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Some Properties of Frame Domination in Graphs

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Abstract: Through this research, a new concept of domination in Graph G = (V, E) named "frame domination" has been defined. The inverse frame domination and connected frame domination have been addressed. This study presents some properties (related to the frame dominating set and frame domination number) which show the concepts of connected and inverse frame domination. Furthermore, some bounds of frame, connected and inverse frame domination are determined. Specifically, the affection of frame domination parameter when a graph is modified by deleting or adding or contracting an edge or deleting a vertex studied deeply through the literature of this study.

Key words: Dominating set, frame dominating set, frame domination number, inverse, affection, domination parameter

INTRODUCTION

Throughout this study, we deal with undirected and simple graphs. For a Graph G = (V, E), let V(G) and E(G) denote its vertex and edge set, respectively. The complement $\overline{G}$ of a simple Graph G with vertex set V(G)is the graph in which two vertices are adjacent if and only if they are not adjacent in G. G[D] is a subgraph of G induced by the vertices in D. G-v and G-e are the subgraphs obtained from G by deleting the vertex v or edge e, respectively. Also, G+e is the graph obtained from G by adding the edge e where $e \in \overline{G}$. Let e = uv be an edge of a graph G, the operation G/e called contraction of the edge e that obtained by removing the edge e from the graph and merges the vertices u, v to one new vertex. All the other edges incident to u or v become incident to the new vertex.

For graph theoretic terminology we refer to Harary (1969). Ore (1962) introduced the concept of domination number $\gamma(G)$ in graph theory. If the induced subgraph G[D] is connected then the dominating set D called connected dominating set which is defined by Walikar *et al*., (1979). The minimum cardinality of connected dominating set in G is called connected domination number and is denoted by γ_c (G). The inverse domination number γ^{-1} (G) is introduced by Kulli and Sigarkanti (1991).

In this study, a new concept of domination in graphs, namely frame domination has been introduced. The frame dominating set and frame domination number have been defined. As well as the inverse frame domination and connected frame domination have been presented. Furthermore, properties and some bounds for frame dominating and inverse frame dominating set are determined. Finally, the affection of frame domination parameter when a graph is modified by deleting or adding or contracting an edge or deleting a vertex studied deeply through the literature of this study.

An excellent treatment of several topics in domination can be found by Haynes *et al*. (1998a, b). For more details about the parameters of domination number (Kiunisala, 2016; Omran and Harere, 2016, 2017; Sampathkumar and Neeralagi, 1985; Sampathkumar and Walikar, 1979). For more clarify we found that it is better to state the following.

Corollary 1.1 (Harary, 1969): "A connected graph is Eulerian if and only if its set of edges can be split up into disjoint cycles".

Definition 1.2 (Haynes *et al*., 1998a): "We say a property P is hereditary if for all sets S that satisfy P, every set S'×S also satisfies P".

Definition 1.3 (Haynes *et al*., 1998b): "We say a property P is super hereditary if, for all sets S that satisfy P, every set S'⊇S also satisfies P".

THE CONCEPT OF FRAME DOMINATION

In this study, we introduce a new definition of domination in graphs, namely frame domination. Also, some properties about frame dominating set and frame domination number are mentioned.

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Definition 2.1: Let, G = (V, E) and $F \subseteq V(G)$. Then, the set F is called a "frame dominating set" if for every $v \in V\text{-}F$ there is a cycle contains v and a vertex in F. The frame domination number $\gamma_f(G)$ is the minimum cardinality of a frame dominating set of G.

Definition 2.2: Let, G = (V, E) and $F \subseteq V(G)$. The set F is called a "connected frame dominating set" if G[F] is a connected graph. The connected frame domination number $\gamma_{cf}(G)$ is the minimum cardinality of a connected frame dominating set of G.

Definition 2.3: Let, G = (V, E) and $F \subseteq V$ (G) is a frame dominating set. The set F' is called an inverse frame dominating set with respect to F, if V-F contains a frame dominating. Also, the minimum cardinality of an inverse dominating set of G is called the inverse frame domination number $\gamma_f^{-1}(G)$. From the definition of frame domination, one can easily calculate the next observation.

Observation 2.4: $\gamma_f(C_n) = \gamma_f(K_n) = \gamma_f(W_n) = \gamma_f(K_{m,n}) = 1$.

Proposition 2.5: Let, G be a graph of order n, then:

- Every graph contains a pendant or an isolated vertex has no frame dominating set
- If a graph G has a frame dominating set and it is disconnected with k components then $\gamma_f(G) \geq k$
- If a graph G is Hamiltonian, then $\gamma_f(G) = 1$
- If a graph is Eulerian, then $\gamma_f(G) \geq 1$
- If a graph G has a frame domination number $\gamma_f(G)$ with m bridges, then $\gamma_f(G) \geq m+1$
- If G has a frame domination number $\gamma_f(G)$ then, $1 \leq \gamma_f(G) \leq \lfloor n/3 \rfloor$

Proof:

- It is clear, since, every vertex of degree less than or equal to one does not belong to any cycle, so, we cannot dominate these vertices by frame dominating set
- It is clear, since, every component contains at least one cycle because the graph G has frame dominating set
- Suppose G is Hamiltonian graph, then there is a cycle passing all vertices in this graph, thus we can choose any vertex to dominate all other vertices of G
- Since, G is Eulerian, then by Corollary 1.1 we get the result
- Each bridge in G is join two disjoint cycles at least and therefore the required is achieved
- The lower bound happened when G has a vertex that belongs to each cycle in G. The upper bound occurs when G is split up into several components such that each component is a triangle

Theorem 2.6: If a graph G of order n has a frame domination number $\gamma_f(G)$ then:

- G has an inverse frame domination number $\gamma_f^{-1}(G)$
- $\gamma_f^{-1}(G) \leq \gamma_f^{-1}(G)$
- $\gamma_f(G) + \gamma_f^{-1}(G)$

Proof: Since, G has a frame dominating set, then every minimal frame dominating set contains only one vertex from each cycle, so, we can choose other minimal frame dominating set contains the other vertex from each cycles, since each cycles contains at least three vertices. Thus, one of the other minimal frame dominating sets that has been chosen is the inverse frame dominating set of G. It is clear that $\gamma_f(G) \leq \gamma_f^{-1}(G)$ by definition of $\gamma_f(G)$.

By proposition 2.3, $1 \leq \gamma_f(G) \leq \lfloor n/3 \rfloor$. So, we can prove by the same manner that $1 \leq \gamma f\text{-}1(G) \leq \lfloor n/3 \rfloor$. Thus, we get the result.

Proposition 2.7: If a graph G of order n has a frame domination number $\gamma_f(G)$ then:

- For each vertex v in G, $2 \leq \deg(v) \leq n\text{-}1$
- If the set F is a γ_{cf} set then there is a path such that this path is common with each cycle in G by only one vertex

Proof: Each vertex belong to one cycle at least, so the lower bound condition has been satisfied and since the graph maybe complete, so, deg(v) = n-1. Therefore, we get the result.

Since, G[F] is connected, then there is a path contains all vertices of F. Now, suppose that this path contains more than one vertex from a cycle which means that F contains two vertices from the same cycle and this is a contradiction with the set F is a minimal. Therefore, the result is a chivied.

Proposition 2.8: If G has a frame domination number $\gamma_f(G)$, then $\gamma_f(G) \leq \gamma(G)$ and equality hold if and only if every cycle in G is a triangle.

Proof: It is obvious, since, every vertex in each frame dominating set dominates all vertices in its open neighborhood. Additionally, this vertex dominates all vertices which are located in the same cycle. Thus, we get the result.

SOME OPERATIONS ON GRAPHS HAVING FRAME DOMINATION

In this study, we introduce some operations in a graph G which has frame domination.

Theorem 3.1: If a graph G has a frame domination number $\gamma_f(G)$ then, G-ν has either no frame domination number or $\gamma_f(G\text{-}\nu)\geq\gamma_f(G)$.

Proof: Let, ν∈V, then there are two cases about G-ν as follows:

Case 1: If all vertices that belong to the open neighborhood of ν contained in some cycles in G-ν, then G-ν has a frame domination number. So, in general $\gamma_f(G\text{-}\nu)\geq\gamma_f(G)$.

Case 2: If there is at least one vertex from open neighborhood in G-ν does not contained in any cycle, then G-ν has no frame dominating set. Therefore, G-ν has no frame domination number.

Theorem 3.2: If G is a graph which has frame domination number $\gamma_f(G)$ and e∈E, then G-e has either no frame domination number or $\gamma_f(G\text{-}e) = \gamma_f(G)$.

Proof: Let, G be a graph has a frame domination number and let uv = e∈E, then there are two cases about G-e as follows.

Case 1: If e is a bridge in G, then G-e has the same frame domination, since each frame dominating set is not influenced by deletion this edge.

Case 2: If there is a u-v path in G-e we can use this path instead of the edge e. Again, each frame dominating set is not influenced by deletion this edge. For special case if e is a common edge for two cycles or more, then there is a cycle that contains all vertices in these cycles. Thus, in these two cases we get $\gamma_f(G\text{-}e) = \gamma_f(G)$. Otherwise, G-e has no frame dominating set. Therefore, G-e has no frame domination number.

Theorem 3.3: If G is a graph has frame domination number $\gamma_f(G)$ and $e\in\overline{G}$, then, $\gamma_f(G+e)\leq\gamma_f(G)$.

Proof: Let, G has a frame domination number and let $uv = e\in\overline{G}$, then there are two cases about adding this edge as follows:

Case 1: If e joins two vertices from the same cycle. Then, this edge becomes common edge between two cycle, so, there is a cycle that contains all vertices in these two cycles and this cycle is the same cycle before adding the edge e. Therefore, $\gamma_f(G+e) = \gamma_f(G)$

Case 2: If the edge e joins two vertices from disjoint cycles, then this edge becomes a bridge between these two cycles and again this edge is not influenced by the frame domination.

Case 3: If the edge e joins two vertices from two different cycles say C_1 and C_2 that joined by bridge $e_1 = u_1\nu_1$ such that u and u_1 belongs to the same cycle say C_1, ν and ν_1 belong to cycle C_2, then the following cases are arise:

- If u is not adjacent to u_1 or ν is not adjacent to ν_1, then the frame domination number is not influenced by this addition
- If u is adjacent to u_1 and ν is adjacent to ν_1, then there is a cycle containing all vertices in the two cycles C_1 and C_2. Thus, $\gamma_f(G+e)<\gamma_f(G)$. Therefore, we get the required for all cases

Theorem 3.4: If a graph G has a frame domination number and e∈E, then G/e has no frame domination number or $\gamma_f(G/e)\leq\gamma_f(G)$.

Proof: Let, G has a frame domination number $\gamma_f(G)$ and let e∈E, then there are two cases about G/e as follows:

Case 1: If e is a bridge in G, then this bridge is join at least two disjoint cycles without loss of generality assume we have two cycles, then in G/e the two vertices u and ν become one vertex. This vertex is a common vertex between the two cycles mentioned later. Thus, this vertex frame dominates all vertices in these cycles. Therefore, $\gamma_f(G/e)<\gamma_f(G)$.

Case 2: If e is not a bridge, then e belongs at least one cycle and therefore, there are two cases depending on the order of this cycle as follows:

- If the order of the cycle is more or equal to four, then after the contraction, the cycle C_n becomes C_{n-1}. Therefore, the frame domination number is still fixed
- If the cycle is of order three and all edges which are not common with any other cycles, then after contracting this edge this cycle turn to a path of order two and each vertex in this path is not contained in any cycle. Thus, G/e has no frame dominating set

Proposition 3.5: If G-ν has a frame domination number, this does not means that G has a frame domination number in general. Figure 1 illustrates this fact.

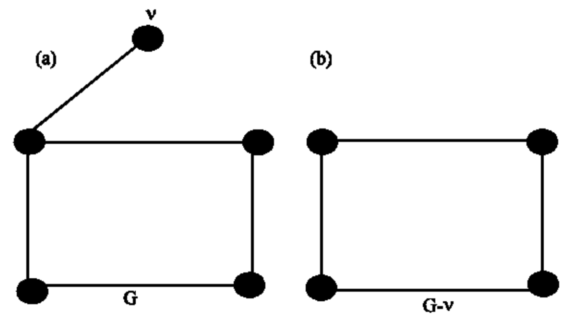

Fig. 1: a, b) G-v Frame domination number

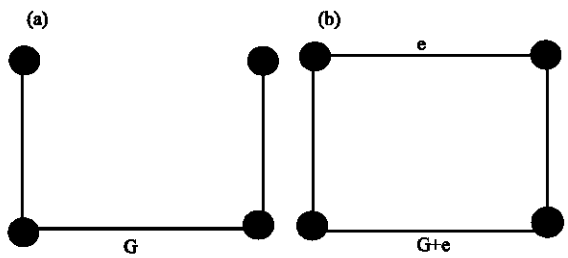

Fig. 2: a, b) G+e frame domination number

If G+e has a frame domination number, this does not means that G has a frame domination number in general as illustrated in Fig. 2.

If G/e has a frame domination number, this does not means that G has a frame domination number in general. Fig. 1 illustrates that when G/e = G-v.

Remark 3.6: If G-e has a frame domination, then by Theorem 3.3 G has a frame domination.

Remark 3.7: Frame domination neither hereditary nor super hereditary.

CONCLUSION

In this study, we introduced a new definition for the concept of domination in graphs, namely frame domination. The frame dominating set and the frame domination number as well as the inverse frame domination have been defined. Also, some operations in frame domination number have been stated and proved. According to the theorems and remarks mentioned before, we can conclude that the frame domination property neither hereditary nor super hereditary.

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