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Vibrational Frequency of Isotropic Square Plate on C-C-S-S Condition

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Abstract: In this study, researchers discuss the frequency modes (first two modes) of non-homogeneous isotropic square plate on C-C-S-S condition where C and S represent clamed and simply supported, respectively. Here, thickness varies linearly in one direction. For non homogneity in the material, we considered circular varation in density. Bi-parabolic temperature variation on the plate is also being viewed. To find the vibrational frequency, Rayleigh Ritz technique has been applied for the different values of plate's parameters.

Key words: Circular variation, bi-parabolic temperature, clamped and simply supported condition, linear thickness, square, vibrational

INTRODUCTION

Liessa (1997a) provided an excellent monograph in which he studied plate vibration on different boundary (clamped, simply supported and free) conditions. Leissa (1997b) also provided recent research in plate vibration. Leissa and Nartia (1980) discussed natural frequency of simply supported circular plate. Kalita and Haldar (2015) applied FSDT to study the vibration of thick rectangular plate. Kim and Dickinson (1990) discussed flexural vibration of thin isotropic and polar orthotropic annular and circular plate. Ansari and Gupta (1999) studied asymmetric free vibration of polar orthotropic circular plate with parabolic variation using Ritz Method. Zhou (2002) discussed vibrations of point supported rectangular plates with variable thickness using a set of static tapered beam functions. Sharma *et al.* (2016a) provided mathematical modeling on vibrational frequency of skew plate with thermal gradient. Sharma *et al.* (2016b-d) provided the free vibration of non-homogeneous trapezoidal plate and orthotropic rectangular plate under thermal gradient. Hosseini-Hashemi *et al.* (2013) provided a mathematical model to study free vibration of stepped circular and annular FG plate. Khanna and Kaur (2016) discussed the effect of structural parameter on vibration of non-homogeneous rectangular plate. Ansari (2016) studied axisymmetirc forced response of polar orthotropic tapered circular using Rayleigh Ritz Method.

The present study provides the frequency modes of non-homogeneous tapered square plate on C-C-S-S condition at different value of thermal gradient, non-homogeneity and tapering constant. All the calculations are carried out for Duralumin material using high level computational software MAPLE. All the findings are presented with the help of tables.

MATERIALS AND METHODS

Equation of motion: Equation 1 of motion for isotropic plate is:

$$\begin{bmatrix} D_1\left(\Phi_{xxxx}+2\Phi_{xxyy}+\Phi_{yyyy}\right)+2D_{1,x} \\ \left(\Phi_{xxx}+\Phi_{xyy}\right)+2D_{1,y}\left(\Phi_{yyy}+\Phi_{yxx}\right)+ \\ D_{1xx}\left(\Phi_{xx}+v\Phi_{yy}\right)+D_{1,yy} \\ \left(\Phi_{yy}+v\Phi_{xx}\right)+2\left(1-v\right)D_{1,xy}\Phi_{xy} \end{bmatrix} -\rho c^2 h\Phi = 0 \quad (1)$$

where, $D_1 = Yh^3/12\,(1-v^2)$ called flexural rigidity. A comma followed by suffix is known as partial derivative with respect to independent variable. For non-homogeneity, we assume that the density varies circularly in one direction as:

$$\rho = \rho_0\left\{1-\alpha_1\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right\} \quad (2)$$

where, α_1 $(0\leq\alpha_1\leq1)$ is known as non-homogeneity constant. Thickness of plate is assumed to be linear in one direction:

$$h = h_0\left(1+\beta\frac{x}{a}\right) \quad (3)$$

where, β $(0\leq\beta\leq1)$ is known as tapering parameter of the plate and $h = h_0$ at $x = 0$. Also, the temperature variation on the plate is assumed to be parabolic:

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$$\tau = \tau_0 \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right) \quad (4)$$

where, τ and τ_0 are known as temperature above the mention temperature at any point on the plate and at origin, i.e., x = y = 0. For engineering material, the modulus of elasticity is:

$$Y = Y_0\left(1-\gamma\tau\right) \quad (5)$$

where, y_0 is the Young's modulus at $\tau = 0$ and γ is known as slope of variation. Substitute Eq. 4 and 5, we get:

$$Y = Y_0\left\{1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right)\right\} \quad (6)$$

where, α ($0\leq\alpha<1$) is known as temperature gradient which is the product of variation of slope and temperature at origin, i.e., $\alpha = \gamma\tau_0$. Using Eq. 3 and 6 the flexural rigidity of the plate becomes:

$$D_1 = \frac{Y_0 {h_0}^3\left[1-\alpha\left\{1-\frac{x^2}{a^2}\right\}\left\{1-\frac{y^2}{a^2}\right\}\right]\left\{1+\beta\frac{x}{a}\right\}^3}{12\left(1-\nu^2\right)} \quad (7)$$

Solution for differential equation: We are using Rayleigh Ritz technique (i.e., maximum kinetic energy T must equal to maximum strain energy V) to solve differential equation, therefore, we have:

$$\delta\left(V-T\right) = 0 \quad (8)$$

Where:

$$T = \frac{1}{2}c^2\int_0^a\int_0^a \rho h \Phi^2\, dydx \quad (9)$$

$$V = \frac{1}{2}\int_0^a\int_0^a D_1\left\{\begin{matrix}\left(\Phi_{xx}\right)^2+\left(\Phi_{yy}\right)^2+2v\Phi_{xx}\\ \Phi_{yy}+2(1-v)\left(\Phi_{xy}\right)^2\end{matrix}\right\}dydx \quad (10)$$

In the present scenario, we are computing frequency modes on C-C-S-S condition. The boundary condition for C-C-S-S is:

$$\Phi = \Phi_x = 0,\ x = 0,\ a \text{ and } \Phi = \Phi_{yy} = 0,\ y = 0,\ a \quad (11)$$

Therefore, two term deflection function Φ which satisfy above boundary condition could be:

$$\Phi = \left[\left(\frac{x}{a}\right)^2\left(\frac{y}{a}\right)^2\left(1-\frac{x}{a}\right)\left(1-\frac{y}{a}\right)\right]\left[H_1+H_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{a}\right)\right] \quad (12)$$

where, H_1 and H_2 represents arbitrary constants. Now converting X and Y in non-dimensional variable as:

$$X = \frac{x}{a},\ Y = \frac{y}{a} \quad (13)$$

Using Eq. 13, 9 and 10 becomes:

$$T = \frac{1}{2}c^2\int_0^1\int_0^1 \rho h \Phi^2\, dYdX \quad (14)$$

$$V = \frac{1}{2}\int_0^1\int_0^1 D_1\left\{\begin{matrix}\left(\Phi_{XX}\right)^2+\left(\Phi_{YY}\right)^2+2v\Phi_{XX}\Phi_{YY}+\\ 2(1-v)\left(\Phi_{XY}\right)^2\end{matrix}\right\}dYdX \quad (15)$$

Substitute Eq. 14 and 15 in Eq. 8, we get:

$$\left(V^*-\lambda^2T^*\right) = 0 \quad (16)$$

Where:

$$V^* = \int_0^1\int_0^1\left[1-\alpha\left\{1-X^2\right\}\left\{1-Y^2\right\}\right]\left\{1+\beta X\right\}^3 \left\{\begin{matrix}\left(\Phi_{XX}\right)^2+\left(\Phi_{YY}\right)^2+\\ 2v\Phi_{XX}\Phi_{YY}+2(1-v)\left(\Phi_{XY}\right)^2\end{matrix}\right\}dYdX$$

$$T^* = \int_0^1\int_0^1\left\{1-\alpha_1\left(1-\sqrt{1-X^2}\right)\right\}\left\{1+\beta X\right\}\Phi^2\, dYdX$$

and λ^2 is known as frequency parameter. Equation 16 consists of two unknowns constants, i.e., H_1, H_2 because of substitution of deflection function Φ. These constants can be determined by:

$$\frac{\partial\left(V^*-\lambda^2T^*\right)}{\partial H_i} = 0,\ i = 1,\ 2 \quad (17)$$

After simplifying Eq. 17, we get

$$a_{i1}H_1+a_{i2}H_2 = 0,\ i = 1, 2 \quad (18)$$

where, a_{i1}, a_{i2} (i = 1, 2) involve parametric constant and frequency parameter. To get frequency modes, the determinant of the coefficient matrix obtain from Eq. 18 must be zero. Therefore:

$$\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix} = 0 \quad (19)$$

Equation 19 is quadratic equation from which we get two roots as λ_1 (1st mode) and λ_2 (2nd mode).

RESULTS AND DISCUSSION

To calculate the modes of frequency on different values of plate's parameter (i.e., thermal gradient, non-homogeneity and taper constant), the following parameters are used:

$$\rho_0 = 2.80\times10^3 \text{ kg/m}^3, v = 0.345, h_0 = 0.01 \text{ m}$$

Table 1 provides the vibrational frequency corresponding to thermal gradient for the following three cases:

- Case 1; $\alpha_1 = \beta = 0$
- Case 2; $\alpha_1 = \beta = 0.4$
- Case 3; $\alpha_1 = \beta = 0.8$

From Table 1, we conclude that frequency mode decreases when the temperature on the plate increases from 0-0.8 for all the above said three cases. When combined value of non-homogeneity constant 1 and tapering parameter increases from 0-0.8 frequency mode increases. Table 2 gives the frequency modes corresponding to thickness parameter for the following:

- Case 4; $\alpha = \alpha_1 = 0$
- Case 5; $\alpha = \alpha_1 = 0.4$
- Case 6; $\alpha = \alpha_1 = 0.8$

From Table 2, we conclude that frequency mode increases along with the thickness parameter for all the three cases. When the combined value of temperature and non-homogeneity α_1 increases from 0-0.8 as in case 4 to case 6 the frequency mode decreases:

- Case 7; $\alpha = \beta = 0$
- Case 8; $\alpha = \beta = 0.4$
- Case 9; $\alpha = \beta = 0.8$

Table 3 gives the frequency modes corresponding to non-homogeneity constant for the following cases. From Table 3, one can easily get that frequency decreases when non-homogeneity increases from 0-1 for all the three cases. On the other hand when the combined value of temperature gradient α and tapering parameter β increases from 0-0.8 the frequency mode increases. The rate of decrement is much smaller because of circular variation in density.

Table 1: Temperature (α) variations vs. vibrational frequency (λ)

	$\alpha_1 = \beta = 0$		$\alpha_1 = \beta = 0.4$		$\alpha_1 = \beta = 0.8$	
α	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	30.18	172.99	36.59	215.22	42.78	254.96
0.2	29.24	171.22	35.54	213.26	41.63	252.79
0.4	28.26	169.44	34.46	211.28	40.43	250.60
0.6	27.24	167.64	33.33	209.28	39.19	248.39
0.8	26.17	165.82	32.16	207.26	37.91	246.16

Table 2: Tapering parameter (β) vs. vibrational frequency (λ)

	$\alpha = \alpha_1 = 0.0$		$\alpha = \alpha_1 = 0.4$		$\alpha = \alpha_1 = 0.8$	
β	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	30.18	172.99	27.04	159.48	24.05	147.90
0.2	34.18	200.63	30.70	185.07	27.41	171.79
0.4	38.29	228.96	34.46	211.28	30.86	196.24
0.6	42.47	257.75	38.29	237.90	34.36	221.07
0.8	46.72	286.87	42.17	264.82	37.91	246.16
1.0	51.00	316.23	46.08	291.96	41.49	271.45

Table 3: Nonhomogeneity (α1) vs. vibrational frequency (λ)

	$\beta = \alpha = 0$		$\beta = \alpha = 0.4$		$\beta = \alpha = 0.8$	
α_1	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	30.18	172.99	36.05	224.79	41.38	277.07
0.2	29.50	167.67	35.23	217.70	40.42	268.19
0.4	28.87	162.84	34.46	211.28	39.53	260.16
0.6	28.28	158.43	33.74	205.43	38.70	252.85
0.8	27.72	154.38	33.06	200.06	37.91	246.16
1.0	27.19	150.64	32.42	195.12	37.17	240.00

CONCLUSION

The present study shows the how circular variation in non-homogeneity affects on frequency of tapered square plate on C-C-S-S conditions. When non-homogeneity increases, the frequency mode decreases with less rate of decrement due to circular variation in density. The other parameter (temperature and thickness) also affects the vibrational frequency. When thickness parameter increases, the frequency mode also increases. The frequency decreases when temperature on the plate increases. The present research also provides some numerical data in the form of modes which is very useful for researchers/scientists to understand the behavior of frequency.

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