Journal of Engineering and Applied Sciences 12 (Special Issue 1): 5679-5683, 2017
ISSN: 1816-949X

The Properties of Fuzzy Green Relations on Bilinear Form Semigroups

Karyati
Department of Mathematics Education, Yogyakarta State University, Yogyakarta, Indonesia

Abstract: The Green relations on semigroups have been introduced by Howie. They are right Green relation R, left Green relation L and (two sided) Green relation I. The right Green relation R is defined as $\{(x, y)\in S\times S|(y)_R\}$ with $\langle x\rangle_R$ denotes the right ideal generated by an element x (or called the principle right ideal generated by x. The definition of the left Green relation L and the Green relation I are similar to the definition of the right Green relation. In this study, we will construct the definition of the fuzzy right Green relation (denoted by R^F), the fuzzy left Green relation (denoted by L^F) and the fuzzy Green relation (denoted by I^F) on a semigroup. First we define a fuzzy ideal (right/left) generated by a fuzzy subset (a fuzzy principle ideal) on a semigroup and their examples. Based on, the fuzzy principle ideal definition, we define a fuzzy (right/left) Green relation on a semigroup. The fuzzy subset μ and ρ are fuzzy (right/left) Green related if and only if the fuzzy (right/left) ideal generated by μ is equal to the fuzzy (right/left) ideal generated by ρ.

Key words: Green relation, fuzzy ideal, fuzzy pricipal ideal, fuzzy Green relation, validity, element x, definition

INTRODUCTION

A non empty subset I of a semigroup S is called a right (left) ideal if $IS\subseteq I$ ($IS\subseteq I$) and an ideal (two sided) if I is both a right ideal and a left ideal. The right (left) generated by $x\in S$ is denoted by $\langle x\rangle_F$ ($\langle x\rangle_L$) and an ideal generated by $x\in S$ is denoted by $\langle x\rangle$. The Green relation on a semigroup has been introduced by Howie (1976). They are right Green relation (R), the left Green relation (L) and the Green relation (I). The Green relation R, L, I are equivalence relations, defined as follow:

$$R = \{(x,y)\in S\times S \,|\, \langle x\rangle_R = \langle y\rangle_R\}$$
$$L = \{(x,y)\in S\times S \,|\, \langle x\rangle_L = \langle y\rangle_L\}$$
$$I = \{(x,y)\in S\times S \,|\, \langle x\rangle = \langle y\rangle\}$$

Some studies related to the fuzzy ideal of semigroups, the fuzzy ideal of semigroups generated by a fuzzy singleton and their properties have been introduced by Karyati (2002). In this study we will discuss how to define the fuzzy Green relations on a semigroup based on the fuzzy (right/left) ideal generated by a fuzzy subset of this semigroup.

MATERIALS AND METHODS

Fuzzy Green relations on semigroup: Asaad (1991), Kandasamy (2003), Malik and Mordeson (1998), a fuzzy subsemigroup μ of a semigroup S is defined as a mapping from S into the interval (0, 1), i.e., $\mu: S\rightarrow(0, 1)$ which fulfils the condition $\mu(xy)\geq\min\{\mu(x), \mu(y)\}$ for all $x, y\in S$. A fuzzy subset μ is called a fuzzy right (fuzzy left) ideal of S, if for every $x, y\in S$ then $\mu(xy)\geq\mu(x)$ ($\mu(xy)\geq\mu(x)$) and μ is called fuzzy ideal of S if μ is both a fuzzy right ideal and a fuzzy left ideal, i.e., $\mu(xy)\geq\max\{\mu(x), \mu(y)\}$ for all $x, y\in S$. Fuzzy subsets λ and μ are called $\lambda\subset\mu$ if and only if $\lambda(x)\leq\mu(x)$ for every $x, y\in S$. A fuzzy relation θ of S is defined as a mapping from $S\times S$ into the closed interval (0, 1).

Definition 2.1: Let S be a semigroup and μ be a fuzzy relation on S. Then, A fuzzy relation μ on S is said to be reflexive if $\mu(x, x) = 1$ for all $x\in S$. A fuzzy relation μ on S is said to be symmetric if $\mu(x, y) = \mu(x, y)$ for all $x, y\in S$. If $\mu_1 = \mu_2$ are two relations on S, then their max-product composition denoted by $\mu_1\circ\mu_2$ is defined as Aktas (2004), Kuroki (1992) and Murali (1989):

$$\mu_1\circ\mu_2(x,y) = \max_{Z\in S}\{\mu_1(x,z), \mu_2(z,y)\}$$

If $\mu_1 = \mu_2 = \mu$ and $\mu\circ\mu\leq\mu$, then the fuzzy relation μ is called transitive. Aktas (2004), Kuroki (1992) and Murali (1989), we give some kinds of relations defined as follow.

Definition 2.2: A fuzzy relation μ on a semigroup S is called a similarity relation if μ is reflexive, symmetric and transitive (Aktas, 2004; Kuroki, 1992).

Definition 2.3: Let S be a semigroup. A fuzzy relation μ on S is called fuzzy left (right) compatible if and only if

$\mu(x, y)\leq\mu(x, ly)$ for all $x, y, l\in S$ ($\mu(x, y))\leq\mu(xt, yt)$ for all $x, y, t\in S$) (Aktas, 2004; Kuroki, 1992; Murali, 1989).

Definition 2.4: A fuzzy relation μ on a semigroup S is called fuzzy compatible if and only if min $\{\mu(a, b), \mu(c, d)\}\leq\mu(ac, bd)$ for all $a, b, c, d\in S$ (Aktas, 2004; Kuroki, 1992; Murali, 1989).

Definition 2.5: A fuzzy compatible relation μ on a semigroup S is called a fuzzy congruence. Karyati (2002), Mary (2011), Rajendran and Nambooripad (2000), Green (right/left) relations on a semigroup are defined as follow (Aktas, 2004; Kuroki, 1992).

Definition 2.6: Let S be a semigroup and S^1 be a monoid generated by S. For element a and b of S, the Green (right/left) relations are defined by Karyati (2002), Mary (2011), Rajendran and Nambooripad (2000):

- a and b are R_related, denoted by aRb, if and only if $S^1a = S^1b$
- a and b are L_related, denoted by aLb, if and only if $aS^1 = bS^1$
- a and b are I_related, denoted by aIb, if and only if aRb and aLb

In other word, a and b are R_related (L_related/I_related), if they generate the same right (left/two sided) ideal. Let S be a semigroup then, we can construct infinite fuzzy subsets of S. Let α be a fuzzy subset of S. There are some fuzzy ideals of S, denoted by R_α^F having a property $\beta_1\supseteq\alpha$. We collect all of fuzzy ideals with this property and we define a set as follow.

Definition 2.7: Let α be a fuzzy subset of a semigroup S. A set of all fuzzy right ideal of S containing α, a fuzzy subset of S, denoted by R_α^F is defined as:

$$R_\alpha^F = \{\beta|\beta \text{ is fuzzy right ideal of S with } \beta\supseteq\alpha\}$$

Defining a fuzzy left ideal containing α, a fuzzy subset S, denoted by L_α^F and defining a fuzzy ideal (two sided) containing α, a fuzzy subset of S, denoted by I_α^F are similar to R_α^F.

Example 2.8: Let S be a set, i.e., S = {a, b, c, d, e}. The set S is a semigroup with respect to a binary operation number defined as the following Cayley (Table 1). The following function is a fuzzy subset of S:

$$\alpha(x) = \begin{cases} 0.5, & x = a, b, c \\ 0.25 & x = d, e \end{cases}$$

Table 1: The Caley table semigroup (S, #)

#	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	b	c
c	a	b	c	a	a
d	a	a	a	d	e
e	a	d	e	a	a

The fuzzy ideals containing the fuzzy subset α are given as follow:

$$\beta_i(x) = \begin{cases} \gamma_i, & x = a \\ 0.5 + \Delta_i, & x = b, c, d, e \end{cases}$$

with $0.5+\Delta_i\leq r_i\leq 1$ and $0\leq\Delta_i\leq 0.5$. Based on the definition of β_i, we can prove that every β_i contains α. Furthermore, we have the following example.

Example 2.9: Based on the example 2.8, we can construct another fuzzy subset of S. One of them is given as follow:

$$\alpha' = \begin{cases} 0.5, & x = a \\ 0, & x = b, c, d, e \end{cases}$$

Generally, we can construct fuzzy ideals of S containing α' as follow:

$$\delta_j(x) = \begin{cases} s_j, & x = a \\ 0.5 + \varepsilon_j, & x = b,c,d,e \end{cases}$$

with $0.5+\varepsilon_j,\leq s_j\leq 1$ and $0\leq\varepsilon_j\leq 0.5$.

Definition 2.10: Let S be a semigroup and α be a fuzzy subset of S. A fuzzy subset $\rho = \langle\alpha\rangle_R^F$, if:

$$\rho\in R_\alpha^F \quad (1)$$

$$\rho\subseteq\beta, \forall\beta\in R_\alpha^F \quad (2)$$

Defining a fuzzy (left) ideal generated by α, denoted by $\langle\alpha\rangle_L^F(\langle\alpha\rangle^F)$ are similar with how we define a fuzzy right ideal generated by α.

Example 2.11: Let ρ be a fuzzy right ideal of a semigroup S' with $\rho(x) = 0.5$ for all x = a, b, c, d, e and ρ be the fuzzy subset α as the Example 2.8. We can prove that $\rho\supset\alpha$ so $\rho\in R_\alpha^F$. For every fuzzy right ideal contains α then:

$$\beta_i(x) = \begin{cases} \gamma_i, & x = a \\ 0.5 + \Delta_i, & x = b,c,d,e \end{cases}$$

with $0.5+\Delta_i \leq r_i \leq 1$ and $0 \leq \Delta_i \leq 0.5$, $\rho \subseteq \beta_i$. The following theorem gives one of the properties of the fuzzy (right/left) ideal generated by α, a fuzzy subset of a semigroup.

Theorem 2.12: Let S be a semigroup, α be a fuzzy subset of S and β_i be a fuzzy right ideal of S such that $\beta_i \supseteq \alpha$ for every i∈N. If, we define a mapping from S into (0, 1), i.e., $\wedge_{\beta_i \in R^F_\alpha} \beta_i : S \rightarrow (0,1)$ with $\wedge_{\beta_i \in R^F_\alpha} \beta_i(x) = \inf\{\beta_i(x)\}$ then $\wedge_{\beta_i \in R^F_\alpha} \beta_i$ is a fuzzy right ideal generated by α.

Proof: Let x, y∈S such that x = y. So, we have $\wedge_{\beta_i \in R^F_\alpha} \beta_i(x) = \inf\{\beta_i(x)\} = \inf\{\beta_i(y)\} = \wedge_{\beta_i \in R^F_\alpha} \beta_i(y)$. Since, $\beta_i \in R^F_\alpha$, we get $0 \leq \beta_i(x) \leq 1$ for every I∈N. Then, we have $0 \leq \inf\{\beta_i(x)\} \leq 1$ or $0 \leq \wedge_{\beta_i \in R^F_\alpha} \beta_i \leq 1$. Hence, the mapping $\wedge_{\beta_i \in R^F_\alpha} \beta_i$ is the fuzzy subset of S.

Since, β_i is a fuzzy right ideal so, we have $\wedge_{\beta_i \in R^F_\alpha} \beta_i(xy) = \inf\{\beta_i(xy)\} \geq \inf\{\beta_i(x)\} = \wedge_{\beta_i \in R^F_\alpha} \beta_i(x)$. It is clear that $\wedge_{\beta_i \in R^F_\alpha} \beta_i$ is a fuzzy right ideal of S. Since, $\beta_i \in R^F_\alpha$ for every i∈N, so we get $\alpha \subseteq \beta_i$ for i∈N. Then, we have $\alpha(x) \leq \beta_i(x)$ for every i∈N and x∈S. Finally, we have $\alpha(x) \leq \beta_i(x)$ or $\alpha \subseteq \wedge_{\beta_i \in R^F_\alpha} \beta_i$.

For the other cases, i.e., when α be a fuzzy subset of S and β_i be a fuzzy left (two sided) ideal of S such that $\beta_i \supseteq \alpha$ for every i∈N, if we define a mapping from S into (0, 1) $\wedge_{\beta_i \in R^F_\alpha} \beta_i : S \rightarrow (0,1)$ with $\wedge_{\beta_i \in R^F_\alpha} \beta_i(x) = \inf\{\beta_i(x)\}$ then $\wedge_{\beta_i \in R^F_\alpha} \beta_i$ is a fuzzy left (two sided) ideal generated by α.

If γ is an arbitrary fuzzy right ideal of a semigroup S generaterd by α, a fuzzy subset of S, then we guarantee γ is equal to $\wedge_{\beta_i \in R^F_\alpha} \beta_i$. This property is given ini the following theorem.

Theorem 2.13: Let S be a semigroup, α be a fuzzy subset of S and β_i be a fuzzy right ideal of S such that $B_i \supseteq \alpha$ for every i∈N. If γ is a fuzzy right ideal of S generated by α, then $\wedge_{\beta_i \in R^F_\alpha} \beta_i = \gamma$.

Proof: Since, γ is a fuzzy right ideal of S generated by α, we have $\gamma \in \langle\alpha\rangle^F_R$. So, we get $\wedge_{\beta_i \in R^F_\alpha} \beta_i(x) = \inf\{\beta_i(x)\} \leq \gamma(x)$. Hence, we have $\wedge_{\beta_i \in R^F_\alpha} \beta_i \subseteq \gamma$. Since, $\wedge_{\beta_i \in R^F_\alpha} \beta_i \subseteq \gamma$ so, we have $\gamma \in \beta_i$ for every $\beta_i \in \langle\alpha\rangle^F_R$. Finally, we have $\gamma \subseteq \wedge_{\beta_i \in R^F_\alpha} \beta_i$.

Similar to Theorem 2.13, let S be a semigroup, α be a fuzzy subset of S and β_i be a fuzzy left (two sided) ideal of S generated by α, then $\wedge_{\beta_i \in R^F_\alpha} \beta_i = \gamma$.

Definition 2.14: Let S be a semigroup and F(S) be the family of all fuzzy subset of S. For every μ, ρ⊂F(S), a mapping $R^F(\mu, \rho)$ from F(S)×F(S) into the closed interval (0, 1) is defined as:

$$R^F(\mu, \rho) = \begin{cases} 1, & \langle\mu\rangle^F_R = \langle\rho\rangle^F_R \\ 0, & \langle\mu\rangle^F_R \neq \langle\rho\rangle^F_R \end{cases}$$

The fuzzy subset in the Definition 2.14 is called a fuzzy right Green Relation and denoted by R^F. The definition of the fuzzy left Green relation L^R and fuzzy Green relation I^F are defined similarly. The fuzzy subset R^F, L^F and I^F are fuzzy relations.

Theorem 2.15: The mapping R^F from F(S)×F(S) into the closed interval (0, 1) defined as: $R^F(\mu, \rho)$-1 if $\langle\mu\rangle^F_R$-$\langle\rho\rangle^F_R$ and $R^F(\mu, \rho) = 0$ if $\langle\mu\rangle^F_R \neq \langle\rho\rangle^F_R$ is a fuzzy relation on F(S).

Proof: Let (μ, ρ) = (μ′, ρ′) so, μ′ = μ′ and ρ = ρ′. If $\langle\mu\rangle^F_R = \langle\rho\rangle^F_R$ then $\langle\mu'\rangle^F_R = \langle\rho'\rangle^F_R$ and we get $R^F(\mu, \rho) = 1 = R^F(\mu', \rho')$. For the other case, if $\langle\mu\rangle^F_R \neq \langle\rho\rangle^F_R$ then $\langle\mu'\rangle^F_R \neq \langle\rho'\rangle^F_R$. Finally, w obtain $R^F(\mu, \rho) = 0 = R^F(\mu', \rho')$. The mapping R^F is a mapping from F(S)×F(S) into the closed interval (0, 1). Thus, we have $R^F(\mu, \rho)$ is a fuzzy subset, i.e., the value of $R^F(\mu, \rho)$ between 0 and 1. So, it is a fuzzy relation.

Similar to Theorem 2.15, the mapping L^F (I^F) from F(S)×F(S) into the closed interval (0, 1) defined as: $L^F(u, p) = 1$ if $\langle\mu\rangle^F_L = \langle\rho\rangle^F_L$ ($\langle\mu\rangle^F_L = \langle\rho\rangle^F_L$) and $L^F(u, p) = 0$) ($I^F(\mu, \rho) = 0$) if $\langle\mu\rangle^F_L \neq \langle\rho\rangle^F_L$ ($\langle\mu\rangle^F_L \neq \langle\rho\rangle^F_L$) is a fuzzy relation on F(S).

Theorem 2.16: The fuzzy relation R^F defined as on the Definition 2.14 is a fuzzy similarity relation on F(S).

Proof: Based on the Definition 2.2, we must prove that R^F is reflexive, i.e., $R^F(\mu, \mu) = 1$. It is always fulfilled that $\langle\mu\rangle^F_R = \langle\mu\rangle^F_R$. Based on the Definition 2.14, we obtain $R^F(\mu, \mu) = 1$. The second one, we must prove that R^F is symmetric, i.e., $R^F(\mu, \rho) = R^F(\rho, \mu)$. If $R^F(\mu, \rho) = 1$, then $\langle\mu\rangle^F_R = \langle\rho\rangle^F_R$ and $\langle\rho\rangle^F_R = \langle\mu\rangle^F_R$. Now, we obtain $R^F(\rho, \mu) = 1$. Finally, we have $R^F(\mu, \rho) = R^F(\rho, \mu)$. We can prove similarly for other case $R^F(\mu, \rho) = 0$. Thirdly, we must proof that R^F is transitive, i.e., $(R \circ R)(\mu, \rho) \geq R(\mu, \rho)$. Based on the definition, we have:

$$(R \circ R)(\mu, \rho) = \max_{\alpha \in F(S)}\{\min\{R(\mu, \alpha), R(\alpha, \rho)\}\}$$

If the case is $\langle\mu\rangle^F_R = \langle\rho\rangle^F_R$ and $\langle\mu\rangle^F_R = \langle\alpha\rangle^F_R$, then we have:

$$(R \circ R)(\mu, \rho) = \max_{\alpha \in F(S)}\{\min\{1, 1\}\} \geq 1 = R(\mu, \rho)$$

If the case is $\langle\mu\rangle^F_R \neq \langle\rho\rangle^F_R$ and $\langle\mu\rangle^F_R = \langle\alpha\rangle^F_R$, then we have:

$$(R \circ R)(\mu, \rho) = \max_{\alpha \in F(S)}\{\min\{0, 1\}\} \geq 0 = R(\mu, \rho)$$

For the other cases, we can prove similarly as the above.

Example 2.17: Based on the Example 2.8 and 2.9, we get:

$$\bigwedge_i \beta_i(x) = \inf\{\beta_i(x)\} = \inf\{0.5 + \Delta_i\} = 0.5$$

So, we obtain $\alpha \subseteq \wedge_i \beta_i$. It is clearly that $\wedge_i \beta_i$ is the smallest fuzzy ideal fuzzy containing α or in other word $\wedge_i \beta_i$ is a fuzzy ideal generated by α and denoted by $\langle \alpha \rangle^F_R$:

$$\bigwedge_i \delta_i(x) = \inf\{\beta_j(x)\} = \inf\{0.5 + \varepsilon_j\} = 0.5$$

So, we obtain $\alpha' \subseteq \wedge_j \delta_j$. It is clearly that $\wedge_j \delta_j$ is the smallest fuzzy ideal fuzzy containing α' or in other word $\wedge_j \delta_j$ is a fuzzy ideal generated by α' and denoted by $\langle \alpha' \rangle^F_R$. Finally, we have $\langle \alpha \rangle^F_R = \langle \alpha' \rangle^F_R$ or in the other word $\langle \alpha, \alpha' \rangle \in R^F$.

Fuzzy Green relation on bilinear form semigroups: Fuzzy right Green relation R^F, fuzzy left Green relation L^F and fuzzy Green relation I^F are fuzzy equivalence relations on F (S (B)), respectively. For every $\alpha \in F$ (S (D)), we define a fuzzy subset $R^F_{(\alpha)}$ which is defined as $R^F_{(\alpha)}(\beta) = R^F(\alpha, \beta)$, for every $\beta \in F$ (S (B)). So, we have $R^F_{(\alpha)}$ is a fuzzy subset on the family of fuzzy subset on F (S (B)). The following proposition is one of the properties of this relation.

RESULTS AND DISCUSSION

Proposition 3.1: For arbitrary μ, $\beta \in F(S(B))$, then for a fuzzy right Green relation R^F we have the following bi-implication:

$$R^F_{(\alpha)} = R^F_{(\beta)} \Leftrightarrow R^F(\alpha, \beta) = 1$$

Proof: The first we assume that $R^F_{(\alpha)}$-$R^F_{(\beta)}$. Therefore, we have:

$$R^F_{(\alpha)}(\beta) = R^F_{(\beta)}(\beta) = R^F(\beta, \beta) = 1$$

Conversely, we assume that $R^F(\alpha, \beta) = 1$. For every $\delta \in F(S(B))$, we have:

$$\begin{aligned} R^F_{(\alpha)}(\delta) &= R^F(\alpha, \delta) \geq (R^F \circ R^F)(\alpha, \delta) \\ &= \sup_{\eta \in F(S(B))} \{\min\{R^F(\alpha, \eta), R^F(\eta, \delta)\}\} \\ &= \min\{R^F(\alpha, \beta), R^F(\beta, \delta)\} \\ &= \min\{1, R^F(\beta, \delta)\} = R^F(\beta, \delta) = R^F_{(\beta)}(\delta) \end{aligned}$$

So, we have $R^F_{(\alpha)}(\delta) > R^F_{(\beta)}(\delta)$ for every $\delta \in F$ (S (B)). It is mean that $R^F_{(\alpha)} \supseteq R^F_{(\beta)}$. On the other, relation R^F is reflective. Hence, we have $R^F(\alpha, \beta) = R^F(\beta, \alpha)$. Now, we obtain:

$$\begin{aligned} &\text{For every } \delta \in F(S(B)) \\ R^F_{(\beta)}(\delta) &= R^F(\mathrm{B}, \delta) \\ &\geq (R^F \circ R^F)(\beta, \delta) \\ &= \sup_{\eta \in F(S(B))} \{\min\{R^F(\beta, \eta), R^F(\eta, \delta)\}\} \\ &= \min\{R^F(\beta, \alpha), R^F(\alpha, \delta)\} \\ &= \min\{1, R^F(\beta, \alpha)\} = R^F(a, \delta) = R^F_{(a)}(\delta) \end{aligned}$$

So, we have $R^F_{(\beta)}(\delta) > R^F_{(\alpha)}(\delta)$ for every $\delta \in F$ (S (B)). It is mean that $R^F_{(\alpha)} \subseteq R^F_{(\beta)}$. Finally, we can prove that $R^F_{(\alpha)} = R^F_{(\beta)}$. The following proposition give the properties of relation L^F and I^F, respectively.

Proposition 3.2: For arbitrary elements α, $\beta \in F(S(B))$, then for fuzzy right Green relation L^F we have the following bi-implication:

$$I^F_{(\alpha)} = I^F_{(\beta)} \Leftrightarrow L^F(\alpha, \beta) = 1$$

Proof: The proof of this proposition is in the same way with the proof of the previous proposition.

Proposition 3.3: For arbitrary elements α, $\beta \in F(S(B))$, then for fuzzy right Green relation I^F we have the following bi-implication:

$$I^F_{(\alpha)} = I^F_{(\beta)} \Leftrightarrow I^F(\alpha, \beta) = 1$$

Proof: The proof of this proposition is in the same way with the proof of the previous proposition. Furthermore, fuzzy subsets $R^F_{(\alpha)}$, $L^F_{(\alpha)}$ and $I^F_{(\alpha)}$ of bilinear form semigroup F(S (B)) are equivalence classes of equivalence relations R^F, L^F and I^F which contain α, respectively. Based on these equivalence classes, we can construct a set as:

$$F(S(B))/R^F = \{R^F_{(\alpha)} \mid \alpha \in F(S(B))\}$$

We can define an operation "*" on F (S (B))/R^F which is defined as:

$$R^F_{(\alpha)} * R^F_{(\beta)} = R^F_{(\alpha\beta)}$$

This operation is a binary operation, i.e., for every $R_{(\alpha)}{}^F = R_{(\beta)}{}^F$ and $R_{(\gamma)}{}^F = R_{(\delta)}{}^F$ we have:

$$R^F(\alpha\gamma, \beta\delta) \geq R^F \circ R^F(\alpha\gamma, \beta\delta) = \sup_{\varepsilon \in F(S(B))} \{\min\{R^F(\alpha\gamma, \varepsilon), R^F(\varepsilon, \beta\delta)\}\} \geq \min\{R^F(\alpha\gamma, \beta\gamma), R^F(\beta\lambda, \beta\delta)\}$$

Hence, we obtain $R^F_{(\alpha\gamma,\ \beta\delta)} = 1$ or $R_{(\alpha\gamma)}{}^F = R_{(B\delta)}{}^F$. The operation '*' is associative, i.e., for every $R_{(\alpha)}{}^F$, $R_{(\beta)}{}^F$, $R_{(\gamma)}{}^F \epsilon F(S(B))/R^F$:

$$\left(R^F_{(\alpha)} * R^F_{(\beta)}\right) * R^F_{(\gamma)} = R^F_{(\alpha\beta)} * R^F_{(\gamma)} * = R^F_{((\alpha\beta)\gamma)} = R^F_{(\alpha(\beta\gamma))} = R^F_{(\alpha)} * R^F_{(\beta\gamma)} = R^F_{(\alpha)} * \left(R^F_{(\beta)} * R^F_{(\gamma)}\right)$$

So, we have proven that $(F(S(B))/R^F, *)$ is a semigroup. In the same way, we can construct another semogroups, i.e. $(F(S(B))/L^F, *)$ and $(F(S(B))/I^F, *)$.

CONCLUSION

Refer to the second section and third section, we conclude to define a fuzzy (right/left) Green relation I^F (R^F/L^F) on a semigroup, the first we define a fuzzy (right/left) ideal generated by an fuzzy subset. We define $(\alpha, \beta) \epsilon I^F$ if and only if α and β generade the same fuzzy ideal. Futhermore, we can define $(\alpha, \beta) \epsilon R^F$ and $(\alpha, \beta) \epsilon L^F$ in the same way, respectively. We have proven that these relations are equivalence relations on a family all fuzzy subsets on a semigroup which denoted by F(S). This properties hold on a bilinear form semigroup. Furthermore we defined a set of all equivalence classes which are denoted by $F(S(B))/R^F$, $F(S(B))/I^F$ and $F(S(B))/L^F$. We have proven that $(F(S(B))/R^F, *)$ is a semigroup. Similarly, we can prove that $(F(S(B))/I^F, *)$ and $(F(S(B))/L^F, *)$ are semigroups. We obtain many properties related to these semigroups, i.e., $R^F_{(\alpha)} = R^F_{(\beta)} \Leftrightarrow R^F(\alpha, \beta) = 1$, $L^F_{(\alpha)} = L^F_{(\beta)} \Leftrightarrow L^F(\alpha, \beta) = 1$ and $I^F_{(\alpha)} = I^F_{(\beta)} \Leftrightarrow I^F(\alpha, \beta) = 1$. The other results, we have obtained: for arbitrary $\alpha, \beta \epsilon F(S(B))$, then for a fuzzy right Green relation R^F we have the following bi-implication $R^F_{(\alpha)} = R^F_{(\beta)} \Leftrightarrow R^F(\alpha, \beta) = 1$. Following to the properties are similar with the previous property $L^F_{(\alpha)} = L^F_{(\beta)} \Leftrightarrow L^F(\alpha, \beta) = 1$ and $I^F_{(\alpha)} = I^F_{(\beta)} \Leftrightarrow I^F(\alpha, \beta) = 1$. We construct a set, i.e., $F(S(B))/R^F = \{R^F_{(\alpha)} | \alpha \epsilon F(S(B))\}$ and defined $R^F_{(\alpha)} * R^F_{(\beta)} = R^F(\alpha, \beta)$, then $(F(S(B))/R^F, *)$ is a semigroup. In the same way, we can construct semigroups $(F(S(B))/L^F, *)$ and $(F(S(B))/I^F, *)$.

REFERENCES

Aktas, H., 2004. On fuzzy relation and fuzzy quotient groups. Int. J. Comput. Cognition, 2: 71-79.

Asaad, M., 1991. Groups and fuzzy subgroups. Fuzzy Sets Syst., 39: 323-328.

Howie, J.M., 1976. An Introduction to Semigroup Theory. 1st Edn., Academic Press, New York, ISBN-10: 0123569508.

Kandasamy, W.V., 2003. Smarandache Fuzzy Algebra. American research Press, Rehoboth Beach, Delaware, ISBN:1-931233-74-8, Pages: 454.

Karyati, K., 2002. [Semi group constructed from bilinear forms]. Master Thesis, Gadjah Mada University, Yogyakarta, Indonesia.

Kuroki, N., 1992. Fuzzy congruences and fuzzy normal subgroups. Inf. Sci., 60: 247-259.

Malik, D.S. and J.N. Mordeson, 1998. Fuzzy commutative algebra. World Scientific Publications, USA.

Mary, X., 2011. On generalized inverses and Green's relations. Linear Algebra Appl., 434: 1836-1844.

Murali, V., 1989. Fuzzy equivalence relations. Fuzzy Sets Syst., 30: 155-163.

Rajendran, D. and K.S.S. Nambooripad, 2000. Bilinear forms and semigroup of linear transformations. South East Asian Bull. Math., 24: 609-616.