

## Effect of Rotation on the Rayleigh-Benard Convection in Nanofluid Layer with Vertical Magnetic Field and Internal Heat Source

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**Abstract:** Effect of rotation on the onset of Rayleigh-Benard convection in a horizontal nanofluid layer with vertical magnetic field and internal heat source is investigated. Linear stability analysis based upon normal mode method is employed to find solution of the horizontal nanofluid layer bounded between free-free, rigid-free and rigid-rigid boundaries. Rayleigh number has been determined using the galerkin method. Graphs have been plotted to study the efficiency of rotation, magnetic field, internal heat source and other nanofluid parameters to the system.

**Key words:** Rotation, magnetic field, internal heat source, nanofluid, Rayleigh-Benard convection

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### INTRODUCTION

Nanofluid is an advanced fluid that is effective in a wide range of engineering applications and the purpose of using nanofluid rather than any fluid is to obtain higher values of heat transfer. The earliest appearance observations of thermal conductivity enhancement in nanofluid were reported by Masuda *et al.* (1993) and the term nanofluid has been proposed by Choi (1995). Nanofluid is a base fluids (e.g., water, ethanol, oil) with a dissolved nano-sized (1-100 nm) particles of a various type of materials. Buongiorno's model for convective transport in nanofluid incorporates the effects of Brownian diffusion and thermophoresis has attracted great interest. The effect of nanoparticles deposition on surface wettability is found to give impact on the boiling heat transfer in nanofluid (Kim *et al.*, 2006). The effect of thermal diffusion has shown a strong effect in a fluid suspension of alumina nanoparticles confined between two differentially heated parallel plates under different gravity conditions (Savino and Paterna, 2008). There is a convincing study that cooperates with the physical performance of thermal conductivity enhancement in nanofluid (Eapen *et al.*, 2010). Nield and Kuznetsov (2010, 2011) studied the onset of convection in a horizontal nanofluid layer of finite depth and double-diffusive convection in a horizontal nanofluid layer. The instability effect because of the internal heat source on horizontal fluid layer has been examined by numbers of author

previously (Friedrich and Rudraiah, 1984; Bhattacharyya and Jena, 1984; Char and Chiang, 1994; Mokhtar *et al.*, 2009, 2010, 2011, Capone *et al.*, 2011; Bhadauria *et al.*, 2011) and for the internal heat source effect on horizontal nanofluid layer has been investigated by Yadav *et al.* (2012a, b, 2013, 2014, 2016) and Nield and kuznetsov (2014). The stabilizing effect of magnetic field has pulled on a great interest because of its capability to elevate the performance of heat transfer (Chandrasekhar, 1961; Rao, 1980; Rudraiah *et al.*, 1986; Wilson, 1994; Yadav *et al.*, 2012a, b; Khalid *et al.*, 2013). Effect of magnetic field on horizontal nanofluid layer has been investigated by these researchers Sheikholeslami *et al.* (2014a, b, 2015), Al-Zamily (2014), Yadav *et al.* (2013, 2014), Gupta *et al.* (2013), Hamad *et al.* (2011), Bansal and Chatterjee (2015), Chand and Rana (2015).

The first landmark of the effect of rotation was examined by Chandrasekhar (1961) on the onset of Benard convection. In 1966, the effect of rotation was continued by Vidal and Acrivos (1966) on the onset of Marangoni convection. McConaghy and Finlayson (1969) extended the previous investigation (Vidal and Acrivos, 1966) on the oscillatory Marangoni convection with coriolis force. The effect of rotation in the instability convection for both buoyancy and surface-tension forces was considered by Namikawa *et al.* (1970). In 1994, Kaddame and Lebon (1994a, b) examined the onset of steady and oscillatory Benard-Marangoni convection with the effect of rotation. Later, Hashim and Sarma (2004) explored the

effect of rotation for the onset of steady and oscillatory Marangoni convection. Nanjundappa *et al.* (2015) extended coriolis force in Benard-Marangoni convection with magnetic field dependent viscosity. Meanwhile, Agarwal (2014) and Yadav *et al.* (2016) used the effect of coriolis force on the nanofluid layer. Very recently, Wakif *et al.* (2016) examined the effect of internal heating in nanofluid layer with the addition effect of coriolis force.

The mentioned literature survey demonstrates no other study has been made on the effect of rotating nanofluid layer with vertical magnetic field and internal heat source on the onset of Rayleigh-Benard convection. Therefore, our present investigation is examined and the obtained results are presented graphically.

**MATERIALS AND METHODS**

**Mathematical formulation:** Consider a horizontal layer of a rotating nanofluid layer of thickness, *d* confined between the planes  $z^* = 0$  and  $z^* = d$  subjected to the uniformly internal heat generation,  $J_0^*$  with the uniform vertical magnetic field,  $h^* = (0, 0, h_0^*)$  heated from below. The nanofluid layer is keep rotating about vertical axis at a constant angular velocity,  $\Omega^* = (0, 0, \Omega)$ . The stability of a horizontal rotating nanofluid layer in the existence of vertical magnetic field and internal heat generation is examined. The dimensional (non-dimensional) variables are marked with asterisks (without asterisks). The temperatures at the bottom and upper wall are taken to be  $T_1^*$  and  $T_u^*$ . The applicable governing equations to describe the Boussinesq flow under this model with the modified Maxwell’s equations are:

$$\nabla^* \cdot V^* = 0 \tag{1}$$

$$\rho_{f0} \left[ \frac{\partial V^*}{\partial t^*} + (V^* \cdot \nabla^*) V^* \right] = -\nabla^* p^* + \mu \nabla^{*2} V^* + (2\rho_0 V^* \times \Omega^*) + \frac{\mu_e}{4\pi} (\nabla^* \times h^*) \times h^* + g \left\{ \phi^* \rho_p + (1 - \phi^*) \rho_{f0} \left[ 1 - \alpha (T^* - T_u^*) \right] \right\} \tag{2}$$

$$(\rho c)_f \left[ \frac{\partial T^*}{\partial t^*} + (V^* \cdot \nabla^*) T^* \right] = \kappa \nabla^{*2} T^* + J_0^* + (\rho c)_p \left[ D_B \nabla^* \phi^* \cdot \nabla^* T^* + D_T \frac{\nabla^* T^* \cdot \nabla^* T^*}{T_u^*} \right] \tag{3}$$

$$\left[ \frac{\partial \phi^*}{\partial t^*} + (V^* \cdot \nabla^*) \phi^* \right] = D_B \nabla^{*2} \phi^* + D_T \frac{\nabla^{*2} T^*}{T_u^*} \tag{4}$$

$$\left[ \frac{\partial h^*}{\partial t^*} + (V^* \cdot \nabla^*) h^* \right] = (h^* \cdot \nabla^*) V^* + \eta \nabla^{*2} h^* \tag{5}$$

$$\nabla^* \cdot h^* = 0 \tag{6}$$

Where:

- $V^* = (u, v, w)$  = Nanofluid velocity
- $t^*$  = Time
- $\rho_{f0}$  = Nanofluid density
- $p^*$  = Pressure
- $\mu$  = Viscosity
- $\mu_e = \phi^* \mu_{ep} + (1 - \phi^*) \mu_{ef}$  = Magnetic permeability
- $h^* = (h_x, h_y, h_z)$  = Magnetic field
- $g$  = Gravity
- $\rho_p$  = Particle density
- $\phi^*$  = Nanoparticles volume fraction
- $\alpha = -\rho_0^{-1} \frac{\partial \rho}{\partial T}$  = Thermal expansion coefficient
- $c$  = Specific heat
- $T^*$  = Temperature
- $\kappa$  = Nanofluid thermal conductivity
- $D_B$  = Brownian diffusion
- $D_T$  = Thermophoretic diffusion coefficient
- $\eta = \phi^* \eta_p + (1 - \phi^*) \eta_f$  = Electrical resistivity

On the steady state, the upper free surface of the fluid layer is flat and stationary. The pressure and temperature field are (Char and Chiang, 1994):

$$p(z) = p_0 - \rho_0 g (z - d) \left[ 1 + \frac{\alpha \Delta T}{2d} (z - d) \right] \tag{7}$$

$$T(z) = -\frac{J_0^*}{2\kappa} z^2 + \left[ \frac{J_0^* d}{2\kappa} - \frac{\Delta T}{d} \right] z + T_0 \tag{8}$$

Where:

- $\Delta T$  = Temperature difference
- $P_0$  = Reference pressure along the nanofluid layer

Dimensionless variables are introduced as below:

$$\begin{aligned} (x, y, z) &= \frac{(x^*, y^*, z^*)}{d}, (u, v, w) = d \frac{(u^*, v^*, w^*)}{\alpha_f} \\ \Psi_z &= d^2 \frac{\Psi_z^*}{\alpha_f}, t = \frac{t^* \alpha_f}{d^2}, p = \frac{p^* d^2}{\mu \alpha_f}, \phi = \frac{\phi^* - \phi_u^*}{\phi_1^* - \phi_u^*}, \\ T &= \frac{T^* - T_u^*}{T_1^* - T_u^*} (h_x, h_y, h_z) = \frac{(h_x^*, h_y^*, h_z^*)}{h_0^*} \end{aligned} \tag{9}$$

Where:

- $\alpha_f = \kappa / \rho c$  = Thermal diffusivity
- $\Psi_z^*$  = Z-component of vorticity due to the rotation

Substitute Eq. 9 into Eq. 1-6 then the following equations are the obtained non-dimensional variables:

$$\nabla \cdot \mathbf{V} = 0 \tag{10}$$

$$\frac{1}{Pr} \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \nabla^2 \mathbf{V} - Rm \hat{e}_z + RaT \hat{e}_z - Rn \phi \hat{e}_z + H \frac{Pr}{Pm} [(\nabla \times \mathbf{h}) \times \mathbf{h}] + \sqrt{Ta} (\mathbf{V} \times \hat{e}_z) \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T = \nabla^2 T + \frac{N_B}{Le} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{Le} \nabla T \cdot \nabla T + [Q(1-2z) - 1] w \tag{12}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 T \tag{13}$$

$$\frac{\partial \mathbf{h}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{h} = (\mathbf{h} \cdot \nabla) \mathbf{V} + \frac{Pr}{Pm} \nabla^2 \mathbf{h} \tag{14}$$

$$\nabla \cdot \mathbf{h} = 0 \tag{15}$$

Where:

- $Pr = \mu/\alpha_f \rho$  = Prandtl number
- $R_m = [\rho_p \phi_0^* + \rho_{b0}(1-\phi_0^*)]gd^3/\mu\alpha_f$  = Basic density Rayleigh number
- $Ra = \rho_b g \alpha d^3 \Delta T/\mu\alpha_f$  = Rayleigh number
- $R_n = (\rho_p - \rho_b)(\phi_1^* - \phi_0^*)/gd^3 \mu\alpha_f$  = Nanoparticle concentration Rayleigh number
- $H = \mu_{ef} h_0^* d^2/4\pi\rho_b \nu \eta_f$  = Nanofluid magnetic number
- $P_m = \mu/\rho_b \eta$  = Magnetic prandtl number
- $Ta = 4\Omega^2 d^4/\mu^2$  = Taylor number
- $N_B = (\rho c)_p/(\rho c)_f(\phi_1^* - \phi_0^*)$  = Modified particle density increment
- $N_A = D_T \Delta T^*/D_B T_c(\phi_1^* - \phi_0^*)$  = Modified diffusivity ratio
- $Le = \alpha_f/D_B$  = Lewis number
- $Q = J_0^* d^2/2\kappa \Delta T^*$  = Internal heat source

We seek a time-independent quiescent solution of Eq. 10-15 with temperature and nanoparticle volume fraction varying only in the z-direction, that is a solution of the form:

$$\mathbf{V} = 0, T = T_b(z), p = p_b(z), \phi = \phi_b(z), \mathbf{h} = \hat{e}_z \tag{16}$$

where the subscript b denotes the basic state. On the basic state we superimposed infinitesimal perturbations in the form:

$$\mathbf{V} = \mathbf{V}', T = T_b(z) + T', p = p_b(z) + p', \phi = \phi_b(z) + \phi', \psi_z = \psi'_{b,z} + \psi'_z, \mathbf{h} = \hat{e}_z + \mathbf{h}' \tag{17}$$

Substitute Eq. 17 into Eq. 10-15 and linearized by neglecting products of primed quantities. The resulting equations are obtained when Eq. 16 are used:

$$\nabla \cdot \mathbf{V}' = 0 \tag{18}$$

$$\frac{1}{Pr} \frac{\partial \mathbf{V}'}{\partial t} = -\nabla p' + \nabla^2 \mathbf{V}' + RaT' \hat{e}_z - Rn \phi' \hat{e}_z + \sqrt{Ta} (\mathbf{V}' \times \hat{e}_z) + H \frac{Pr}{Pm} [(\nabla \times \mathbf{h}') \times \hat{e}_z] \tag{19}$$

$$\frac{\partial T'}{\partial t} - [Q(1-2z) - 1] w' = \nabla^2 T' + \frac{N_B}{Le} \left( \frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z} \right) - \frac{2N_A N_B}{Le} \frac{\partial T'}{\partial z} \tag{20}$$

$$\frac{\partial \phi'}{\partial t} + w' = \frac{1}{Le} \nabla^2 \phi' + \frac{N_A}{Le} \nabla^2 T' \tag{21}$$

$$\frac{\partial \mathbf{h}'}{\partial t} = \frac{\partial w'}{\partial z} \hat{e}_z + \frac{Pr}{Pm} \nabla^2 \mathbf{h}' \tag{22}$$

$$\nabla \cdot \mathbf{h}' = 0 \tag{23}$$

The parameter  $R_m$  is not included in these and subsequent equations. It is just the measure of the basic static pressure gradient. Applying the curl operator twice on Eq. 19 together with Eq. 18 and Eq. 23 then combine the resulting Eq. 19 with Eq. 22 in the z-component, we obtain:

$$\left[ \nabla^4 - \frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 \right] w' - RaT' \nabla_p^2 + Rn \phi' \nabla_p^2 - H \frac{Pr}{Pm} \nabla^2 \left( \frac{\partial^2 w'}{\partial z^2} \right) - \sqrt{Ta} \frac{\partial \psi'_z}{\partial z} = 0 \tag{24}$$

$$\left[ \nabla^2 - \frac{1}{Pr} \frac{\partial}{\partial t} \right] \psi'_z + \sqrt{Ta} \frac{\partial w'}{\partial z} = 0 \tag{25}$$

In Eq. 24 is the horizontal 2-dimensional Laplacian operator. The proposed normal mode technique is in the form:

$$(w', T', \phi', \psi'_z) = [W(z), \theta(z), \phi(z), \Psi(z)] e^{[i(a_x x + a_y y) + \sigma t]} \tag{26}$$

Where:

- $(a_x, a_y)$  = Wave vector on the (x, y) plane
- $\alpha^2 = \alpha_x^2 + \alpha_y^2$  = Square wavenumber
- $\sigma$  = Growth parameter

Substituting Eq. 26 into Eq. 20-21 and Eq. 24-25 by neglecting terms of the second and higher orders in the perturbations.

$$(D^2 - a^2)^2 W - HD^2 W - a^2 Ra \theta + a^2 Rn \phi - \sqrt{Ta} D \Psi = 0 \tag{27}$$

$$\left[1 - Q(1 - 2z)\right] W + \left[ D^2 + \frac{N_B}{Le} D - \frac{2N_A N_B}{Le} D - a^2 \right] \theta - \frac{N_B}{Le} D \phi = 0 \tag{28}$$

$$W - \frac{N_A}{Le} (D^2 - a^2) \theta - \frac{1}{Le} (D^2 - a^2) \phi = 0 \tag{29}$$

$$\sqrt{Ta} DW + (D^2 - a^2) \Psi = 0 \tag{30}$$

The appropriate boundary conditions are:

$$W = DW = \theta = \xi = \phi = \Psi = DP = 0 \quad \text{at } Z = 0 \tag{31}$$

At the upper free boundary:

$$W = D^2 W = D\theta = \xi = \phi = \Psi = 0, \quad \text{at } Z = 1 \tag{32}$$

At the upper rigid boundary:

$$W = DW = D\theta = \xi = \phi = DP = 0, \quad \text{at } Z = 1 \tag{33}$$

Where  $D = d/dz$  and  $a = \sqrt{a_x^2 + a_y^2}$ .

Galerkin-type weighted residuals method is employed to find an approximate solution to the system of Eq. 27-30. The variables are written in a series of basis function as:

$$W = \sum_{i=1}^N A_i W_i, \theta = \sum_{i=1}^N B_i \Theta_i, \phi = \sum_{i=1}^N C_i \Phi_i, \Psi = \sum_{i=1}^N D_i \Psi_i \tag{34}$$

Substitute Eq. 34 into Eq. 27-30 and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of  $4N$  linear algebraic equations in the  $4N$  unknowns. The vanishing of the determinant of coefficients produces the eigenvalue equation for the system.

### RESULTS AND DISCUSSION

The linear stability analysis is carried out to investigate the effect of the onset of rotating nanofluid

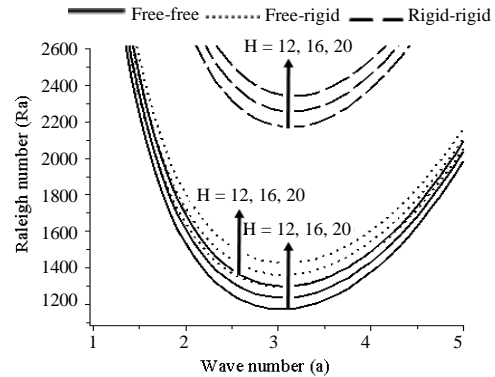


Fig. 1: The effect of magnetic field, H on the Rayleigh number, Ra against wavenumber (a)

layer with vertical magnetic field and internal heat generation. The various boundary conditions that have been used are free-free, rigid-free and rigid-rigid boundaries. The model employed for nanofluid affiliates the effect of Brownian motion and thermophoresis. In this research, all the parameters considered are aligned with those proposed by Chand and Rana (2015). The response of the critical Rayleigh number,  $Ra_c$  to the changes of the difference physical parameters;  $Ta$ ,  $H$ ,  $Q$ ,  $Rn$ ,  $N_B$  and  $N_A$  are studied.

We compared our results with Nield and Kuznetsov (2011) in the omission of Taylor number,  $Ta$ , magnetic field;  $H$  and internal heat source;  $Q$ . The obtained results equivalent with (Nield and Kuznetsov, 2011) where the value of  $Ra_c = 657.5$  for free-free,  $Ra_c = 1140$  for free-rigid and  $Ra_c = 1750$  for rigid-rigid boundaries.

Figure 1 represents the stability curves of Rayleigh number,  $Ra$  against wavenumber,  $a$  for various values of magnetic field,  $H = 12, 16, 20$  in various horizontal boundary conditions. From the Fig. 1, the increasing values of magnetic field,  $H$  is to shift the curves to a higher region and hence delays the onset of convection. The reason behind this is the Lorenz force, a force exerted by a magnetic field on a moving electric charge that produces a resistance on the onset of heat transfer (Yaduv *et al.*, 2015). The system is found to be more stable in rigid-rigid boundaries compared to free-free and rigid-free boundaries. From the graph, it is obviously that the rigid-rigid curves dominated the upper space of the graph.

Figure 2 represents the plot of variation values of Rayleigh number,  $Ra$  against wavenumber,  $a$  for different values of Taylor number,  $Ta = 1000, 2000, 3000$  in various horizontal boundary conditions. The Rayleigh number,  $Ra$  increase with an increase in the Taylor number,  $Ta$  and

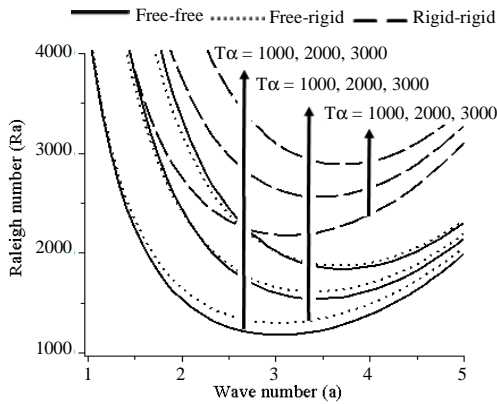


Fig. 2: The effect of Taylor number,  $Ta$  on the Rayleigh number,  $Ra$  against wavenumber ( $a$ )

this indicates the existence of Coriolis force due to rotation is to inhibit the onset of Rayleigh-Benard convection. The mechanism of the rotation is to inhibit the motion to the horizontal plane thus suppress the vertical motion and thermal convection (Yadav *et al.*, 2016).

For variation values of Rayleigh number,  $Ra$  against wavenumber  $a$  for different values of internal heat source,  $Q = 0.5, 1, 1.5$  are shown in Fig. 3 in various horizontal boundary conditions. The Rayleigh number,  $Ra$  decrease with an increase in the internal heat source,  $Q$  and this implies the effect of internal heat source,  $Q$  is to accelerate the onset of Rayleigh-Benard convection. This implies that the increase in internal heat source,  $Q$  is to increase in the distribution of basic temperature gradient in which leads to the increase in the rate of disturbances in the nanofluid layer and destabilize the system.

Figure 4 and 5 show the effects of Lewis number,  $Le = 0.4, 0.6, 0.8$  and modified diffusivity ratio,  $N_A$  on the onset of convection. It is observed that both effects of Lewis number,  $Le$  and modified diffusivity ratio,  $N_A$  promote the onset of convection in a rotating nanofluid layer. It is because the role of both the thermophoresis and Brownian motion is to enhance the onset of heat transfer in nanofluid layer in which can be observed at any volume fraction of nanoparticles. As the reaction of modified particle density,  $N_B$  in nanofluid layer, there is no significant effect observed in the increment of the  $N_B$  values. It is because of the low value of  $N_B$  which presents only in the perturbed energy equation, thus the effect of parameter  $N_B$  on the onset of convection in nanofluid layer will be very small which can be omitted (Nield and Kuznetsov, 2011), (Chand and Rana, 2015) and (Yadav *et al.*, 2016).

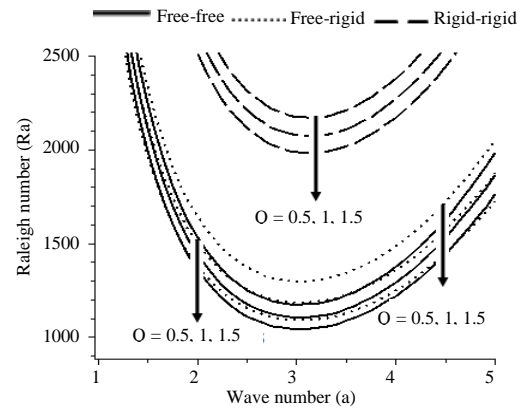


Fig. 3: The effect of internal heat generation,  $Q$  on the Rayleigh number,  $Ra$  against wavenumber ( $a$ )

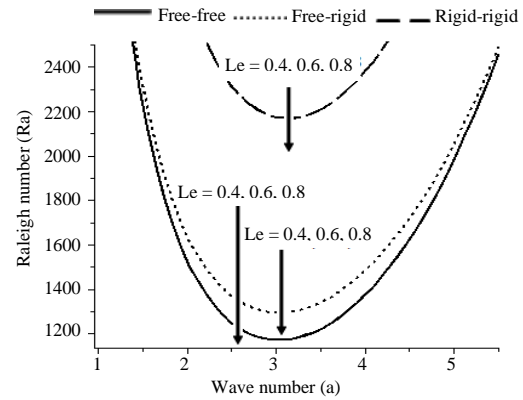


Fig. 4: The effect of Lewis number,  $Le$  on the Rayleigh number,  $Ra$  against wavenumber ( $a$ )

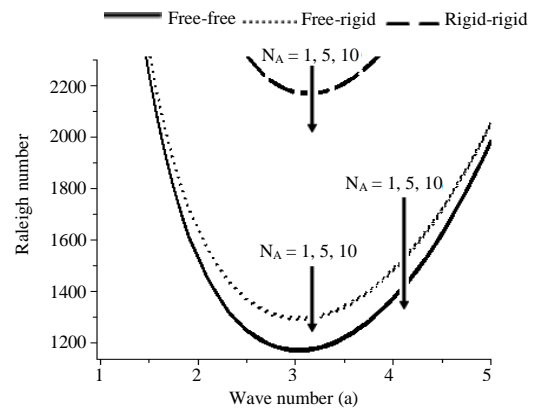


Fig. 5: The effect of modified diffusivity ratio,  $N_A$  on the Rayleigh number,  $Ra$  against wavenumber ( $a$ )

Figure 6 shows the effect of selected values of concentration nanofluid Rayleigh number,  $Rn = 1, 5, 9$ .

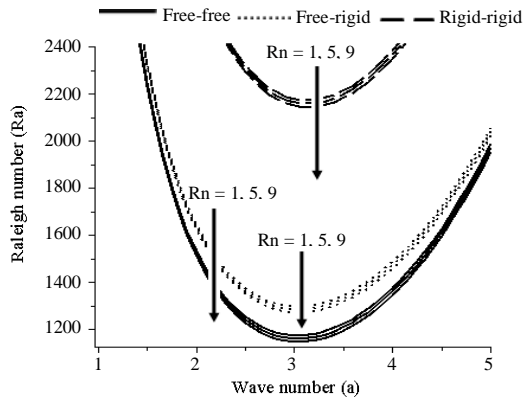


Fig. 6: The effect of concentration nanofluid Rayleigh number,  $R_n$  on the Rayleigh number,  $R_a$  against wavenumber (a)

Based from the observation as the concentration nanofluid Rayleigh number,  $R_n$  increases the Rayleigh number,  $R_a$  increases. This leads to advance the onset of convection and thus destabilizes the system. An increasing values of concentration nanofluid Rayleigh number,  $R_n$  increase the volumetric fraction of nanoparticles and thus increase the Brownian motion and thermophoretic diffusion of the nanoparticles which induce to destabilize the system.

Figure 7 represents the critical Rayleigh number,  $R_{ac}$  as a function of the magnetic field,  $H$  for the selected values of internal heat source,  $Q = 0.1, 0.5$ . From the respective figure, the effect of magnetic field,  $H$  slightly induces the values of critical Rayleigh number,  $R_{ac}$  for internal heat source,  $Q$ . As expected the existence of magnetic field,  $H$  has a positive significant effect to stabilize the nanofluid layer in the presence of internal heat source,  $Q$ .

Figure 8 plot of critical Rayleigh number,  $R_{ac}$  for various values of modified diffusivity ratio,  $N_A = 1, 10$  against the Taylor number,  $Ta$ . Clearly, the effect of modified diffusivity ratio,  $N_A$  is known to hasten the onset of convection, but the process can be slowed down with the existence effect of Taylor number,  $Ta$ .

The interaction between the effect of concentration nanofluid Rayleigh number,  $R_n = 1, 3$  on the critical Rayleigh number,  $R_{ac}$  with Taylor number,  $Ta$  can be seen in Fig. 9. In the respective figure, it is observed that the presence of nanofluid Rayleigh number,  $R_n$  has a destabilizing effect where the increased values of concentration nanofluid Rayleigh number,  $R_n = 1, 3$  decreased the critical Rayleigh number,  $R_{ac}$ . But by adding the effect of Taylor number,  $Ta$  the process of heat transfer can be delayed.

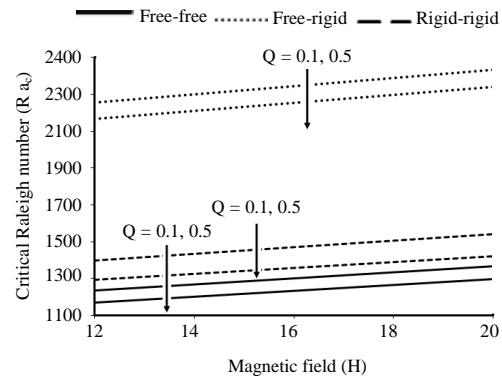


Fig. 7: The variation critical Rayleigh number,  $R_a$  and magnetic field,  $H$  with different values of internal heat source ( $Q$ )

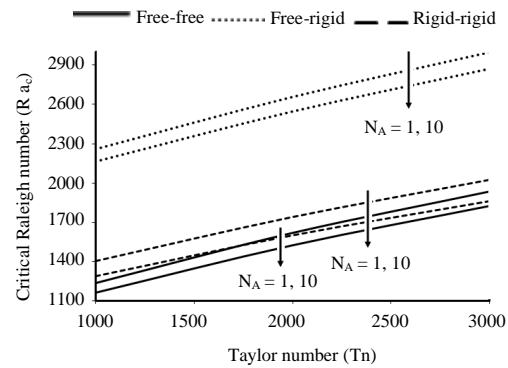


Fig. 8: The variation critical Rayleigh number,  $R_a$  and Taylor number,  $Ta$  with different values of modified diffusivity ratio ( $N_A$ )

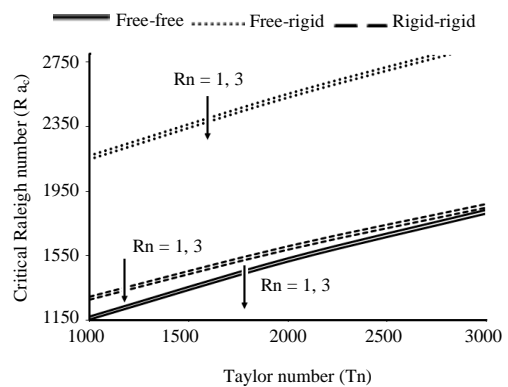


Fig. 9: The variation critical Rayleigh number,  $R_a$  and Taylor number,  $Ta$  with different values of concentration nanofluid Rayleigh number ( $R_n$ )

The presence of internal heat source,  $Q = 3, 9$  to the system for the selected values of Taylor number,  $Ta$  is plotted in Fig. 10. The figure shows that the effect of

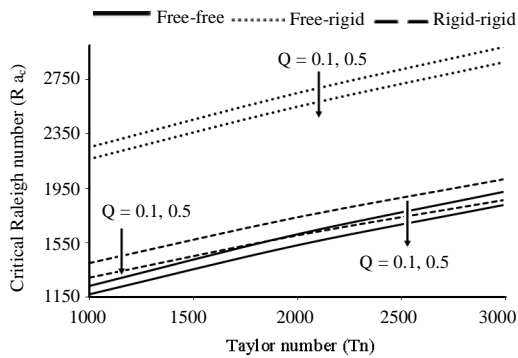


Fig. 10: The variation critical Rayleigh number,  $Ra_c$  and Taylor number,  $Ta$  with different values of internal heat source ( $Q$ )

Taylor number,  $Ta$  slightly increases the values of critical Rayleigh number,  $Ra_c$  for internal heat generation,  $Q$ . Therefore, this observation shows that the increasing values of destabilizing internal heat source,  $Q$  can be delayed through the effect of Taylor number,  $Ta$  on the system.

### CONCLUSION

The stability of a horizontal layer of a rotating nanofluid layer in the presence of vertical magnetic field and internal heat generation is investigated. Linear stability analysis has been made using normal mode technique for horizontal nanofluid layer heated from below in free-free, rigid-free and rigid-rigid boundaries. Then, the effects of various parameters are presented graphically. The main conclusions are; rigid-rigid boundaries are the most stable compared to free-free and rigid-free boundaries. Taylor number,  $Ta$  and magnetic field,  $H$  inhibits the onset of convection of the system. Internal heat source,  $Q$  accelerates the onset of heat transfer. Nanofluid parameters: Lewis number,  $Le$ , concentration nanofluid Rayleigh number,  $Rn$  and modified diffusivity ratio,  $N_A$  both have a destabilizing effect to the system when their values are increased.

### ACKNOWLEDGEMENT

The researchers would like to thank the Ministry of Higher Education (MOHE) for FRGS Vote No. 5524311 and UPM Vote No. 9428400. Researchers are also thankful to the worthy referees for their valuable comments and suggestion for the improvement of the quality of the study.

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