

Optimization of Time and Cost in Critical Chain Project Scheduling Problem Using Genetic Algorithm

¹Mohammad Javad Taheri Amiri, ²Farshidreza Haghighi, ³Ehsan Eshtehardian,
⁴Milad Hematian and ⁵Hojjat Kordi

¹Department of Construction Engineering and Management,

²Department of Civil Engineering, Babol University of Technology, Babol, Iran

³Department of Project and Construction Management, Tarbiat Modares University, Tehran, Iran

⁴Department of Industrial Engineering,

Mazandaran University of Science and Technology, Babol, Iran

⁵Department of Construction Engineering and Management, Islamic Azad University of Sari, Sari, Iran

Abstract: Time and costs are considered as the major criteria in a project and project managers always seek to finish projects in the shortest possible time spending the lowest costs to obtain success in projects. One of the major challenges in this regard is to select a proper approach in order to achieve the above goals. Since, costs and time are regarded as two major aspects in doing construction projects, the purpose of this research is to optimize these two factors. In the process of optimizing the period of doing the project, a tolerance is considered for the project costs, so that the optimal conditions in the acceptable range for the costs and time of doing the project are specified. The genetic algorithm has been utilized in this research for optimization of the project time and costs. The critical chain method has also been used for project scheduling.

Key words: Project management, time and cost management, critical chain, genetic algorithm, optimization

INTRODUCTION

In view of the greatness and complexity of projects today, it seems hardly plausible to achieve the project goals without scheduling. One of the important parts of project management is the skill to evaluate, schedule and supervise projects. The purpose of project management is to schedule projects operationally and technically in order to obtain the best product at the lowest costs in the shortest time. The scheduling procedure, therefore, is one of the most important principles in the success of construction projects and avoidance of delays created in projects (Nazarpour *et al.*, 2014).

In general, executive management and scheduling of activities and use of the demanded resources in a project requires a variety of analyses, one of which is modeling for correct selection of the costs and time of doing the project and demanded amount and time for using the resources. This will greatly help manage projects optimally and make decisions in critical conditions. In an atmosphere where the competition between companies becomes closer every day and slight differences in offering prices in tenders leads to

success or failure in the tender, it is of great importance to present a schedule in accordance with the reality that can include all economic facts in a project model.

Not only does this matter when prices are offered for a project before the implementation begins but presence of a flexible schedule can help a company confront with various problems, frequently out of its control, also after the task begins. A flexible schedule is capable of considering the necessary changes in costs, time and resources using the relationship between costs and time in a project and of providing users with various proper solutions so that they can have a proper estimation of the executive time and costs and amount of resources needed in the project prior to its implementation. In a problem involving optimization or time-cost tradeoff of the executive methods suitable for doing the series of activities in a project, the objective functions should be selected from the beginning to the end such that the time and costs of project implementation are minimized. Based on the above points, the present research has explained the bi-objective model for time-cost tradeoff using the genetic algorithm.

Literature review: According to Azaron *et al.* (2005) presented a multi-objective model for the time-cost tradeoff in the PERT network. Activity times follow the erlang distribution in this problem. A genetic algorithm has been presented for solving the problem. The results obtained from the genetic algorithm were compared to those from the approximation method used in previous studies which demonstrated the efficiency of this method in solving time-cost tradeoff problems in PERT networks (Azaron *et al.*, 2005).

According to Anagnostopoulos and Kotsikas (2010) presented a simulated annealing algorithm to minimize total costs in the time-cost tradeoff problem. The two factors of quality of solution and computational time have been considered for evaluation of the proposed algorithm. The computational results demonstrate the technique presented for solving problems in real conditions (Anagnostopoulos and Kotsikas, 2010).

According to Chen and Tsai (2011) investigated the problem of time-cost tradeoff of a project network in the fuzzy environment. Bi-level mathematical programming has been used for specification of the upper and lower limits of total fuzzy cost. The membership function of total fuzzy cost and optimal time for each activity are obtained through specification of different α values. The time-cost tradeoff problem was solved with a number of fuzzy parameters for validation of the proposed approach (Chen and Tsai, 2011).

According to Klansek and Psunder (2012) presented a mixed-integer nonlinear programming optimization model for the nonlinear time-cost tradeoff problem. Prerequisite relationships have been postulated in this study between different project activities, project time constraints and budget constraints. The objective function of the problem is to minimize total project costs and to obtain the optimal time-cost curves of the project. The computational results demonstrate that the proposed mathematical model is capable of solving the problem accurately (Klansek and Psunder, 2012).

According to Ke and Ma (2014) modeled the time-cost tradeoff problem in the fuzzy environment. For this purpose, three time-cost tradeoff models were presented based on the fuzzy theory. Fuzzy simulation techniques and the genetic algorithm have also been used in the study for development of the search method. Several examples were finally solved for evaluating the efficiency of the proposed method (Ke and Ma, 2014).

Genetic algorithm: The genetic algorithm is one of the heuristic methods in the optimization problem

which is rooted in the survival of the fittest law and the algorithm is actually a virtual simulation of Darwin's theory of gradual evolution, stating that weaker creatures disappear while stronger creatures survive. Genetic algorithms, which operate based on the idea of evolution in the nature (Sivanandam and Deepa, 2008) search for the final solution over a population of potential solutions.

In each generation, the best in that generation are selected and they generate a new set of offspring after reproduction. In this process, fitter individuals are more likely to survive into later generations. At the beginning of the algorithm, a number of individuals randomly form the initial population and the objective function is evaluated for each of them. If the condition for obtaining the solution is not met, the next generation is generated through selection of parents based on their fitness values and the offspring is mutated with a constant probability. The fitness values for the new offspring are then calculated and the new population is formed through replacement of parents by the offspring and the process is repeated until the termination condition is met.

The major advantages of this method over the common methods include: parallel search instead of sequential search, absence of need for any auxiliary information such as the problem solving method, nondeterministic of the algorithm, easy implementation and obtaining multiple optimal alternatives (Goldberg, 1989). The algorithm is employed in a variety of problems such as system optimization, identification and control, image processing and combinatorial problems, specification of the topology of and training artificial neural networks and decision-and rule-based systems. In this research, the algorithm has been used for optimization of time and costs considering budget constraints.

Problem definition: In this study, the two objectives of time and costs have been considered for evaluation of project scheduling with the critical chain method. The time objective of the project has been to minimize project termination time where the critical chain method has been used for project time calculations. Furthermore, the cost objective of the project is to minimize final project costs, including direct and indirect costs. Direct costs include the sum of costs concerning renewable and nonrenewable resources for all project activities and indirect costs include constant costs of the company during project implementation. For presentation of the objective function of the proposed problem, the concerning notation is first presented.

MATERIALS AND METHODS

Proposed model for critical chain scheduling: As stated, the project scheduling problem has been considered in this study as bi-objective; each of the objectives has been examined below.

Analysis of the objective function of time in critical chain project scheduling: The process of critical chain project scheduling and controlling is very complicated and the implementer and owner of the project always focus on total project time; therefore, the objective of project scheduling in the critical chain is to minimize time which is calculated from the following equation:

$$\min T = E_{eo} + PB \tag{1}$$

where project buffer has been calculated based on the cut and paste method.

Analysis of the objective function of cost in critical chain project scheduling: Total multi-project management costs include the costs of renewable and non-renewable resources. Based on the defined notations, the objective function of project costs is obtained from the following equation in critical chain multi-project scheduling:

$$\min C = \sum_{j=1}^J \left(\sum_{k=1}^K r_{jk} C_k t_j + \sum_{p=1}^P \pi_{jp} C_p \right) \tag{2}$$

Utility function of the proposed model: As stated, T and C represent time and cost for multi-objective optimization in project scheduling with the critical chain method which can be decomposed into weighted polynomials based on the multi-objective utility function decomposition theorem as shown in the following equation:

$$u(T,C) = \alpha_T \times u(T) + \alpha_C \times u \tag{3}$$

$$\alpha_T, \alpha_C \geq 0 \tag{4}$$

$$\alpha_T, \alpha_C = 0 \tag{5}$$

The quadratic function diagram of the utility function can be used as solution space and all utility functions are convergent. The utility value of total project time (D) was taken to be 1; therefore:

$$U(T) = \begin{cases} \varphi_T - \beta_T (T - D), T \in [0, 2D]^2 \\ 0, T \notin [0, 2D] \end{cases} \tag{6}$$

Total project management costs include the costs of renewable and nonrenewable resources and the utility value is 1; therefore:

$$U(C) = \begin{cases} \varphi_C - \beta_C (C - (1-\eta)U)^2, C \in [0, 2(1-\eta)U] \\ 0, C \notin [0, 2(1-\eta)U] \end{cases} \tag{7}$$

Multi-objective optimization model: A multi-objective optimization model has been presented for solving the proposed problem. In view of the two features of time and costs, mainly considered by organizations and contractors, a utility function has been presented for the two objectives. The proposed optimization model is, therefore as follows:

$$\max u(T,C) \tag{8}$$

$$\text{s.t. } E_j - E_{(j-1)} \geq t_j \tag{9}$$

$$PB = \frac{T_{ts}}{2} \tag{10}$$

Equation 8 demonstrates the objective function of the problem which is to maximize multi-objective optimization utility in critical chain multi-project scheduling. Equation 9 demonstrates the prerequisite and post requisite relationships in the project which require that a post requisite activity cannot be implemented until the current activities end and an activity cannot stop once it begins due to persistence of the activity. Equation 10 demonstrates how project buffer is calculated.

RESULTS AND DISCUSSION

Genetic algorithm design: After the proposed mathematical model was presented, a genetic algorithm was presented for solving the problem. Next, it was stated how the samples are generated and characterized.

Problem generation method: The genetic algorithm method has been used in this research for optimization of critical chain project scheduling. In this problem, we are seeking the best sequence of activities using this algorithm, so that the best utility value is met. Attempts have also been made to avoid the occurrence of local optima using this method. The following assumptions have been made for solving the model using the genetic algorithm method:

- Number of generations: maxit = 100
- Size of the initial population: npop = 100
- Crossover rate: 70% of the initial population
- Mutation rate: 30% of the initial population

The genetic algorithm first forms and schedules the initial chromosomes which consist of the sequence of performing the activities, based on the prerequisite relationships. Finally, a schedule of activity sequences that provides the highest utility is selected as the optimal schedule.

Sample example: A sample example is presented for validation of the model. The project contains 7 activities, the precedence relationships of which are presented as in Table 1 and Fig. 1. In this research, 4 types of renewable resources and 2 nonrenewable resources have been used for solving the optimization model, the change ranges of which for each activity are displayed as follows (Table 2): based on the ranges considered, an activity with low time and costs may be selected whereas such a state is impossible in real life. For avoidance of infeasible solutions created in this way, the time and cost ranges were divided into two groups such that if the time of an activity is generated from the smaller range, the cost of the activity will be selected certainly from the larger range.

Furthermore, for calculation of the utility function, the optimal time and costs need to be predetermined by the experts. The proposed optimal values for the above two features, therefore are: $U = 80000$; $D = 80$. Different scenarios have been suggested for solving the problem using the genetic optimization algorithm method based on the time and cost importance factors. Based on Table 3, the problem was solved several times for each scenario and average project completion time, total project costs and final utility were presented. It is observed that the cost of doing the project increases as cost importance factor decreases. Furthermore, the time and costs of each activity has been reported separately for each scenario in Table 4.

Next, analysis of the sensitivity and trend of time and cost changes with respect to their importance factors has been investigated. Figure 2 displays the change trend of the objective function of time with respect to different scenarios. As observed, project termination time increases in general as the importance factor of the time function increases. The trend however has been steady in some of the scenarios and decreasing in one scenario.

Table 1: Precedence relationships concerning the project

Activity name	Precedence relation
A	-
B	A
C	A
D	A
E	B-C
F	D
G	E-F

Table 2: Time and cost change ranges for each activity

Activities	Time range	Cost range
A	14-24	12000-23000
B	15-25	1000-3000
C	15-33	3200-4500
D	12-20	30000-45000
E	22-30	10000-20000
F	14-24	18000-40000
G	9-18	22000-30000

Table 3: Results obtained from consideration of different scenarios based on the time and cost importance factors

Scenario	Time		Cost		Utility
	coefficient	coefficient	Time	Cost	
1	0.1	0.9	73.50	99233	0.9563
2	0.2	0.8	73.50	99433	0.9515
3	0.3	0.7	66.75	100416	0.9462
4	0.4	0.6	68.25	101909	0.9378
5	0.5	0.5	69.75	103508	0.9486
6	0.6	0.4	71.25	104166	0.9563
7	0.7	0.3	72.75	105613	0.9670
8	0.8	0.2	75.00	109285	0.9701
9	0.9	0.1	76.50	110806	0.9834

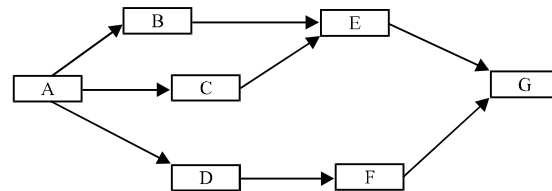


Fig. 1: Node network concerning the project

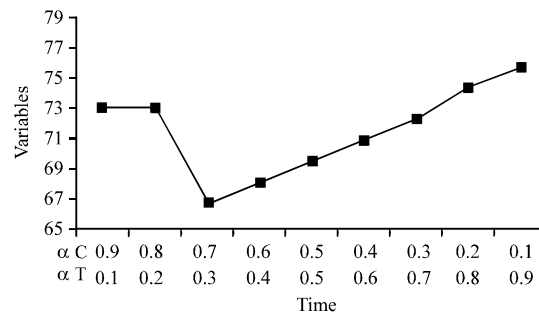


Fig. 2: Change trend of the objective function of time with respect to different scenarios

The change trend of the objective function value of cost with respect to different scenarios is displayed in Fig. 3. It is observed that the cost objective function

Table 4: Time and costs obtained for each activity in different scenarios

Scenario	Time	Cost	Utility	Activity	1	2	3	4	5	6	7
1	73.50	99233	0.9563	Time	24	16	29	17	29	23	16
				Cost	13100	2141	3612	30098	10125	18046	22111
2	73.50	99433	0.9515	Time	24	16	29	17	29	23	16
				Cost	13300	2141	3612	30098	10125	18046	22111
3	66.75	100416	0.9462	Time	20	24	22	17	29	23	16
				Cost	12083	1134	4067	30145	11483	18796	22708
4	68.25	101909	0.9378	Time	21	15	26	18	28	23	16
				Cost	13691	2341	3331	30297	11013	19124	22112
5	69.75	103508	0.9486	Time	22	19	29	19	27	24	15
				Cost	13651	2398	3429	31208	11715	18861	22246
6	71.25	104166	0.9563	Time	24	24	26	17	28	21	17
				Cost	12040	1028	3626	30728	14743	18078	23923
7	72.75	105613	0.967	Time	23	16	29	18	29	23	16
				Cost	13128	2863	3544	30550	11994	20698	22836
8	75.00	109285	0.9701	Time	24	24	33	19	29	22	14
				Cost	12016	1922	3416	31543	10624	20651	29113
9	76.50	110806	0.9834	Time	22	20	32	17	30	22	18
				Cost	12671	2678	3567	35466	14045	19484	22895

Table 5: Deviation values obtained from different scenarios in view of the optimal value considered

Scenario	Expected time deviation	Expected cost deviation
1	6.50	19233
2	6.50	19433
3	13.25	20416
4	11.75	21909
5	10.25	23508
6	8.75	24166
7	7.25	25613
8	5.00	29285
9	3.50	30806

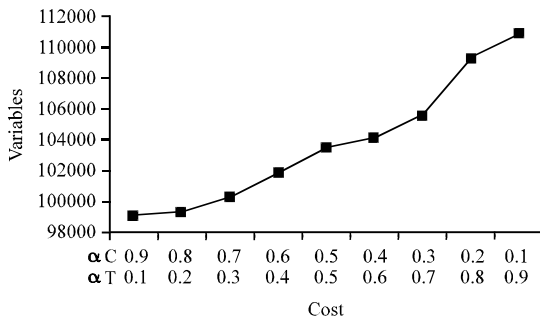


Fig. 3: Change trend of the objective function of cost with respect to different scenarios

value has dramatically increased as the importance factor of cost has decreased; the change trend seems correct, since the cost value of implementing the project increases as the importance of the objective function decreases and the objective function of time is focused on.

As observed in Fig. 4, the utility value obtained from doing the project increases as the importance factor of time increases and the importance factor of cost decreases. Even though the utility value first decreased as the importance factor of cost decreased, the final utility increased as the value then decreased further.

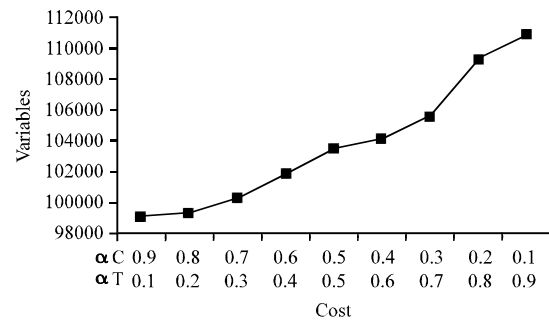


Fig. 4: Change trend of the utility function with respect to different scenarios

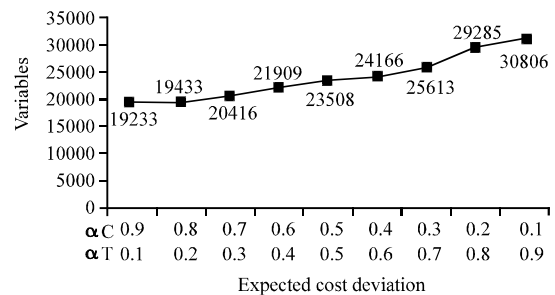


Fig. 5: Deviation value of the costs from the expected optimal value

Next, the value of deviation from the expected time and cost with respect to different scenarios was calculated for further investigation and analysis of the proposed problem. Table 5 displays the values of the deviations with respect to each of the scenarios. As observed in Fig. 5, the value of deviation from the expected cost increases as the importance factor of the cost function decreases and the change trend seems to make sense since the other function is focused on. On the

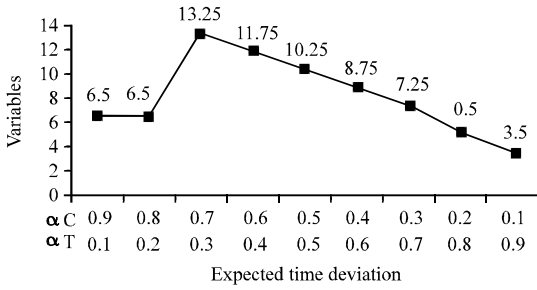


Fig. 6: Deviation value of the times from the expected optimal value

other hand, the value of deviation from the expected time decreases at the same time as the importance factor of time increases as displayed in Fig. 6. It seems through examination of the simultaneous changes in these two importance factors and the change trends of deviation from the expected value that the proposed model is correct and efficient and can help the decision-maker select one of the states in light of the change trends.

CONCLUSION

The project scheduling problem has been investigated in this research using the critical chain method. For this purpose, a mathematical model based on the utility function was presented and a genetic algorithm was developed for solving the problem due to its complexity. For evaluation of the proposed algorithm, a case example was presented and solved in different states through scenario building. Furthermore, the sensitivity of the functions of time and cost to their importance factors was analyzed. Finally, the deviation values from the optimal values were calculated based on different scenarios and the change trends were shown.

In this study only the two objectives of time and cost were considered for the project scheduling problem while the problem can also be studied from other perspectives such as quality. On the other hand, the solution approach in this research has been to use the utility function; multi-objective solution methods and subsequently, multi-objective optimization algorithms such as NSGAII and MOPSO and algorithms like that can be used in future studies.

NOMENCLATURE

- J = Number of project activities, $j = 1, 2, 3, \dots, J$
- K = Renewable resources, $k = 1, 2, 3, \dots, K$
- r_{jk} = Value of renewable resource k required for performing activity, $j_k = 1, 2, 3, \dots, K$ and $j = 1, 2, 3, \dots, J$

- C_k = Cost of each unit of renewable resource, $k = 1, 2, 3, \dots, K$
- t = Activity time, $j = 1, \dots, J$
- P = Non-renewable resources, $p = 1, 2, 3, \dots, P$
- nr_{jp} = Value of non-renewable resource P required for performing activity, $j_j = 1, 2, 3, \dots, J$ and $p = 1, 2, 3, \dots, P$
- C_p = Cost of each unit of non-renewable resource p , $p = 1, 2, 3, \dots, P$
- E_{e0} = Time when the final virtual activity ends
- PB = Project buffer
- T = Total project time from the critical chain method
- r_k = Value of available unusable resource
- $U(T)$ = Time-utility function
- $U(C)$ = Cost-utility function
- α_T = Time weight coefficient
- α_C = Cost weight coefficient
- D = Optimum total project time
- U = Optimum total project costs

REFERENCES

Anagnostopoulos, K.P. and L. Kotsikas, 2010. Experimental evaluation of simulated annealing algorithms for the time-cost trade-off problem. *Applied Math. Comput.*, 217: 260-270.

Azaron, A., C. Perkgoz and M. Sakawa, 2005. A genetic algorithm approach for the time-cost trade-off in PERT networks. *Appl. Math. Comput.*, 168: 1317-1339.

Chen, S.P. and M.J. Tsai, 2011. Time-cost trade-off analysis of project networks in fuzzy environments. *Eur. J. Oper. Res.*, 212: 386-397.

Goldberg, D.E., 1989. *Genetic Algorithm in Search Optimization and Machine Learning*. 1st Edn., Addison-Wesley, New York, ISBN: 10: 0201157675, pp: 432.

Ke, H. and J. Ma, 2014. Modeling project time-cost trade-off in fuzzy random environment. *Appl. Soft Comput.*, 19: 80-85.

Klansek, U. and M. Psunder, 2012. MINLP optimization model for the nonlinear discrete time-cost trade-off problem. *Adv. Eng. Software*, 48: 6-16.

Nazarpour, H., M.J.T. Amiri and M. Hemmatian, 2014. Prioritizing delay causes in construction projects in Mazandaran province (Iran) and presenting solutions for improving it. *Adv. Social Humanities Manage.*, 1: 16-25.

Sivanandam, S.N. and S.N. Deepa, 2008. *Introduction to Genetic Algorithms*. Springer, USA., ISBN: 354073189X, Pages: 442.