a function over the domain of integers (usually $t = 0, 1, \dots, N-1$ where N is denoted as the number of values in the time series). Discrete Wavelet Transform (DWT) is widely used in engineering and computer applications.

Multi-level wavelet transform can be defined as a Discrete Wavelet Transformation Model that transforms the data repeatedly. The algorithm of multilevel wavelet transform is as follows (Widiartha and Wijaya, 2006):

- The data are initially transformed using DWT and produces approximation and detail coefficients
- The transformation coefficients are transformed again by using DWT resulting transform coefficients approximation and the second detail
- If the length of the level is three, then the transformation process is repeated three times (repeat step two until the length is equal to the level of three). This process is continued until the specified level is achieved

The maximum length of the level of multi-level wavelet transformation of a signal is as follows:

$$\text{Level}_{\max} = \frac{\ln\left(\frac{\text{length of data (signal)}}{\text{length of filter-1}}\right)}{\ln(2)} \quad (1)$$

In this research, the transformation used is the Haar wavelet transformation. Haar wavelet is a simple wavelet type and can be applied to one-dimensional signal transformation. The Haar wavelet is equal to wavelet Db1 (Daubechies order 1). Length of filter Haar wavelet is 2.

Wavelet Weighted Mamdani Fuzzy Model: In this study, we propose a new procedure to construct Weighted Mamdani Fuzzy Model for time series data. The procedure is as follows:

- Decomposing the time series data with wavelet transform
- Determining the DWS based on the significant decomposition
- Determining the ACF and PACF of the data DWS to determine the input of fuzzy model
- Letting the input obtained is lag-n, then we obtain N pairs of input-output as training data

$$(x_{(t-1)p}, x_{(t-2)p}, \dots, x_{(t-n)p}; x_{tp})$$

Where:

$$x_{(t-i)p} \in [\alpha, \beta] \subset R, i = 0, 1, 2, \dots, n, p = 1, 2, 3, \dots, N$$

Defining M fuzzy sets A_i on $[\alpha, \beta]$, $i = 1, 2, \dots, M$ which are normal, complete and consistent. Building fuzzy rule of each pair of input-output and obtained fuzzy IF-THEN rules as follows: There are p_1 fuzzy rules in the from:

$$\text{IF } x_{(t-1)} \text{ is } A_{i_1} \text{ and } x_{(t-2)} \text{ is } A_{i_2} \text{ and } \dots \text{ and } x_{(t-n)} \text{ is } A_{i_n}, \\ \text{THEN } x_t \text{ is } A_j \text{ with } j, i_1, i_2, \dots, i_n \in \{1, 2, \dots, M\}$$

Determining the degree of each fuzzy rule. If there are conflicting rules, then the chosen rule is the rule which has the highest degree.

Determining the weight of each fuzzy rule. Based on all training data, the sets of fuzzy rules are obtained as follows, there are p_1 fuzzy rules in the form:

$$\text{IF } x_{(t-1)} \text{ is } A_1^1 \text{ and } x_{(t-2)} \text{ is } A_2^1 \text{ and } \dots \text{ and } x_{(t-n)} \\ \text{is } A_n^1, \text{ THEN } x_t \text{ is } A^1$$

There are p_2 fuzzy rules in the form:

$$\text{IF } x_{(t-1)} \text{ is } A_1^2 \text{ and } x_{(t-2)} \text{ is } A_2^2 \text{ and } \dots \text{ and } x_{(t-n)} \\ \text{is } A_n^2, \text{ THEN } x_t \text{ is } A^2$$

There are p_T fuzzy rules in the form:

IF $x_{(t-1)}$ is A_1^T and $x_{(t-2)}$ is A_2^T and ... and $x_{(t-n)}$ is A_n^T ,
 THEN x_t is A^T with $p_1 + p_2 + \dots + p_T \leq N$

Constructing the fuzzy rule bases of the total T fuzzy rules where the weight of jth fuzzy rule is:

$$m_j = \frac{P_j}{\sum_{i=1}^T P_i} \quad (1)$$

Constructing Wavelet Weighted Fuzzy Model. If we select singleton fuzzifier, multiplication fuzzy inference engine and center average defuzzifier then the output of Wavelet Weighted Fuzzy Model is:

$$y = \frac{\sum_{i=1}^T m_i y_i (\mu_{A_{i1}}(x_1) \mu_{A_{i2}}(x_2) \dots \mu_{A_{in}}(x_n))}{\sum_{i=1}^T m_i (\mu_{A_{i1}}(x_1) \mu_{A_{i2}}(x_2) \dots \mu_{A_{in}}(x_n))} \quad (2)$$

RESULTS AND DISCUSSION

Application of the proposed method to forecast Jakarta composite index: In this study, we apply the proposed method to predict JCI. The predicting steps of JCI are as follows.

Identify the data: The data is grouped into training and testing data. The first 700 data are for training and the rest 600 data are for testing. The plot of JCI data is shown in Fig. 2.

Differencing data to get stationary data: The data becomes stationary after once differencing process is applied as shown in Fig. 3.

Decompose stationary data using wavelet transformation: Wavelet transformation process uses DWT Mother Haar level 10. The correlation coefficient between the decomposition data to the original data is determined as shown in Table 1.

Based on Table 1, DWs is formed by DW1 and DW2 since those two coefficients are the most significant. Then, time series of DWs data is shown in Fig. 4. The ACF plot of time series in Fig. 4 is determined to get the number of inputs. Based on Fig. 5 there are 5 inputs.

Determine the universal set of input and output: Based on the DWs, universal set of inputs and outputs is (-246.227,158.7233).

Table 1: Correlation coefficient

Decomposition results	Correlation coefficient
DW1	0.674
DW2	0.514
DW3	0.433
DW4	0.201
DW5	0.165
DW6	0.102
DW7	0.100
DW8	0.054
DW9	0.015
DW10	0.013
Approximation	-0.004

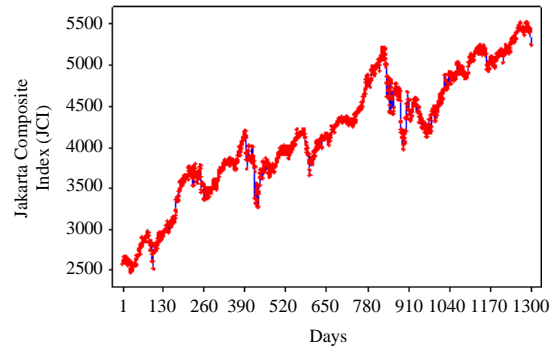


Fig. 2: Plot of time series data of JCI

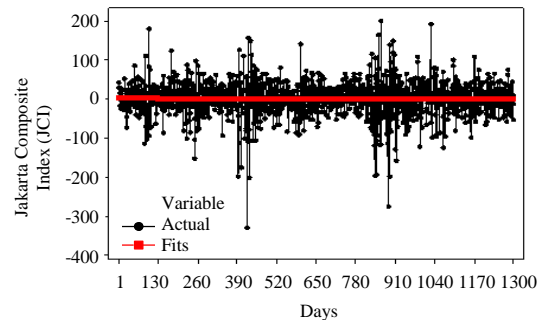


Fig. 3: Stationary test; trend analysis plot for JCI, linear trend model, $Y_t = 3.07 - 0.001328 * t$; accuracy: measure: MAPE: 192.23; MAD: 31.50; MSD: 1999.29

Determine the fuzzy sets on the universal set of input and output: Based on the histogram of DWs as shown in Fig. 6 there are 17 fuzzy sets defined on the input and output. In this study, we use triangular membership function as shown in Fig. 7.

Determine the weighted fuzzy rules. Fuzzy rules are built from training data, then the weight of each rule is computed by Eq. 1. Thus, there are 621 fuzzy rules. Perform fuzzy inference using Mamdani method. Defuzzify with Eq. 2, Table 2 gives the comparison of accuracy of the 3 models. Based on Table 2 for training data, the values of MSE and RMSE of Wavelet Weighted Fuzzy

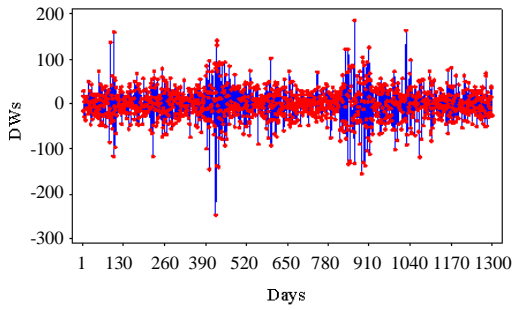


Fig. 4: Plot time series of DWs

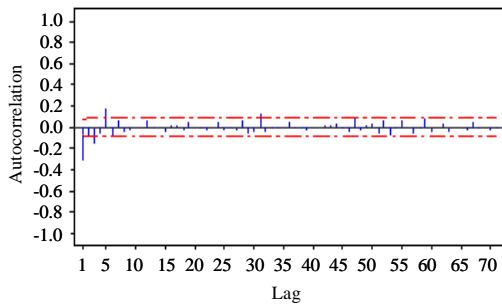


Fig. 5: ACF test for time series of DWs; autocorrelation function (with 5% significance limits for the autocorrelation)

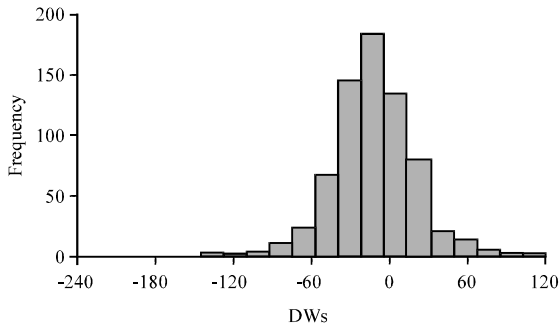


Fig. 6: Histogram of DWs data

Table 2: Predicted results

Methods	MAPE (%)	MSE	RMSE
Training			
Weighted fuzzy model	0.8746	1882.1801	43.3841
Wavelet fuzzy model	0.4971	792.3062	28.1479
Wavelet weighted fuzzy model	0.5034	763.4869	27.6313
Testing			
Weighted fuzzy model	0.7535	2498.1903	49.9819
Wavelet fuzzy model	0.7474	2746.5569	52.4076
Wavelet weighted fuzzy model	0.7153	2142.5460	46.2876

Model are smaller than those of Weighted Fuzzy Model and Wavelet Fuzzy Model. However, Wavelet Fuzzy Model gives the smallest value of MAPE. For testing

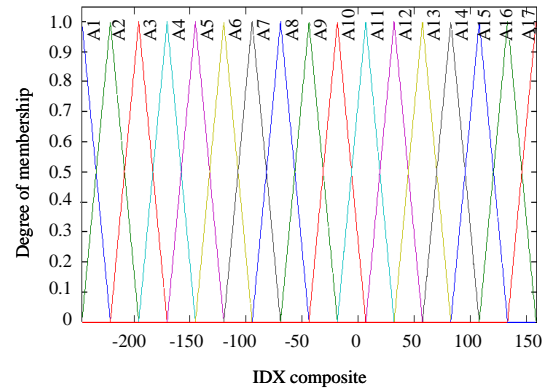


Fig. 7: Membership functions of fuzzy sets

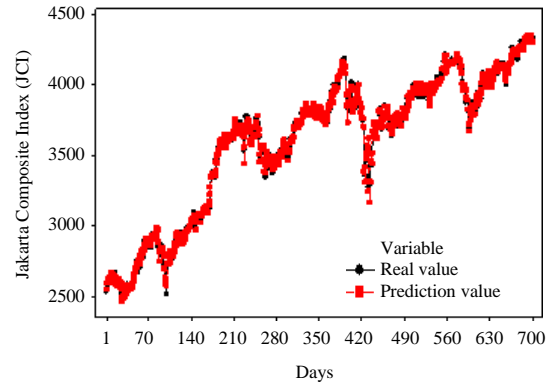


Fig. 8: Plot of real and forecasting values of JCI using wavelet weighted fuzzy model for training data

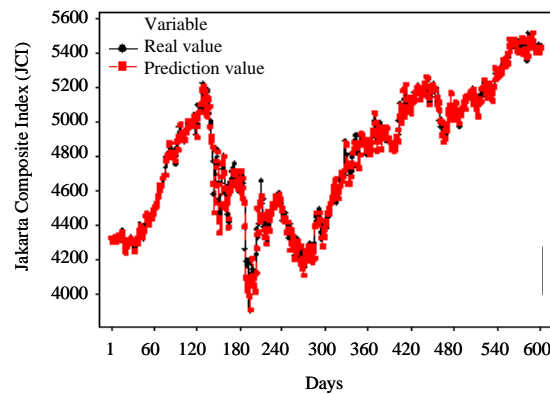


Fig. 9: Plot of real and forecasting values of JCI using wavelet weighted fuzzy model for testing data

data, Wavelet Weighted Fuzzy Model gives the smallest values of MAPE, MSE and RMSE. The comparison of real values and forecasting values of JCI using Wavelet Weighted Fuzzy Models can be seen in Fig. 8 and 9.

CONCLUSION

In this study, a new procedure to construct wavelet weighted Mamdani fuzzy model was established and applied to predict JCI. The result showed that the wavelet weighted Mamdani fuzzy model gives better accuracy than Wavelet Fuzzy Model and Weighted Fuzzy Model for the training data when viewed from the MSE and RMSE values. For the testing data, Wavelet Weighted Fuzzy Model gives better accuracy than Wavelet Fuzzy Model and Weighted Fuzzy Model when viewed from the values of MAPE, MSE and RMSE. So, wavelet weighted fuzzy model has high capability for modeling JCI.

SUGGESTION

In the future research, to improve the accuracy of prediction, we are going to establish a procedure to get optimal wavelet weighted one-order Sugeno fuzzy model and apply it to predict the JCI value.

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