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A Performance Comparative Evaluation for NHPP Software Reliability Model Considering Makeham and Extended Weibull Distribution

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Abstract: In paper research on reliability model consist of the software special feature seeing Makeham and extended Weibull distribution that controls the distribution of life from the development of software about performance comparison was planned. Using the finite-failure non-homogeneous poisson process property, the software failure model was planned. Using the maximum likelihood estimation, the parameter estimation scheming was lead. Ultimately, from comparison about the mean value function, extended Weibull distribution model can be realized that the closer to the true value than the Makeham distribution model. As compared to the true value, extended Weibull distribution model was completed overestimation and the Makeham distribution model was completed underestimation. In terms of reliability, Makeham distribution model than extended Weibull distribution displays the high tendency. Thus, in terms of reliability, Makeham distribution model than extended Weibull distribution model can be refereed more trustworthy model in this field.

Key words: NHPP, model selection, Makeham and extended Weibull distribution, Laplace trend test, trustworthy, underestimation

INTRODUCTION

A studies for this field about Non-Homogenous Poisson Process (NHPP) (Gokhale and Trivedi, 1999; Goel and Okumoto, 1979; Tae-Hyun, 2015) from the fault discovery procedure if a fault happens, directly can be removed during the correcting procedure and has the supposition that new errors have not faced. Goel and Okumoto (1979) was projected specified model with exponential properties. Through this model, the overall sum of faults can be organized S-shaped or exponential-designed type about mean value functions. Hence in research, special feature about reliability software model using the Makeham and extended Weibull distribution that controls the life property since the procedure of software product investigating was premeditated.

MATERIALS AND METHODS

Failure time model using NHPP: The Non-Homogeneous Poisson Process (NHPP) Model (Tae-Hyun, 2015) contains of characteristics property about mean value m(t) and characteristics property about intensity pattern λ(t):

$$m(t) = \int_0^t \lambda(\omega)d\omega, \quad \frac{dm(t)}{dt} = \lambda(t) \tag{1}$$

Therefore, N(t) yields Probability Density Function (Kim, 2015) (PDF) of poisson distribution using component m(t):

$$p(N(t) = n) = \frac{[m(t)]^n}{n!}e^{-m(t)}, \ n = 0, 1, \cdots, \infty \tag{2}$$

This period-related model based on NHPP used the possibility of failure time. About this model, the mean value function was recognized to following structure (Kuo and Yang, 1995):

$$m(t) = -\ln(1-F(t)) \tag{3}$$

Specifically, the intensity function (Kim, 2015) can be transformed the hazard function (h(t)):

$$\lambda(t) = m'(t) = f(t)/(1-F(t)) = h(t) \tag{4}$$

The data structure $\{t_i, i = 1, 2, ...\}$ embodies directive statistic of the times amongst succeeding software disappointment. Consequently, the failure last time x_n can be expressed from the nth failure time (Kim, 2015):

$$x_n = \sum_{i=1}^{n} t_i \ (i = 1,2,\cdots, n;\ 0 \le x_1 \le x_2 \le \cdots \le x_n) \tag{5}$$

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The characteristics property using likelihood pattern of $x_1, x_2, ..., x_n$ can be stated to next shape (Gokhale and Trivedi, 1999; Kim, 2015):

$$f_{x_1, x_2, ..., x_n}(x_1, x_2, ..., x_n) = L(\Theta \mid \underline{x}) = e^{-m(x_n)} \prod_{i=1}^{n} \lambda(x_i) \quad (6)$$

In these settings, the reliability for restricted process (Kim, 2015) $\hat{R}(\xi \mid x_n)$ can be stated next calculation formula using the work time ζ:

$$\hat{R}(\xi \mid x_n) = e^{-\int_{x_n}^{x_n+\xi} \lambda(t)\, dt} = \exp[-\{m(\xi+x_n)-m(x_n)\}] \quad (7)$$

Software reliability NHPP Model using Makeham and extended Weibull distribution

Makeham NHPP Model: The Makeham distribution (Manton *et al.*, 1986) is a significant life distribution and has been extensively used to fit actuarial data and model adult lifetimes by actuaries.

The characteristics property for intensity pattern using $\lambda(t)$ and using mean value pattern $m(t)$ were recognized following property:

$$\lambda_{Mam}(t) = a e^{bt},\ a, b, t > 0,\ m_{Mam}(t) = \int_0^t a\, e^{-b\omega} d\omega = \frac{a}{b}\left(e^{bt} - 1\right) \quad (8)$$

In the sculpture of Eq. 8 a and b are embodied parameter. From Eq. 8, the likelihood function using infinite failure NHPP can be derived as next form:

$$L_{NHPP}(\Theta \mid \underline{x}) = \prod_{i=1}^{n} a e^{b x_i} \exp\left[\frac{-a}{b}\left(e^{b x_n} - 1\right)\right] \quad (9)$$

$\underline{x} = (x_1 \le x_2 \le x_3, \cdots, \le x_n)$, Θ is parameter space. So as to the parameter estimation, the log likelihood forming can be conducted as next formula using Eq. 9:

$$\ln L_{NHPP}(\Theta \mid \underline{x}) = n \ln a + b \sum_{i=1}^{n} x_i - \frac{a}{b}\left(e^{b x_n} - 1\right) \quad (10)$$

The estimator $\hat{a}_{MLE}$ and $\hat{b}_{MLE}$ must be pleased the following calculation:

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial a} = \frac{n}{a} - \frac{1}{b}\left(e^{b x_n} - 1\right) = 0 \quad \left(\text{finally, } \hat{a} = \frac{nb}{e^{b x_n} - 1}\right) \quad (11)$$

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial b} = b^2 \sum_{i=1}^{n} x_i + a e^{b x_n} - a b x_n e^{b x_n} - a = 0 \quad (12)$$

Extended Weibull NHPP Model: The extended Weibull Model (Manton *et al.*, 1986) reduces to the Weibull form. The extended Weibull is a distribution that is extensively used in the field of software reliability and social sciences. The intensity function $\lambda(t)$ and mean value forming $m(t)$ were acknowledged next connection:

$$\lambda_{Edw}(t) = a t^{b-1},\ a, b, t > 0, \quad m_{Edw}(t) = \int_0^t \lambda(\omega)\, d\omega = \frac{a}{b} t^b \quad (13)$$

In Eq. 13, a and b are embodied parameter. Using Eq. 13, the likelihood forming using infinite failure NHPP can be derived as next form:

$$L_{NHPP}(\Theta \mid \underline{x}) = \prod_{i=1}^{n} a\, x_i^{b-1} \exp\left[-\frac{a}{b} x_n^b\right] \quad (14)$$

$\underline{x} = (x_1 \le x_2 \le x_3, ..., x_n)$, $\Theta = \{a, b\}$ is parameter space. So as to the parameter estimation, the log-likelihood forming can be resulted as next formula through Eq. 14:

$$\ln L_{NHPP}(\Theta \mid \underline{x}) = n \ln a + (b-1) \sum_{i=1}^{n} x_i - \frac{a}{b} x_n^b \quad (15)$$

The estimator $\hat{a}_{MLE}$ and $\hat{b}_{MLE}$ must be adapted the following characteristic:

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial a} = \frac{n}{a} - \frac{1}{b} x_n^b = 0 \quad \left(\text{finally, } \hat{a} = \frac{nb}{x_n^b}\right) \quad (16)$$

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial b} = \sum_{i=1}^{n} x_i + \frac{a}{b^2} x_n^b - \frac{a}{b^2} x_n^b \ln x_n = 0 \quad (17)$$

RESULTS AND DISCUSSION

Illustration: In this study, by means of the failure time structure, the characteristic for software reliability model considering the Makeham and extended Weibull distribution that controls the life distribution was considered. Table 1 is structure of the software failure time (Kosznik-Biernacka, 2007). The Laplace trend test (Kim, 2016), preferentially should be preceded to control the usefulness of the data. The significances of this test in Fig. 1 show that estimation value of Laplace influences have the property from 2 and -2. Because the special feature of the reliability growth display, it accomplishes to be the unsafe value is not found. Consequently, the calculation of the reliability using this data may be possible (Chiu *et al.*, 2008; Kanoun and Laprie, 1996).

In this study, consequence of parameter estimation was summarizing in Table 2. These calculations for the solution numerically because the initial values were specified 10^{-3} and 5.000 and acceptance value using the width of intermission value (10^{-5}) were stated were implemented repetition of 150 times inspecting appropriate limit value using C-language.

The Mean Square Error (MSE) (Kim, 2016) signifies difference of the actual value and estimated value. It was recognized next structure:

$$MSE = \frac{\sum_{i=1}^{n}[m(x_i)-\hat{m}(x_i)]^2}{n-k} \quad (18)$$

R^2 can be specified from the foretelling power of the difference for the predicted values (Chiu *et al*., 2008):

$$R^2 = 1- \frac{\sum_{i=1}^{n}[m(x_i)-\hat{m}(x_i)]^2}{\sum_{i=1}^{n}\left(m(x_i)-\sum_{j=1}^{n}m(x_j)/n\right)^2} \quad (19)$$

Communication that $m(x_i)$ means the total number of the faults noticed within period (0, $t(x_i)$] and $\hat{m}(x_i)$ estimating number of the failures at period x_i can be gained from the estimating mean value is the projected. From Eq. 19 n is the totality number of observed data. Also, k indicates the number of parameter.

For the software model assessment from Table 2, MSE displays the situation of the extended Weibull distribution model than Makeham distribution model has a small value. So, the occasion of extended Weibull distribution model is significantly better than Makeham distribution model. Likewise, R^2 displays the situation of the extended Weibull distribution model than Makeham distribution model has a high value. Thus, the situation of extended Weibull distribution model than Makeham distribution model is the usefulness model. The MSE value according to each number of failures is displayed in Fig. 2. While the beginning of failure number for MSE value extended Weibull distribution model shows high shape. Partially, in the second half number of failures display, Makeham distribution model displays high shape.

A characteristics property about mean value was provided using Fig. 3. From this figure, the characteristics

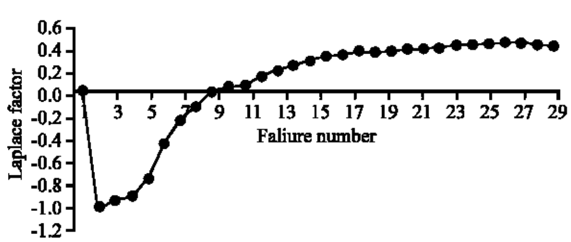

Fig. 1: Outline of the test of Laplace trend

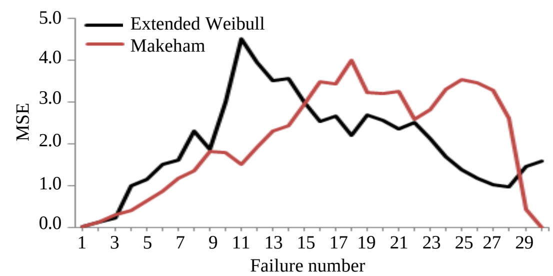

Fig. 2: Outline of the mean square error

Table 1: Information of the failure time data

Failure number	Failure time (h)
1	0.0094
2	0.0500
3	0.4064
4	4.6307
5	5.1741
6	5.8808
7	6.3348
8	7.1654
9	7.2316
10	8.2604
11	9.2962
12	9.3812
13	9.5223
14	9.8783
15	9.9346
16	10.0192
17	10.4077
18	10.4791
19	11.0706
20	11.3250
21	11.5284
22	11.9226
23	12.0294
24	12.0740
25	12.1835
26	12.3549
27	12.5381
28	12.8049
29	13.4615
30	13.8530

Table 2: Estimated value of MLE, MSE and R^2 about Makeham and extended Weibull distribution model

			Model comparison	
Models	Maximum likelihood estimation		MSE	R^2
Makeham	$\hat{a}_{MLE} = 0.82487$	$\hat{b}_{MLE} = 0.08196$	62.6252	0.8864
Extended Weibull	$\hat{a}_{MLE} = 4.06725$	$\hat{b}_{MLE} = 1.2533$	60.7138	0.9107

MLE: Maximum Likelihood Estimation; MSE: Mean Square Error; R^2: Coefficient of determination

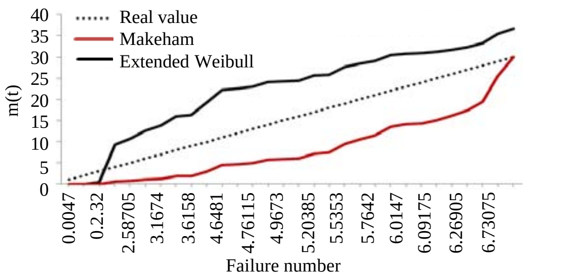

Fig. 3: Outline of the mean value function of respectively model

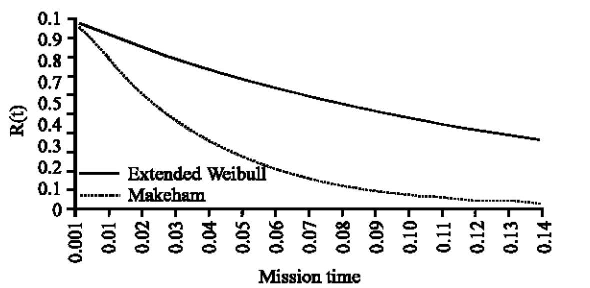

Fig. 4: Outline of the reliability of respectively model

property about mean value have propensity of the increasing specific character. Ultimately, from comparison about the mean value function, extended Weibull distribution model can be seen that the closer to the true value than the Makeham distribution model. As compared to the true value, extended Weibull distribution model was done overestimation and the Makeham distribution model was done underestimation.

In Fig. 4, using comparison of performance, the situation of the reliability for presumed work period (mission time) shows that extended Weibull distribution model and Makeham distribution model have property of the non-reduced pattern. Specifically, the characteristics property about the reliability has been subtle to work period.

In situation of reliability, Makeham distribution model than extended Weibull distribution shows the high trend. Hence, in situation of reliability, Makeham distribution model than the extended Weibull distribution model can be adjudicated more trustworthy model in this field.

CONCLUSION

In research, the reliability software trustworthiness model using Makeham and the extended Weibull distribution that controls the life distribution from the procedure of software performance comparison was planned. The resulting conclusions were gained. In due course, from the between of the foretold principles with the real observations and the foretold rule of the difference the foretold values, some situation of the extended Weibull distribution model than Makeham distribution model is the helpfulness model.

A characteristics property about mean value have propensity of the increasing specific character. Ultimately, from comparison about the mean value function, the extended Weibull distribution model can be perceived that the closer to the observed value than Makeham distribution model. As compared to the observed value (real value), the extended Weibull distribution model was completed overestimation and Makeham distribution model was done underestimation. In terms of the reliability, Makeham distribution model than the extended Weibull distribution displays the high trend. Thus, also in terms of the reliability, Makeham distribution model than the extended Weibull distribution model can be refereed more helpfulness. Through this coursework, it is utilized as material of performance comparison.

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