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Convergence and Stability Analysis of Kolmogorov System Solutions in Infinite-Dimensional Space

[1]V.L. Vorontsova, [1]A.V. Vorontsova, [2]O.V. Druzhinina and [3]E.V. Lisovsky
[1]Institute of Economics Management and Finance, Kazan Federal University, Kazan, Russia
[2]Federal Research Center Computer Science and Control of RAS, Moscow, Russia
[3]Kaluga Branch of the Bauman, Moscow State Technical University (MSTU), Moscow, Russia

Abstract: The study studied the issues of convergence and stability of some calculation system solutions for linear differential equations, namely Kolmogorov's calculation systems in infinite-dimensional space on the basis of local integrability, non-negative coefficients and diagonal dominance properties. The conditions for operators were found with which they solve some problems of these system solution convergence and stability. On the basis of the local integrability, non-negative coefficients and diagonal dominance properties the sufficient conditions were obtained which guarantee the stability and the convergence of Kolmogorov's countable system solutions. The results of the study may be applied during the analysis of technical system various models, particularly the telecommunication system models. Besides, the results of Kolmogorov's system analysis can be used for biological system modeling. The study develops the approach to the qualitative research of Kolmogorov's systems based on the distribution of differential equation qualitative theory in infinite-dimensional spaces on Kolmogorov's systems that allowed to obtain a number of new results. Strict substantiations of the statements are presented concerning the qualitative behavior of solutions for some calculation systems of linear differential equations. One may formulate similar statements for infinite reproduction and death systems which are the particular cases of Kolmogorov's systems as the results of obtained statements.

Key words: Infinite-dimensional systems of differential equations, Kolmogorov's systems, stability, convergence, logarithmic norm

INTRODUCTION

The urgency of the problem concerning explicit dependency obtaining between the input and output characteristics for the network models of mass servicing and reliability is associated with an intensive development of telecommunication and computer networks as well as other technical systems. A continuous modernization of computer networks and communication components requires the ability to react promptly on changes. There is a natural need to observe the behavior of the systems at different parameter values using the example of mathematical models.

There is the need to study the behavior of Kolmogorov's systems in many mass servicing models, reliability and security theory revealed during the last years. In modern data communication systems (including the internet) the streams are non-stationary ones and moreover, they have dependent intervals between the arrival of applications. Therefore, there is a need for the study of mass servicing systems with such flows. Moreover, the focus is on the choice of such non-stationary characteristics of streams whose asymptotic properties could be obtained for a network with a structure of sufficiently general form.

These asymptotic properties can be used to study a wide range of models for complex stochastic systems and during the planning of computational experiments. Asymptotic methods play an important role during the study of many mathematical models including those that describe the operation of various types of mass servicing systems. If direct calculations are difficult because of the need to produce a large amount of computations or if such calculations by modern mathematical methods are not possible explicitly, then the thing is about asymptotic methods.

The properties of differential equation solutions in infinite-dimensional spaces were considered from different points of view (Nemytsky and Stepanov, 1949; Shestakov, 2007; Crane, 1967; Daletsky and Crane, 1970; Curtain and

Corresponding Author: V.L. Vorontsova, Institute of Economics Management and Finance, Kazan Federal University, Kazan, Russia

Pritchard, 1978; Crane and Khazan, 1983; Druzhinina and Shestakov, 2002; Shestakov and Druzhinina, 1999) and in other researches. The qualitative properties of Kolmogorov's system solutions (Kolmogoroff, 1931) and their modifications were studied by Gnedenko and Makarov (1971), Zeifman *et al*. (2011) and Boykov (2008) and in other researches.

The study deals with the convergence and the stability of solutions for some countable systems of linear differential equations, namely Kolmogorov's counting systems (called K systems for brevity) and the calculation systems of reproduction and death.

The issues related to the obtaining of problem solutions referred to mass servicing systems lead in particular to the need of special Hadamard integrals calculation.

There is a number of theoretical and practical problems which lead to the need of equation solutions with fractional integration operators (Vorontsova and Gorskay, 2015a-c; Druzhinina *et al*., 2015; Galimyanov *et al*., 2015).

MATERIALS AND METHODS

The following K-system is considered:

$$y_i' = -a_{ii}(t)y_i + \sum_{j\neq i} a_{ij}(t)y_j,\ i = 1, 2, ... \tag{1}$$

assuming that the following conditions are satisfied. P_1: the ratios $a_{ij}(t)$ included in Eq. 1 are the linear combinations of the final number locally integrable at $t\in R^+ :: = [0, \infty]$ functions. P_2: $A_{ij}(t)$ ratios are not negative, i.e.:

$$a_{ij}(t) \geq 0\ \forall i, j, \forall t \in R^+ \tag{2}$$

P_3:1 the ratios $A_{ij}(t)$ satisfy the following Eq. 3:

$$a_{ii}(t) \geq \sum_{j\neq i} a_{ij}(t)\ \forall i, \forall t \in R^+ \tag{3}$$

or the Eq. 4:

$$a_{ii}(t) = \sum_{j\neq i} a_{ij}(t)\ \forall i, \forall t \in R^+ \tag{4}$$

A special case of K-system (Eq. 1) is the following system of reproduction and death:

$$y_1' = -\lambda_1 y_1 + \mu_2 y_2 \tag{5}$$

$$y_k' = \lambda_{k-1} y_{k-1} - (\lambda_k + \mu_k) y_k + \mu_{k+1} y_{k+1}$$

Obtained from Eq. 1 under the following conditions: $a_{ij} = \lambda_j+\mu_j$ at $i = j$, $a_{ij} = \lambda_j$ at $i = j+1$, $a_{ij} = \mu_j$ at $i = j-1$, $a_{ij} = 0$ at $|i-j|>1$, at that $\mu_1 = 0$ and other ratios meet the conditions (Eq. 2-4). Let's denote the space of summable sequences $x = \{x_i\}$ with a standard via l_1:

$$\|x\| = \sum |x_i|$$

In the area l_1 the system (Eq. 1) can be represented in the form of an evolutional (Eq. 6):

$$y' = A(t)y \tag{6}$$

Due to the condition P_1 on the functions $a_{ij}(t)$ the operator A (t) is a limited one in the space l_1 and its standard $\|A(t)\|$ is locally summable. The operator V: $l_1\rightarrow l_1$ is called a positive one, if $x\in l_{1+} \Rightarrow \forall x \in l_{1+}$ where l_{1+} is the cone of vectors with non-negative coordinates (in the space l_1). If the solution of Eq. 6 is represented in the following form:

$$y(t) = U(t, s)y(s),\quad t \geq s$$

the operator U (t, s) is called the shift operator of Eq. 6. The shift operator U (t, s) of the Eq. 6 is a positive one at $t\geq s$. The general index of the Eq. 6 is the upper limit:

$$\kappa_g = \overline{\lim_{t, s\rightarrow\infty}}\ s^{-1} \ln \|U(t+s, t)\|)$$

The simplex H_1 (and simplex H_2 correspondingly) is the set of vectors $y = (y_1, y_2, ...)^T$ from the cone l_{1+} such that:

$$\sum_{i=1}^{\infty} y_i \leq 1 \left(\text{thus } \sum_{i=1}^{\infty} y_i = 1 \right)$$

The system (Eq. 1) is H_1-invariant (accordingly H_2-invariant) if it meets the condition (Eq. 3) (and the condition (Eq. 4) accordingly). Indeed, if y $(0)\in H_1$ (H_2) then according to Eq. 3:

$$\sum_{i=1}^{\infty} y_i'(t) \leq 0$$

Thus:

$$\sum_{i=1}^{\infty} y_i(t) \leq 1 \quad \forall t \in R^+$$

Thus, according to the condition (Eq. 4):

$$\sum_{i=1}^{\infty} y_i(t) = 1 \quad \forall t \in R^+$$

Let's assume that the operator $A_1(t)$: P→P, P⊂l_1 is H_1 (H_2)-excitation of K-Eq. 6 if: a) the equation y' = A_1y or y' = -A_1y is H_1 (H_2)-invariant of K-equation, б) the equation y' = (A+A_1)y is H_1(H_2)-invariant K-equation. If each number ε>0 has the numbers δ>0, t_0 such that H_1 (H_2)-excitations A_1, $\|A_1\|<\delta$ as an excited, so as a non-excited equation are uniformly exponentially stable in H_1 (H_2) within P space and the norm of solution difference with the same initial conditions less than ε at all $t \geq t_0$, then H_1(H_2)-invariant Eq. 6 is called H_1(H_2)-stable one.

The following theorem on uniform correctness of Cauchy problem concerning K-Eq. 6 for the case of an unbounded operator A(t) in the space l_1 takes place.

Theorem 1: Suppose that: the determination area Dom (A(t)) of the operator A(t) does not depend on time t, the operator A(t) is strongly continuously differentiable on the determination domains Dorm (A(t)), The operator A(t) assume the decomposition of A = A_1+A_2 where the operators A_s (s = 1, 2) have the following form $A_s(a^{(s)}_{ij})$, (i, j = 1, 2, ...), $a^{(1)}_{ij}$, if i≤j and $a^{(1)}_{ij}$ = 0 if i>j, 4, $\|A_2\|l_1<\infty$. Then a) at each s∈[0, τ) and for each point y_0∈Dom (A) there is a single solution y (t, s) of the Eq. 6 with the initial condition on the segment [s, τ], б) y (t, s) and y' (t, s) functions are continuous according to the sum of variables at 0≤s≤t≤τ, в) $y^{(n)}_0 \to 0$, if $y^{(n)}_0$∈Dom (A) and $y^{(n)}(t,s) \xrightarrow[t,s]{} 0$ if $0 \leq s \leq t \leq \tau$.

Proving: Suppose that the theorem conditions are fulfilled. As is known (Crane, 1967; Daletsky and Crane, 1970) the evaluation $\|R^n(A, \lambda)\| \leq M (\mathrm{Re}\lambda-\omega)^{-n}$ takes place if and only if $\|U(t)\| \leq M \exp(-\omega t)$. Considering the reductions of the operator A_1 (its matrix), we see that the logarithmic measure of each truncation is a non-positive one and so at λ>0 we have the following:

$$\left\|U_n(t,s)\right\| \leq 1, \left\| R\left(A_1^{(n)}, \lambda\right)\right\| \leq \lambda^{-1}, \ \forall \lambda > 0$$

Obviously, during the transition from the index n to the index n+1, the first n of resolvent columns (upper triangular matrix) do not change. Consequently, we will have the following in the space l_1:

$$\left\|R(A_1, \lambda)\right\| = \sup_n \left\|R\left(A_1^{(n)}, \lambda\right)\right\| \leq \lambda^{-1}, \forall \lambda > 0$$

The operator $A_1(t)$ is strongly continuously differentiable into Dom(A_1(t)) thus according to [3, theorem 11.3.11] the cauchy problem for the equation y' = A_1(t)y is uniformly correct and, therefore, the cauchy problem for the equation y' = A(t)y will be uniformly correct one for each segment [0, τ]. Theorem 1 is proved. In order to study the qualitative properties of the systems Eq. 1, 5 they use the concept of the logarithmic norm (Daletsky and Crane, 1970; Boykov, 2008). For the operator of K-system the logarithmic rate is determined by the following equation:

$$\mu(A) = \sup_i \left(-a_{ii} + \sum_{j \neq i} a_{ji} \right) \tag{7}$$

Let's consider the K-system in the space l_1 for which the conditions are performed imposed on the coefficients of the system: the basic condition P_1, the condition of coefficient non-negativity P_2 and the condition of diagonal dominance P_3.

It is easy to show that K system is stable in the space l_1. Indeed, the logarithmic norm μ(A) of K-system operator is not positive one that is μ(t, s). This fact follows from a logarithmic norm (Eq. 7) definition and diagonal dominance conditions. Obviously, the following inequality is fair for the shift operator U(t, s) of K-system:

$$\left\|U(t,s)\right\| \leq 1 \quad \forall t \geq s$$

Therefore, K is system is stable according to Lyapunov.

Theorem 2: Let, the functions a_{ij} (t) are continuously differentiable at t∈P^+, the following estimations take place:

$$\sup\left\|A(t)\right\| \leq N_1 < \infty, \ \sup\left\|A'(t)\right\| < N_2 < 1 \tag{8}$$

There is such a natural number k that:

$$\exists b > 0, a_{kk}(t) \geq \sum_{i \neq k} a_{ik}(t) + b \ \forall t \in R^+ \tag{9}$$

$$a_{ii+1}(t) \geq a > 0, i \geq 1 \ \forall t \in R^+ \tag{10}$$

Then in order to solve y (t) of K-system the following property takes place:

$$y_i(t) \to 0 \text{ at } t \to +\infty, i = k, k+1, \ldots \tag{11}$$

Proof: Under the conditions of the theorem it is sufficient to consider only the solutions of K-system coming from the cone l_{1+} (due to the positive development of a shift operator U(t, s)). The following inequality takes place:

$$\sup_{t\geq t_0}|x'(t)| \leq 4\sup_{t\geq t_0}|x(t)|\cdot\sup_{t\geq t_0}|x''(t)| \quad (12)$$

If x(t) is a scalar function. The limitation$\|y(t)\|$ on R^+ follows from theorem conditions. The differentiation result according to t of K-system, allows to obtain the following equation:

$$y'' = A'(t)y + A(t)y'$$

We have the following estimations for the standards of $\|y'\|$ and $\|y''\|$ derivatives:

$$\|y'(t)\| = \|A(t)y\| \leq N_1\|y(t)\| \quad (13)$$

$$\begin{aligned}&\|y''(t)\| \leq \|A'(t)y\| + \|A(t)y'\| \leq \\ &N_2\|y(t)\| + N_1\|y'(t)\| \leq (N_2 + N_1^2)\|y(t)\|\end{aligned} \quad (14)$$

According to Eq. 13 and 14 the standards of the first and the second derivatives are limited on R^+. We have the following from Eq. 9:

$$\sum_{i=1}^{\infty} y_i' \leq -by_k$$

While the standard $\|y(t)\|$ is limited, then:

$$\int_0^{\infty} y_k(s)ds < \infty$$

Fitting the inequality (Eq. 12) to the integral:

$$x(t) = \int_0^{t} y_k(s)ds$$

and letting t_0 to infinity we obtain that $y_k(t)\to 0$ at $t\to+\infty$. Suppose that $x(t) = y_k(t)$ in (12) and letting t to infinity we obtain that $y'_k(t)\to 0$ at $t\to+\infty$. Let's determine now that $y_{k+1}(t)\to 0$ at $t\to+\infty$. Indeed, in the right side of the inequality:

$$y_k' = a_{k1}y_1 + \ldots - a_{kk}y_k + a_{kk+1}y_{k+1} + \ldots \quad (15)$$

All terms are negative except for one. A negative term in (Eq. 15) and the left side (Eq. 15) tend to zero at $t\to+\infty$ and thus $a_{kk+1}y_{k+1}\to 0$ at $t\to+\infty$. Due to the condition (Eq. 10) $y_{k+1}(t)\to 0$ at $t\to+\infty$. Continuing the same reasoning, we find that there is the conclusion (Eq. 11). Theorem 2 is proved.

An infinite-dimensional reproduction and death system (Eq. 5) where the functions $\lambda_i(t)$ and $\mu_j(t)$ are non-negative and are the linear combinations of a finite number of locally integrable functions, the theorems on the convergence and stability of solutions on the assumption that the conditions for the existence of mean value for the functions $\lambda_i(t)$ and $\mu_j(t)$. The evidence are based on the results of Shestakov (1990, 2007), Druzhinina and Shestakov (2002) on the application of the Theorem 1 and on the properties of the logarithmic standard for the system (Eq. 5).

RESULTS AND DISCUSSION

They examined the issues of convergence and stability for Kolmogorov's systems in an infinite-dimensional space on the basis of local integrability, non-negative coefficients and diagonal dominance properties. The conditions for the operators were found with which some problems of solution convergence and stability are solved for these systems.

They obtained sufficient conditions which guarantee the stability and the convergence of solutions for Kolmogorov calculation systems based on the properties of local integrability, non-negative coefficients and diagonal dominance. The theorem about a uniform correctness of the Cauchy problem for an unbounded operator A(t) case in the space l1 was proved. The conditions of Lyapunov's stability were analyzed concerning the Kolmogorov's system solutions using logarithmic norm properties.

Summary: The study results may be applied during the analysis of technical system various models, particularly the telecommunications system models. Besides, the results of Kolmogorov's system analysis can be used during biological system modeling. It is possible to formulate similar statements as the consequences from the obtained allegations for infinite reproduction and death systems which are the particular cases of Kolmogorov's systems. The research is partially supported by the RFBR (Project No. 15-07-08795).

CONCLUSION

The study develops the approach to a qualitative research of Kolmogorov's systems based on the distribution of the qualitative theory methods for differential equations in infinite-dimensional spaces on Kolmogorov's systems that allowed us to obtain a number of new results. Strict substantiation of allegations concerning the qualitative performance of considered system solutions.

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