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Mathematical Modeling and Fuzzy PI Control of BLDC Motors for EV Application

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Abstract: Brushless DC Motor (BLDCM) is being predominantly utilized in Electric Vehicles (EVs). BLDCM is easily affected by variations of parameter and disturbances because of its nonlinear nature. The tuning of parameters of typical PI controller is complicated to obtain enhanced response of BLDCM for EV applications. Hence, the typical PI controller is inadequate to regulate BLDCM. To vanquish this difficulty, a Fuzzy PI (FPI) controller is intended to regulate the speed of BLDCM. In general, Fuzzy Logic Controller (FLC) gives an improved speed response at the time of startup while the typical PI controller provides good response at steady state condition. In this study, the typical PI is combined with FLC to obtain a FPI controller with merits of both. Mathematical modeling of BLDCM with its controller is also explained. And also gives the study of various controllers like typical PI, FLC and FPI for BLDCM. To validate the control characteristics of the typical PI, FLC and FPI, simulations have been done in MATLAB/Simulink environment. Simulation results prove the superiority of the intended FPI for BLDC motors.

Key words: BLDC motor, Fuzzy Logic Controller (FLC), Electric Vehicle (EV), Hysteresis Current Controller (HCC), mathematical modeling, environment

INTRODUCTION

Due to increasing of the energy crisis and the environmental pollution, many countries have been trying to curtail their usage of conventional energy sources like oil, coal, etc. Hence, Electric Vehicles (EVs) are becoming more popular rather than conventional vehicles. The BLDC motor is more prominent in different industries because of their higher power density, high efficiency, less maintenance. Due to these benefits, BLDC motors are well suitable for EV applications (Nikam *et al*., 2012; Wu *et al*., 2005).

The functioning of BLDCM is depends on rotor position, the hall sensors are used to detect the position of rotor. For providing proper commutation of current, BLDCM needs an inverter and for identifying the position of rotor a position sensor is also required. BLDCM is usually governed by a three phase VSI. Depends on the position of rotor, the switches of inverter are commutated sequentially for every 60°. BLDCM is also known as electronic commutator motor because commutation is occurred electronically for commutating the stator currents. This eradicates the problem associated with brushes and arrangement of commutator like sparking and wearing out of brushes.

Various simulation models are suggested for analysis of BLDCM (Rodriguez and Emadi, 2007; Bist and Singh, 2014). For analysis of performance and control system design, mathematical model of BLDCM is essential. While evolving mathematical model, structural characteristics and functioning modes of BLDCM has to be realized. In practice, for representation of BLDCM needs an efficient modeling, proper selection of control design, tuning of parameters, etc. The BLDC motor is highly nonlinear in nature. It is usually very hard to get a meticulous mathematical model for BLDCM while applying traditional strategies. Moreover, properties of BLDCM are generally not familiar and time varied.

For attaining most favorable response, a mastered knowledge of system should be needed for fine tuning the parameters of BLDCM controller. In present days, for regulating the speed of BLDCM, there are various speed governing strategies (Shanmugasundram *et al*., 2014; Rodriguez and Emadi, 2007; Shen *et al*., 2005). The traditional PI controller fails to allow most useful overall performance throughout such changes in working conditions such as load variations and parameter adjustments. It has ensued in an increased attention towards intelligent controllers. Therefore, an intelligent controller like FLC is entailed for enhancing the response.

In this study, to obtain a superior speed response of BLDCM, a FPI speed controller is proposed. It combines merits of FLC and PI speed controller for obtaining

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betterspeed response. The performance of PI, FLC and FPI speed controller for a BLDC motor at different loads has been observed. Comparison is made among the three controllers to evaluate a better controller.

MATERIALS AND MEHTODS

Mathematical modeling of BLDC motor: Figure 1 depicts the BLDC motor drive model. The basic voltage equations of armature winding for BLDCM are derived as follows:

$$v_a = Ri_a + L\frac{di_a}{dt} + e_a \quad (1)$$

$$v_b = Ri_b + L\frac{di_b}{dt} + e_b \quad (2)$$

$$v_c = Ri_c + L\frac{di_c}{dt} + e_c \quad (3)$$

Where:

- L = Armature self-inductance
- R = The armature resistance of stator phase winding
- v_a, v_b, v_c = Terminal phase voltages
- i_a, i_b, i_c = Input currents of motor
- e_a, e_b, e_c = Trapezoidal back EMF of corresponding phases

Thus, the voltage equation matrix for the BLDC motor can be represented as:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (4)$$

As it has been assumed that saturation and iron losses of the motor are negligible, the resistances of all stator windings are equal and self-inductances of all phases are equal. Mutual inductances between phases are identical to one another and they are represented as:

$$L_{aa} = L_{bb} = L_{cc} = L$$

And:

$$L_{ab} = L_{ba} = L_{cb} = L_{ac} = L_{ca} = M \quad (5)$$

Hence, Eq. 4 can be rewritten as :

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (6)$$

In 3-phase BLDCM, back EMF waveform is depends on rotor position, thus equation of each back EMF phase can be as represented as follows:

$$e_a = K_b f_{as}(\theta)\omega \quad (7)$$

$$e_b = K_b f_{as}\left(\theta + \frac{2\pi}{3}\right)\omega \quad (8)$$

$$e_c = K_b f_{as}\left(\theta + \frac{2\pi}{3}\right)\omega \quad (9)$$

Where:

- K_b = Back EMF constant of one phase [V/ rad/sec]
- θ = Electrical rotor angle [in electrical degrees]
- ω = Speed of rotor [rad/sec]
- f_{as}, f_{bs}, f_{cs} = Trapezoidal functions

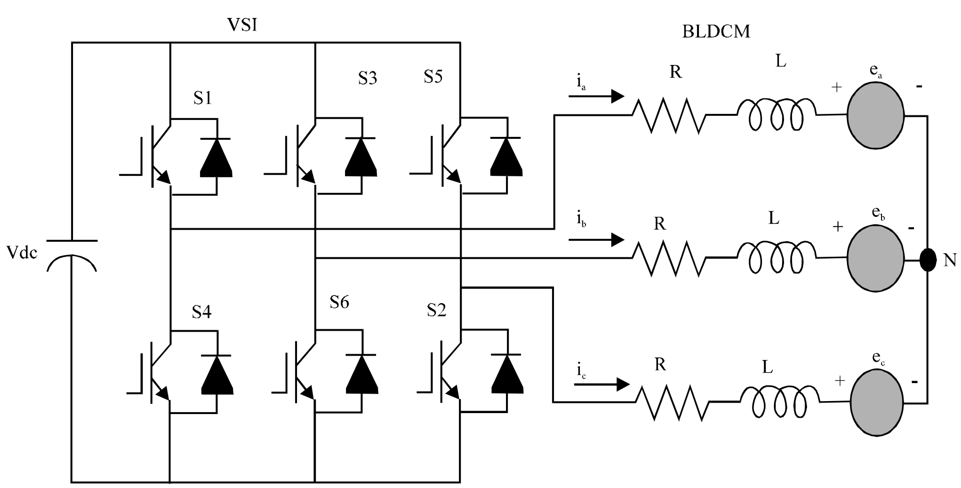

Fig. 1: Model of BLDC motor drive

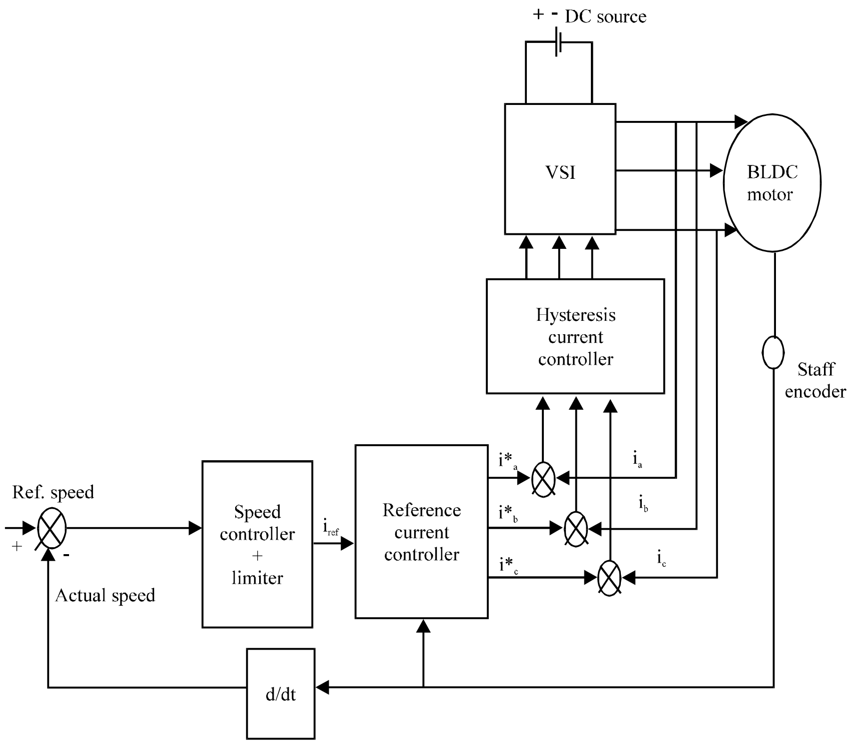

Fig. 2: Closed loop control of a BLDCM

The trapezoidal function can $f_{as}(\theta)$ be represented as:

$$f_{as}(\theta)=\begin{cases}\frac{6}{\pi}\theta & \text{where } 0<\theta<\frac{\pi}{6}\\ I & \text{where } \frac{\pi}{6}<\theta\le\frac{5\pi}{6}\\ 1-\frac{\left(\theta-\frac{5\pi}{6}\right)}{\pi/6} & \text{where } \frac{5\pi}{6}<\theta\le\frac{7\pi}{6}\\ -1 & \text{where } \frac{7\pi}{6}<\theta\le\frac{11\pi}{6}\\ -1+\frac{\left(\theta-\left(\frac{11\pi}{6}\right)\right)}{(\pi/6)} & \text{where } \frac{11\pi}{6}<\theta\le 2\pi\end{cases} \quad (10)$$

The functions $f_{bs}(\theta)$ and $f_{cs}(\theta)$ can be expressed as:

$$f_{bs}(\theta)=f_{as}\left(\theta+\frac{2\pi}{3}\right) \quad (11)$$

$$f_{cs}(\theta)=f_{as}\left(\theta-\frac{2\pi}{3}\right) \quad (12)$$

The electrical rotor angle is related with mechanical rotor angle and number of pole pairs P as follows:

$$\theta=\frac{P}{2}\theta_m \quad (13)$$

where, θ_m is rotor angle in mechanical degrees. The electromagnetic torque T_e can be stated as the following Eq. 14:

$$T_e=\frac{e_a i_a+e_b i_b+e_c i_c}{\omega} \quad (14)$$

The equation of a motor with inertia J, friction coefficient B and load torque is expressed as:

$$T_e-T_l=J\frac{d\omega}{dt}+B\omega \quad (15)$$

Controller description

Closed loop controller: Figure 2 depicts an elementary building blocks of BLDCM. Here, BLDCM is provided by a three phase VSI. In fact, output of speed controller is given for that Reference Current Generator (RCG) which generates reference currents and then these currents are compared with actual currents of BLDCM, there is some difference between them as an error. This error is given to hysteresis current controller. Here, gating signals are generated for firing the inverter switches. These gating signals should maintain constant within electrical degrees of one electrical cycle.

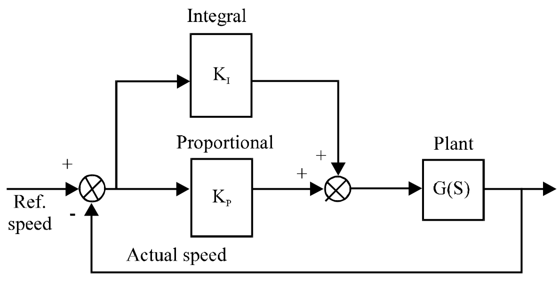

Fig. 3: PI speed controller

PI speed controller: The principal activity of speed controller has to be to give a reference torque which can be modified into reference current. It can be given to reference current generator. The controller's output should be constrained to some acceptable value in fulfillment of BLDCM rating to yield reference torque.

In industrial applications, PI controller could be the most prominent controller which is utilized in extensive range. The output from PI is expressed through subsequent equation:

$$v_c(t) = K_P e(t) + K_i \int_0^t e(t)dt \quad (16)$$

Where:

$v_c(t)PI$ = Controller's output
K_p = Proportional gain
K_i = Integral gain
$e(t)$ = Error signal

The schematic diagram of PI speed controller is projected in Fig. 3. In case of speed control of BLDCM input of PI speed controller would be speed error (e) while, PI speed controller's output can be used to generate reference currents.

Fuzzy logic controller: FLC is identified as powerful controller because of its capability to cope with nonlinearities. FLC is not going to call for an accurate mathematical model of system. FLC would be designed in accordance with experience of people. The first step of FLC design is fuzzification in which convert input data into linguistic elements. These elements can be expressed by using Membership Functions (MFs). The prevalent forms of MFs are triangular and trapezoidal shapes. These MFs can be chosen through user expertise (Fig. 4).

Afterwards, by making use of rule base an inference will be formulated. Rule base continues to be developed to regulate output elements. It is prone to be a simple if-then rule made up of an assumption and conclusion. Immediately after determining the effects of each and every rule, they can be joined to acquire the ultimate result. This approach is referred to as inference. For an intended FLC, a rule base is depicted in Table 1. MFs could be stated as Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM) and Positive Big (PB).

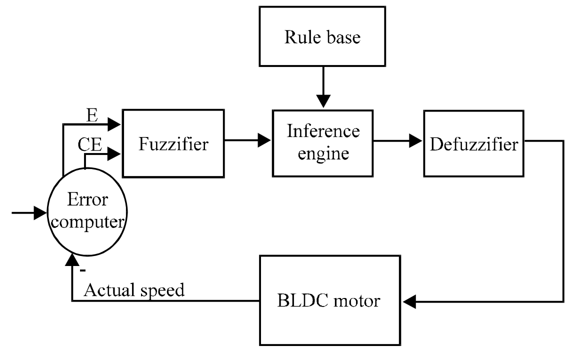

Fig. 4: Fuzzy logic controller

Table 1: Rules for fuzzy logic controller

	Change in error						
Error	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	Z
NM	NB	NB	NB	NM	NS	Z	PS
NS	NB	NB	NM	NS	Z	PS	PM
Z	NB	NB	NS	Z	PS	PM	PB
PS	NM	NS	Z	PS	PM	PB	PB
PM	NS	Z	PS	PM	PB	PB	PB
PB	Z	PS	PM	PB	PB	PB	PB

In the event of regulating BLDCM, speed error (E) and change in speed Error (CE) c actually selected as inputs to FLC. Output of FLC is obtained in terms of E and CE. For possessing essentially the most prosperous control execution, five triangular and two trapezoidal membership functions had been picked. The reverse term of fuzzification would be defuzzification. Rules for FLC develop desired output in a linguistic element. This can be altered to crisp output. By employing of 'centroid' defuzzification scheme, crisp output has long been acquired. Figure 5 elucidates MFs of input and output element for impressive regulation of a BLDCM.

FPI controller: Figure 6 depicts the schematic diagram of the FPI controller. The PI has supercilious performance as contrasted to FLC under steady state conditions. The FLC will produce better response during starting but features a large steady state error. The main objective of this fuzzy PI speed controller would be to employ the proper features of PI and FLC. It will produce a greater response compared to PI and FLC.

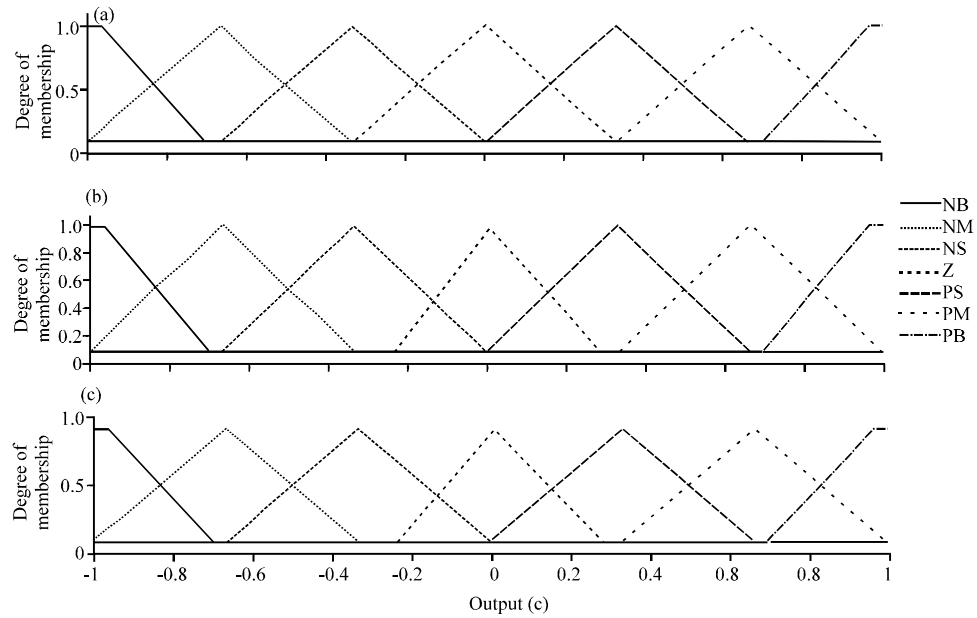

Fig. 5: MFs for: a) Speed error; b) Change in speed error and c) Output

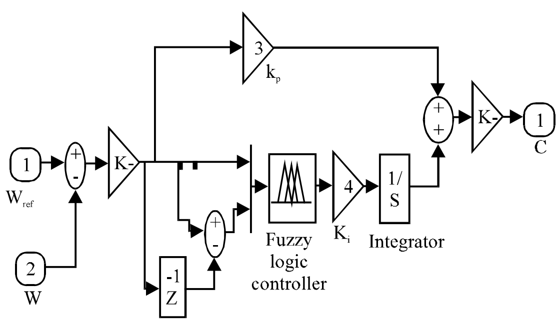

Fig. 6: Fuzzy PI speed controller

Table 2: BLDC motor parameters

Variables	Values
Voltage	120 V
Resistance per phase	4.98 Ω
Inductance per phase	5.05 mH
No. of poles	4
Rated speed	3000 rpm

RESULTS AND DISCUSSION

To validate the applicability of suggested FPI controller, simulation is accomplished on a BLDCM in the MATLAB/Simulink environment. The intention of this simulation would be to analyze the functionality of BLDCM employing the PI controller and evaluating its effectiveness with FLC and the FPI speed controller. Figure 7 represents the simulation model of speed regulation of BLDCM in MATLAB/Simulink. The motor paramaters are shown in Table 2.

Case A: Constant load of 10 Nm at constant speed of 3000 rpm: Figure 8 depicts the effectiveness of BLDCM with PI speed controller and Hysteresis Current Controller (HCC) with a hysteresis band of 0.05 A. For tuning the Kp and Ki values, Ziegler-Nichol's Method is used. Here Kp, Ki is tuned to a value of 0.9 and 4.8, respectively. The BLDCM is fixed to get a reference speed of 3000 rpm. An unexpected load of 10 Nm is employed in the instant of 0.2 sec. It could be noticed that the BLDCM takes 0.1141 secto achieve the reference speed as shown in Fig. 8a. Here, PI speed controller has a considerable quantity of undershoot and attains the reference speed with higher settling time. Figure 8b shows the armature current of phase A and back EMF waveform of phase A is also shown in Fig. 8c. From Fig. 8d it could be noticed that torque developed by BLDCM consists of higher ripples.

Figure 9 depicts the performance of BLDCM with FLC and HCC with a hysteresis b and of 0.05 A. From Fig. 9a, speed response could be noticed that BLDCM takes 0.00793 sec to achieve the reference speed. Even though, the FLC has zero peak overshoot but this could be maintain specific speed error as a result this provides reduced speed which is less than reference speed.

Figure 9b shows the armature current of phase A and back EMF waveform of phase A is shown in Fig. 9c. It could be also evident that torque developed by BLDCM contains a smaller amount of ripple as shown in Fig. 9d. It

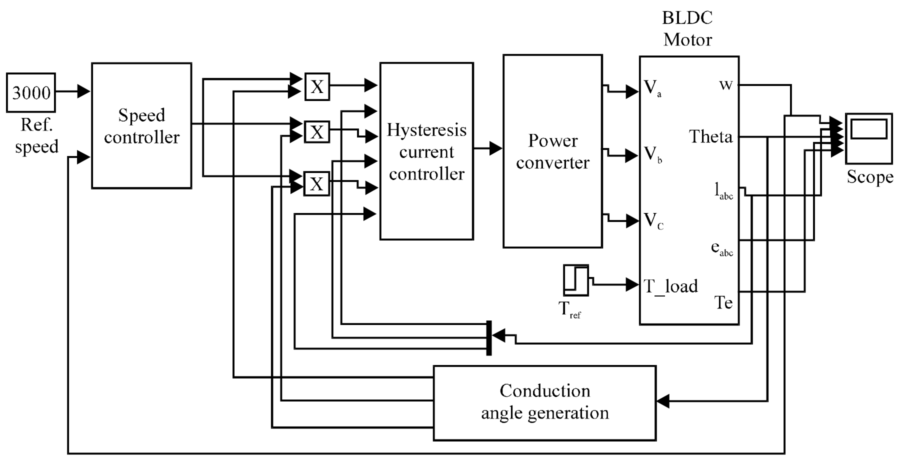

Fig. 7: Simulation model of BLDC motor drive

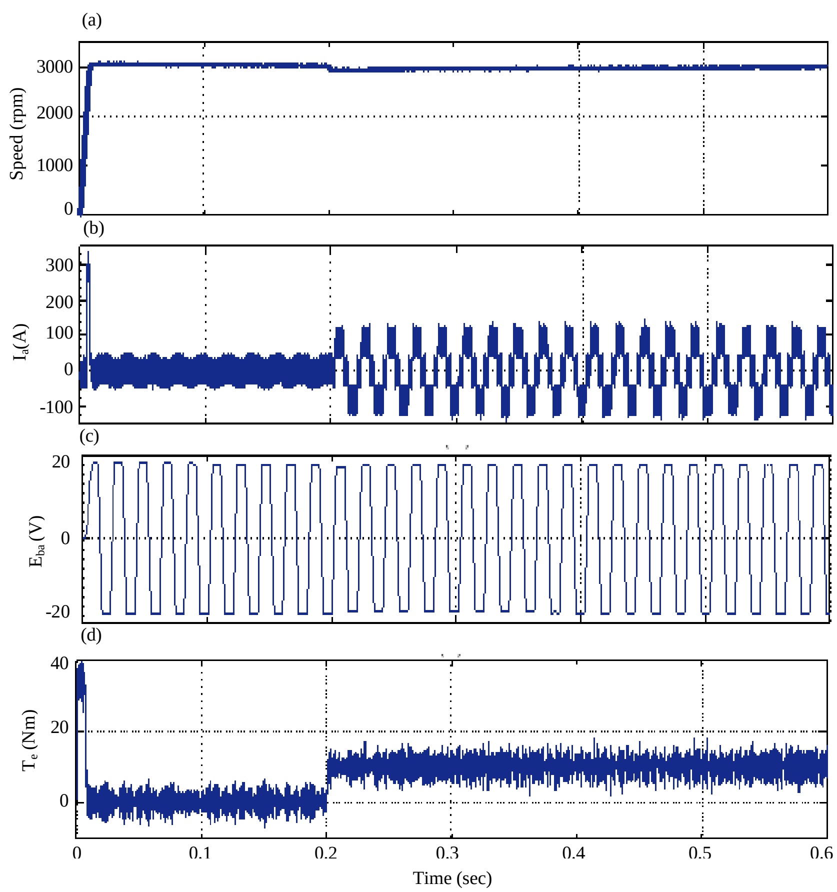

Fig. 8: Performance of BLDC motor with PI speed controller at a load torque of 10 Nm applied at 0.2 sec: a) Speed response; b) Phase A current; c) Back EMF and d) Torque

can be a relatively smaller amount than torque ripples by employing the PI controller. Figure 10 describes the performance of the BLDCM with FPI controller and HCC at a hysteresis band of 0.05A. The FPI controller exhibits the lower undershoot and less settling time. It is observed from the speed response as shown in Fig. 10a-c illustrates armature current of phase A and back EMF waveform of phase A. From Fig. 10d, it could be seen that the torque

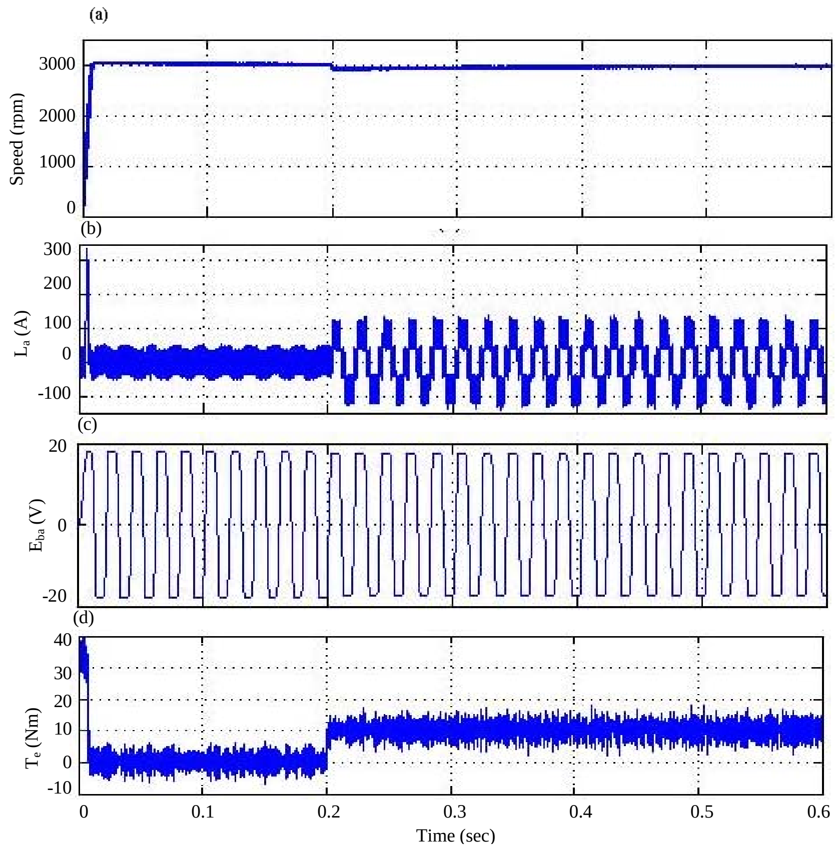

Fig. 9: Performance of BLDC motor with Fuzzy controller at of 10 Nm applied at 0.2 sec: a) Speed response; b) Phase A current; c) Back EMF and d) Torque

made by BLDCM has less ripple. It is comparatively a reduced amount of torque ripples than by using PI and FLC.

Case B: Speed changes from 1500-3000 rpm at a constant load of 10 Nm: Figure 11 depicts the performance of BLDCM through constant load and variable speed by using PI controller. The BLDCM is fixed to get a reference speed of 1500 rpm nearly 0.2 sec, that is then altered to 3000 rpm. An abrupt load of 10 Nm is employed in the instant of 0.05 sec. It can be seen that BLDCM takes approximately 0.00577 sec to attain the first reference speed of 1500 rpm as shown in Fig. 11a. It could be also noticed that the electromagnetic torque rises to a value of 40 Nm at the time of 0.2 sec when there can be an alter in reference speed. The torque generated by motor contains high ripples as depicted in Fig. 11b.

Figure 12 depicts the characteristics of the BLDC motor during constant load and variable speed changes from 1500-3000 rpm with FLC. As illustrated in Fig. 12a, it can be noticed that BLDCM takes 0.0056 sec to get the first reference speed of 1500 rpm. It can be also noticed that torque generated by BLDCM carries ripples. It could be relatively a smaller amount than the torque ripples by employing the PI controller as depicted in Fig. 12b.

Figure 13 illustrates the performance of BLDCM through constant load and variable speed alters from 1500-3000 rpm with FPI controller. By employing FPI controller, BLDCM takes 0.00421 sec to attain the first reference speed of 1500 rpm as depicted in Fig. 13a. The torque generated by BLDCM with FPI contains relatively smaller amount of ripples than PI and FLC as shown in Fig. 13b.

Figure14 depicts the comparison of BLDCM speed responses with PI, FLC and FPI controller. From Fig. 14, it can be observed that during start-up, FLC and FPI have less settling time than PI speed controller. Once, an abrupt load of 10 Nm is employed to the system at the time of 0.3 sec, the PI speed controller has a substantial quantity of unders hoot. Also, it can attain the desired speed with higher settling time. Here, the FLC has smaller settling time but it maintains a speed error. Due to this reason, FLC can be attaining a reduced actual speed than the desired speed. Moreover, FPI and typical PI controller have zero steady state speed error. Thus, the FPI

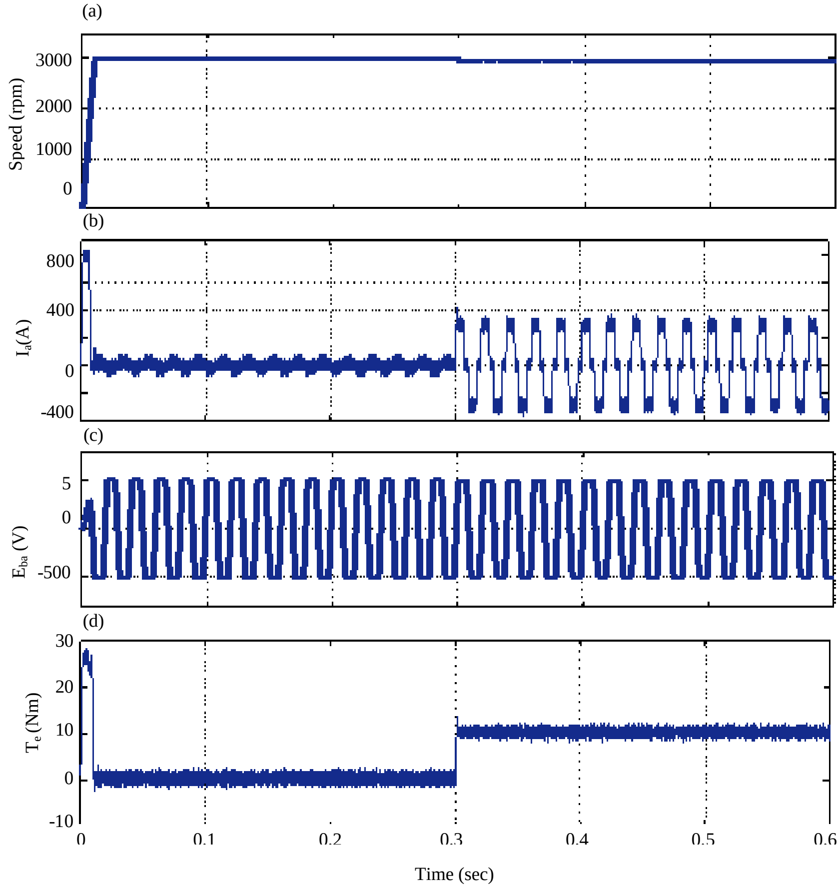

Fig. 10: Performance of BLDC motor with fuzzy PI speed controller at of 10 Nm applied at 0.2 sec: a) Speed response; b) Phase A current; c) Back EMF and d) Torque

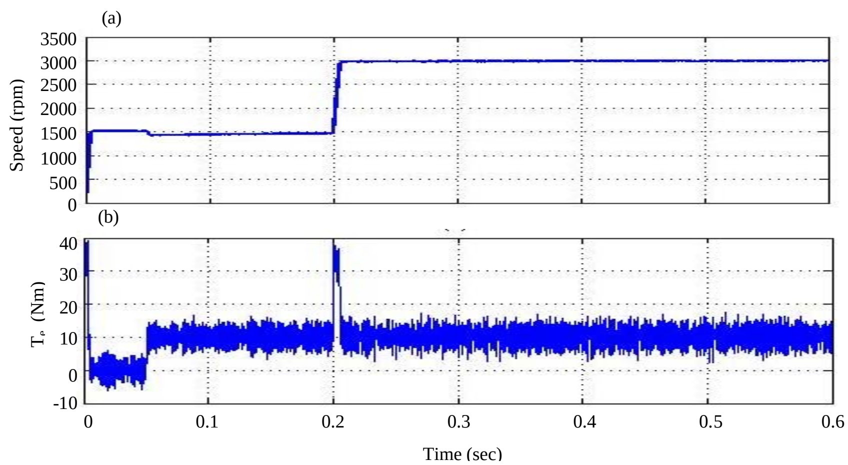

Fig. 11: Performance of the BLDC motor during constant load of 10 Nm and variable speed changes from 1500-3000 rpm using PI speed controller: a) Speed response and b) Torque

controller has maintained steady state accuracy. It can be also improves the dynamic response of BLDCM.

Table 3 illustrates a comparison among the three speed controllers. It can be clearly seen from Table 3 that PI speed controller has high peak overshoot on load

Table 3: Comparison table

	At a load torque of 10 Nm			At a load torque of 20 Nm		
Controller	PI	Fuzzy	Fuzzy-PI	PI	Fuzzy	Fuzzy-PI
Peak overshoot (%)						
At 2000 rpm	1.7	0	0.86	1.7	0	0.86
At 3000 rpm	2.1	0	1	2.1	0	1
Rise time (sec)						
At 2000 rpm	0.004657	0.0041741	0.003283	0.0046498	0.004639	0.003293
At 3000 rpm	0.006899	0.006897	0.0044823	0.0068993	0.0068986	0.004584
Settling time (sec)						
At 2000 rpm	0.0412	0.005534	0.0048669	0.05	0.005705	0.049808
At 3000 rpm	0.1141	0.0079294	0.005731	0.1472	0.008133	0.0058713
Steady state error (rpm)						
At 2000 rpm	0	20	0	0	60	0
At 3000 rpm	0	20	0	0	60	0

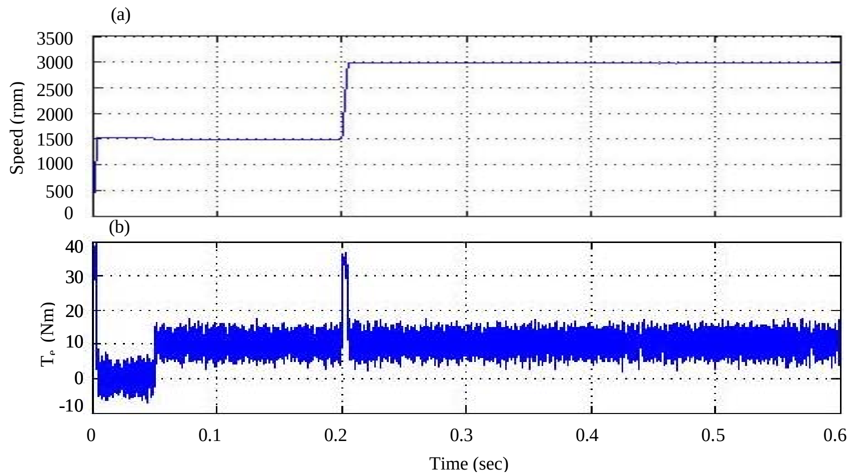

Fig. 12: Performance of the BLDC motor during constant load of 10 Nm and variable speed changes from 1500-3000 rpm using fuzzy logic controller: a) Speed response and b) Torque

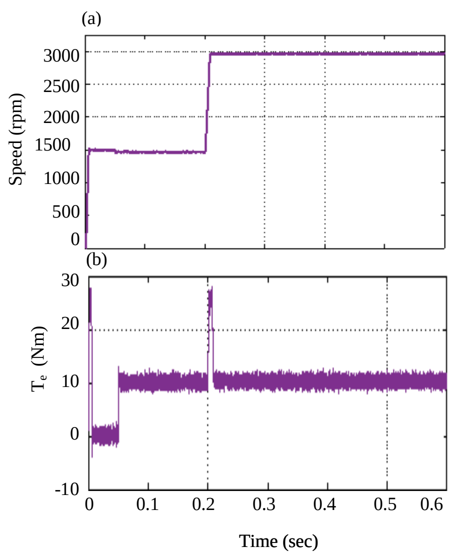

Fig. 13: Performance of the BLDC motor during constant load of 10 Nm and variable speed changes from 1500-3000 rpm using fuzzy PI speed controller: a) Speed response and b) Torque

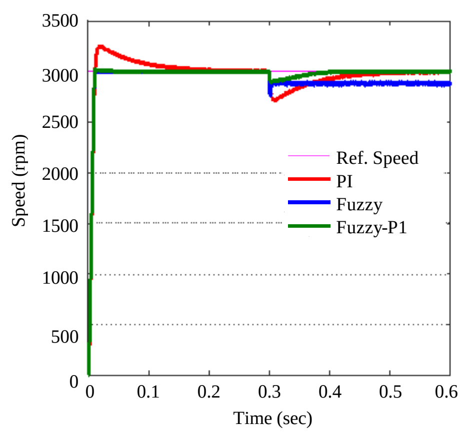

Fig. 14: Comparison of speed response of PI, fuzzy logic controller and fuzzy PI speed controllers for 3000 rpm at of 10 Nm applied at 0.3 sec

torque conditions. Also, the PI speed controller has comparatively higher settling time. FLC offers an improved response, without having peak overshoots. But

FLC resolves with a significant steady state speed error during load changes. The FPI provides relatively superior response with less settling time and zero peak overshoot.

CONCLUSION

In this study, a FPI controller is intended for achieving better dynamic response of BLDCM for EV applications. Mathematical model and performance of the BLDCM with conventional PI speed controller, FLC and FPI are discussed and simulated in the MATLAB/SIMULINK. The behavior of BLDCM with various speed controllers such as PI, FLC and FPI Speed Controller is compared. From simulation results, it can be noticed that PI speed controller featured as having initial Peak overshoot, slow response but maintains the steady state accuracy. Whereas, FLC has zero Peak overshoot and quicker response. However, it resolves with a significant steady state error. The proposed FPI controller has better dynamic response when compared with a typical PI and FLC. Finally, the proposed FPI controller gives BLDC motor with a sustained speed regulation.

REFERENCES

Bist, V. and B. Singh, 2014. An adjustable-speed PFC bridgeless buck-boost converter-fed BLDC motor drive. IEEE. Trans. Ind. Electron., 61: 2665-2677.

Nikam, S.P., V. Rallabandi and B.G. Fernandes, 2012. A high-torque-density permanent-magnet free motor for in-wheel electric vehicle application. IEEE. Trans. Ind. Appl., 48: 2287-2295.

Rodriguez, F. and A. Emadi, 2007. A novel digital control technique for brushless DC motor drives. IEEE. Trans. Ind. Electron., 54: 2365-2373.

Rodriguez, F. and A. Emadi, 2007. A novel digital control technique for brushless DC motor drives. IEEE. Trans. Ind. Electron., 54: 2365-2373.

Shanmugasundram, R., K.M. Zakariah and N. Yadaiah, 2014. Implementation and performance analysis of digital controllers for brushless DC motor drives. IEEE. ASME. Trans. Mechatron., 19: 213-224.

Shen, J.X., Z.Q. Zhu, D. Howe and J.M. Buckley, 2005. Fuzzy logic speed control and current harmonic reduction in permanent magnet brushless AC drives. IEE. Proc. Electr. Power Appl., 152: 437-446.

Wu, H.X., S.K. Cheng and S.M. Cui, 2005. A controller of brushless DC motor for electric vehicle. IEEE. Trans. Magn., 41: 509-513.