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A New Perspective to a Fuzzy Distance Function and its Applications on Fuzzy Due Dates and Cost Function

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Abstract: A Single Machine Scheduling (SMS) problems can do what necessary help and discernment to finding a solution, knowledge, control and design more complicated multi-machine scheduling problems. In this study, we suggest n-jobs be performed on one machine included the triangular fuzzy due date. The various date for each job is regard which meets the request of a customer with more gratification standard. The major aim of this study is to test the prospect of employ fuzzy distance function concepts which mention in Lam and Cai to solve SMS many aims functions one of them nonlinear lateness. We use different Local Search Algorithms (LSAs) to compare Bee Colony Algorithm (BCA), Particle Swarm Optimization (PSO) and Tabu Search (TS). All algorithms are tested and computational experiments are given in tables.

Key words: Single machine scheduling, triangular fuzzy number, fuzzy distance function, fuzzy scheduling problem, fuzzy due date, local search algorithms

INTRODUCTION

The universal scheduling or arrange in a particular order of problem may be detail as: Let, n jobs to be complete one at a time on each of m machines. For the time being, in ambitious and elastic store constitute of machines is high-priced as the applied science changes very extremely and the ancient machines can't accept the demands of the newfangled market. Furthermore, put on more than one machines of itself type can accelerate the work but needs progressively embalmment and overlooking. Also, the composition of machines requires more space to set up which also raise the inactive cost of the enterprise. The fuzzy oncoming, perform an alternative path to pattern imprecision and doubt which is more active, principally when no historical acquaintance is obtainable. Initially, it was inserted as display planner and calculus for ambiguous or vague notions. The fuzzy set theory supplies a conceptual setting that implements, so efficiently in decreasing the scheduling case computational intricacy with respect to itself case educe by the probability theory. It should be known that such an imprecision is due to the individual and specific estimate rather than the impact of unmanageable proceedings (Lam and Cai, 2002).

The employ of the fuzzy sets theory in handling different scheduling problems has been so effective by Prade (1970), published the earliest study on fuzzy scheduling. Especially, where judgment and conjecture play an important role such as shopper demand in Ishii and Tada (1995) processing time in Kuroda and Wang (1996), output due dates in Hong and Chuang (1999) or job precedence relations. The objectives in Ishibuchi *et al*. (1994a, b) and in Han *et al*. (1994) are to maximize the total grade of contentment with respect to the fuzzy due dates. Ishii and Tada (1995) believed a single machine problem minimizing the maximum lateness of job with fuzzy priority relations. Kuroda and Wang (1996) resolved the fuzzy job shop scheduling problem. While Stanfield *et al*. (1996) requests to find the optimal schedule through those that do not overtake the maximum reasonable possibility of lateness in a problem including fuzzy due dates. Itoh and Ishii (1999) suggested a single machine scheduling sample dealing with fuzzy processing times and due dates. Konno and Ishii (2000) considered a single machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. Konno and Ishii (2000) proposed a single machine scheduling problem dealing with fuzzy processing times and due date. Konno and Ishii (2000) debated an open

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shop scheduling trouble with fuzzy permissible time and fuzzy purse constraint. Litoiu and Tadei (2001) sitting some new samples for real-time task scheduling with fuzzy deadlines and processing times. Lam and Cai (2002) inserted the direct popularization of the classical lateness measure with the due dates being fuzzy numbers. Specifically, they gave the case where a due date is appeared by a triangular membership function. They offered a notion of a fuzzy lateness function to measure the lateness of jobs with fuzzy due dates and then outspread it for a more general non-decreasing lateness cost function. Peng and Liu (2004) suggested a functional application. In addition to single-objective scheduling models, they believed the multi-objective flow shop machine scheduling problems and subedited as three-objective samples which not only minimize the fuzzy maximum tardiness but also minimize the fuzzy maximum completion time. Cheachan *et al.* (2011) suggested a SMS problem dealing with fuzzy processing times and due date. Cheachan *et al.* (2011) discuss a various approach to the SMS under a fuzzy environment with bi-objective criteria. Mohammed *et al.* (2015) believed an $\tilde{L}(\tilde{C}_i, \tilde{D}_i)$ problem with fuzzy processing time and fuzzy due date to minimize the cost function: $\sum_i x_i C_i^2 + L_{max}(C_i, \tilde{D}_i)$. In this study, we are attentive in the direct popularization of the classical multi-objective function gauge with adding the concept that shows by Lam and Cai (2002).

Preliminaries: In this study, we give some basic concepts that we needed than later.

Definition; Lam and Cai (2002): Let R be a real line. A fuzzy set $\tilde{A}$ from R into [0, 1] which satisfy the following conditions:

- There exists $x_0 \in R$ such that $\tilde{A}(x_0)=1$
- $\tilde{A}(\lambda x+(1-\lambda)y)\geq \min\{\tilde{A}(x), \tilde{A}(y)\}$ where, x, y∈R and λ∈[0, 1] is said to be a fuzzy number

Note (Lam and Cai, 2002):

- The set $(\tilde{A})=\{x\in R \mid \tilde{A}(x)>0\}$ is called support of a fuzzy number $\tilde{A}$, we assume support of fuzzy number $\tilde{A}$ as a closed bounded subset [0, 1] of R
- The set $\tilde{A}_\alpha=\{x\in R \mid \tilde{A}(x)\geq \alpha\}$ is called an α-cut of a fuzzy number $\tilde{A}$, we assume on α-cut of a fuzzy number $\tilde{A}$ as a closed interval $[\underline{a}_\alpha, \overline{a}_\alpha]$
- The fuzzy number $\tilde{A}$ is defined:

$$\tilde{A}(x)=\begin{cases}0 & \text{for} \quad x<a\\ f_{\tilde{A}}(x) & \text{for} \quad a\leq x<c\\ 1 & \text{for} \quad c\leq x\leq d\\ g_{\tilde{A}}(x) & \text{for} \quad d<x\leq b\\ 0 & \text{for} \quad b<x\end{cases}$$

Where:

$f_{\tilde{A}}$ = The nondecreasing function
$g_{\tilde{A}}$ = The nonincreasing function

The distance between fuzzy numbers is defined as follows:

$$\tilde{d}(\tilde{A}, \tilde{B})=\tilde{d}_\tau(\tilde{A}, \tilde{B})+\tilde{d}_\varepsilon(\tilde{A}, \tilde{B})$$

$$\tilde{d}_\tau(\tilde{A}, \tilde{B})=\frac{1}{2}\int_0^1\left\{(\underline{a}_\alpha-\underline{b}_\alpha)^+ +(\overline{a}_\alpha-\overline{b}_\alpha)^+\right\}d_\alpha,(x)^+=\begin{cases}x & \text{if} \quad x\geq 0\\ 0 & \text{if} \quad x<0\end{cases}$$

$$\tilde{d}_\varepsilon(\tilde{A}, \tilde{B})=\frac{1}{2}\int_0^1\left\{(\underline{a}_\alpha-\underline{b}_\alpha)^- +(\overline{a}_\alpha-\overline{b}_\alpha)^-\right\}d_\alpha,(x)^-=\begin{cases}x & \text{if} \quad x\geq 0\\ 0 & \text{if} \quad x<0\end{cases}$$

Where:

$[\underline{a}_\alpha, \overline{a}_\alpha]$ = The α-cut of $\tilde{A}$
$[\underline{b}_\alpha, \overline{b}_\alpha]$ = The α-cut of $\tilde{B}$

Problem formulation: Assume, we have n independent jobs to be treated on a single machine. The job i, i = 1, ..., n requires p_i and w_i collective of processing time and weight, respectively. Each job is assigned with a fuzzy due date $\tilde{D}_i$ which is a Triangular Fuzzy Number (TFN). We heading the state where $\tilde{D}_i$ is a TFN which is realized in expressions of three-digit $[d_i^l, d_i^c, d_i^u]$ as follows:

$$\tilde{D}_i(x)=\begin{cases}0 & \text{if} \quad x<d_i^l\\ \dfrac{x-d_i^l}{d_i^c-d_i^l} & \text{if} \quad d_i^l\leq x<d_i^c\\ \dfrac{d_i^u-x}{d_i^u-d_i^c} & \text{if} \quad d_i^c\leq x\leq d_i^u\\ 0 & \text{if} \quad d_i^u\leq x\end{cases}$$

The potential domain of the fuzzy due date is $[d_i^l, d_i^u]$ when the extreme value place at the point d_i^c. Consequently, the domain $[d_i^l, d_i^u]$ and the point d_i^c are claimed the base and the core of $\tilde{D}_i$, respectively.

Assuming the due date is a crispy number d_i then the cost function, we are attentive to the discussion has the following form:

$$H_i(C_i, d_i)=\begin{cases}(C_i-d_i)^2 & \text{if} \quad C_i\geq d_i\\ C_i-d_i & \text{if} \quad C_i<d_i\end{cases}$$

where, C_i and w_i are respectively, the completion time and weights of the job i, i = 1, ..., n. If the due date is a fuzzy number, then $\tilde{H}_i(C_i, \tilde{D}_i)$ is a fuzzy function and our trouble is to locate the job orders to treat the jobs, so that, $\sum_i w_{s(i)}C_{s(i)}^2+\max\tilde{H}_i(C_i, \tilde{D}_i)$ using the traditional notion, we denote the problem formulated in this study as: $1|\tilde{D}_i=TFN|\sum_i w_i C_i^2+\max\tilde{H}_i(C_i, \tilde{D}_i)$.

Model development: In this study, we give the formal of a cost function $\sum_i w_i C_i^2 + \max \tilde{H}_i(C_i, \tilde{D}_i)$ which is depended on work. We start to give formal for $\tilde{H}_i(C_i, \tilde{D}_i)$. Lam and Cai (2002) gave the notation of use fuzzy distance in $\tilde{H}_i(C_i, \tilde{D}_i)$ without details here, we give some details of drive $\tilde{H}_i(C_i, \tilde{D}_i)$ which depends on the ideas that appeared by Cheachan *et al.* (2011) and Mohammed *et al.* (2015) and find the final formula of $\tilde{H}_i(C_i, \tilde{D}_i)$ which is needed later. By substituting "fuzzy distance $\tilde{d}$ by $\tilde{H}, \tilde{A}$ with C_i, the crisp completion time of the job i and $\tilde{B}$ with $\tilde{D}_i$ fuzzy due-date of the job i", the fuzzy function $\tilde{H}_i(C_i, \tilde{D}_i)$ can be evaluated as follows:

$$\tilde{H}_i(C_i, \tilde{D}_i) = \frac{1}{2}\int_0^1 \left\{H_i^+(C_i - \underline{d}_{i\alpha}) + H_i^+(C_i - \overline{d}_{i\alpha})\right\} d_\alpha + \frac{1}{2}\int_0^1 \left\{H_i^-(C_i - \underline{d}_{i\alpha}) + H_i^-(C_i - \overline{d}_{i\alpha})\right\} d\alpha$$

Where:

$$H_i^+(x) = \begin{cases} H_i(x) & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases} \quad \text{and} \quad H_i^-(x) = \begin{cases} 0 & \text{if} \quad x \geq 0 \\ H_i(x) & \text{if} \quad x < 0 \end{cases}$$

And $[\underline{d}_{i\alpha}, \overline{d}_{i\alpha}]$ is the α-cut of $\tilde{D}_i$. Hence:

$$\tilde{H}_i(C_i, \tilde{D}_i) = \frac{1}{2}\int_0^1 \left\{H_i^+\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + H_i^+\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\right\} da +$$
$$\frac{1}{2}\int_0^1 \left\{H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + H_i^-\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\right\} da$$

We must discuss the following four cases: If $C_i<d_i^l$ then $C_i-((d_i^c-d_i^l)\alpha+d_i^l)<0$ and $C_i+(d_i^u-a_i^c)\alpha-d_i^u<0$. In this case:

$$\tilde{H}_i(C_i, \tilde{D}_i) = \frac{1}{2}\int_0^1 \left\{H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + H_i^-\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\right\} da =$$
$$\frac{1}{2}\int_0^1 \left\{\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + \left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\right\} da =$$
$$\frac{1}{2}\left(2C_i - \frac{1}{2}d_i^l - \frac{1}{2}d_i^u - d_i^c\right) = C_i - \frac{1}{4}\left(d_i^l + d_i^u + 2d_i^c\right)$$

If $d_i^l \leq C_i < d_i^c$ then $C_i+(d_i^u-d_i^c)\alpha-d_i^u<0$. In this case:

$$\tilde{H}_i(C_i, \tilde{D}_i) = \frac{1}{2}\int_0^1 \left\{\begin{matrix} H_i^+\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + \\ H_i^+\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\end{matrix}\right\} da + \frac{1}{2}\int_0^1 \left\{\begin{matrix} H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) + \\ H_i^-\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right)\end{matrix}\right\} da =$$

$$\frac{1}{2}\left(\begin{matrix} \int_0^{\frac{C_i - d_i^l}{d_i^c - d_i^l}} H_i^+\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da + \\ \int_{\frac{C_i - d_i^l}{d_i^c - d_i^l}}^1 H_i^+\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da \end{matrix}\right) + \frac{1}{2}\left(\begin{matrix} \int_0^{\frac{C_i - d_i^l}{d_i^c - d_i^l}} H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da + \\ \int_{\frac{C_i - d_i^l}{d_i^c - d_i^l}}^1 H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da \end{matrix}\right) + \frac{1}{2}\int_0^{\frac{C_i - d_i^l}{d_i^c - d_i^l}} H_i^-\left(C_i - \left(-\left(\begin{matrix} d_i^u - \\ d_i^c \end{matrix}\right)a + d_i^u\right)\right) da =$$

$$\frac{1}{2}\int_0^{\frac{C_i - d_i^l}{d_i^c - d_i^l}} H_i^+\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da + \frac{1}{2}\int_{\frac{C_i - d_i^l}{d_i^c - d_i^l}}^1 H_i^-\left(C_i - \left(\left(d_i^c - d_i^l\right)a + d_i^l\right)\right) da + \frac{1}{2}\int_0^1 H_i^-\left(C_i - \left(-\left(d_i^u - d_i^c\right)a + d_i^u\right)\right) da =$$

$$\frac{1}{2}\int_{0}^{\frac{C_i-d_i^l}{d_i^c-d_i^l}}\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)^2 da+\frac{1}{2}\int_{\frac{C_i-d_i^l}{d_i^c-d_i^l}}^{1}H_i^-\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)da+\frac{1}{2}\left[\left(\left(C_i-d_i^c\right)a+\frac{\left(d_i^u-d_i^c\right)a^2}{2}\right)\right]_0^1=$$

$$\frac{\left(C_i-d_i^l\right)^3}{6\left(d_i^c-d_i^l\right)}+\frac{1}{4}\left(\frac{2\left(d_i^c-d_i^l\right)\left(C_i-\frac{d_i^l}{2}-\frac{d_i^c}{2}\right)\left(C_i-d_i^l\right)^2}{d_i^c-d_i^l}\right)+\frac{1}{4}\left(2C_i-d_i^u-d_i^c\right)=\frac{\left(C_i-d_i^l\right)^3}{6\left(d_i^c-d_i^l\right)}+\frac{\left(C_i-d_i^l\right)^2}{4\left(d_i^c-d_i^l\right)}+\frac{1}{4}\left(2C_i-d_i^u-d_i^c\right)=$$

If $d_i^c \le C_i < d_i^u$ then $C_i-(d_i^c-d_i^l)\alpha-d_i^l \ge 0$. In this case:

$$\tilde{H}_i\left(C_i,\tilde{D}_i\right)=\frac{1}{2}\int_0^1\left\{\begin{matrix}H_i^+\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+\\H_i^+\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\end{matrix}\right\}da+\frac{1}{2}\int_0^1\left\{\begin{matrix}H_i^-\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+\\H_i^-\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\end{matrix}\right\}da=$$

$$\frac{1}{2}\int_0^1\left\{H_i^+\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+H_i^+\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\right\}da+\frac{1}{2}\int_0^1 H_i^-\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)da=$$

$$\frac{1}{2}\int_0^1\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)^2 da+\frac{1}{2}\int_{\frac{d_i^u-C_i}{d_i^u-d_i^c}}^{1}\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)da+\frac{1}{2}\int_0^{\frac{d_i^u-C_i}{d_i^u-d_i^c}}H_i^-\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)da=$$

$$\frac{-1}{2\left(d_i^c-d_i^l\right)}\left(\frac{\left(C_i-d_i^c\right)^3}{3}-\frac{\left(C_i-d_i^l\right)^3}{3}\right)+\frac{-1}{2\left(d_i^u-d_i^c\right)}\left(\frac{\left(C_i-d_i^c\right)^3}{3}\right)+\frac{1}{2}\left(\left(C_i-d_i^u\right)\left(\frac{d_i^u-C_i}{d_i^u-d_i^c}\right)+\frac{d_i^u-C_i}{2\left(d_i^u-d_i^c\right)}\right)=$$

$$-\frac{1}{4}\frac{\left(C_i-d_i^u\right)^2}{d_i^u-d_i^c}+\frac{\left(C_i-d_i^c\right)^3}{6\left(d_i^u-d_i^c\right)}+\frac{1}{6}\left(\left(2C_i-d_i^l-d_i^c\right)^2-\left(C_i-d_i^l\right)\left(C_i-d_i^c\right)\right)$$

If $C_i \le d_i^u$ then $C_i-(d_i^c-d_i^l)\alpha-d_i^l>0$ and $C_i+(d_i^u-d_i^c)\alpha-d_i^u>0$. Hence:

$$\tilde{H}_i\left(C_i,\tilde{D}_i\right)=\frac{1}{2}\int_0^1\left\{\begin{matrix}H_i^+\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+\\H_i^+\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\end{matrix}\right\}da+\frac{1}{2}\int_0^1\left\{\begin{matrix}H_i^-\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+\\H_i^-\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\end{matrix}\right\}da=$$

$$\frac{1}{2}\int_0^1\left\{\begin{matrix}H_i^+\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)+\\H_i^+\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)\end{matrix}\right\}da=\frac{1}{2}\int_0^1\left\{\begin{matrix}\left(C_i-\left(\left(d_i^c-d_i^l\right)a+d_i^l\right)\right)^2+\\\left(C_i-\left(-\left(d_i^u-d_i^c\right)a+d_i^u\right)\right)^2\end{matrix}\right\}da=$$

$$\frac{-1}{2\left(d_i^c-d_i^l\right)}\left(\frac{\left(C_i-d_i^c\right)^3}{3}-\frac{\left(C_i-d_i^l\right)^3}{3}\right)+\frac{1}{2\left(d_i^u-d_i^c\right)}\left(\frac{\left(C_i-d_i^c\right)^3}{3}-\frac{\left(C_i-d_i^u\right)^3}{3}\right)=$$

$$\frac{1}{6}\left(2\left(C_i-d_i^c\right)^2+\left(C_i-d_i^l\right)^2+\left(C_i-d_i^u\right)^2+\left(C_i-d_i^c\right)\left(C_i-d_i^l\right)+\left(C_i-d_i^c\right)\left(C_i-d_i^u\right)\right)$$

MATERIALS AND METHODS

Local search techniques: In this part, we use local search techniques which is helpful material to find a solution to SMS problem $1|\tilde{D}_i = TFN|\sum_i w_i C_i^2 + \max \tilde{H}_i(C_i, \tilde{D}_i)$ such as BCA, PSO and TS by Pham *et al*. (2008) and Cheachan *et al*. (2013).

Basic description of PSO: PSO is a swarm intelligence meta-heuristic inspired by the group behavior of animals, for example bird flocks or fish schools. In PSO algorithms, the population $P = \{p_1, ..., p_n\}$ of the feasible solutions is often called a swarm. The feasible solutions $p_1, ..., p_n$ are called particles. The PSO method views the set R of feasible solutions as a "space" where the particles "move".

Swarm topology: Each particle i has its neighborhood N_i (subset of P). The structure of the neighborhoods is called the swarm topology which can be represented by a graph. Usual topologies are: fully connected topology and circle topology. Characteristics of particle i at iteration t:

- x_i (t), the position (a d-dimensional vector)
- p_i (t), the "historically" best position
- l_i (t), the historically best position of the neighboring particles; for the fully connected topology, it is the historically best known position of the entire swarm
- V_i (t), the speed; it is the step size between x_i (t) and x_i (t+1) at the beginning of the algorithm, the particle positions are randomly initialized and the velocities are set to 0 or to small random values
- c_1 c_2, ..., acceleration coefficients; usually between 0 and 4

Update of the speed and the positions of the particles many versions of the particle speed update exist, for example: $v_i(t+1) = v_i(t)+c_1u_1(p_i(t)-x_i(t))+c_2u_2(l_i(t)-x_i(t))$. The symbols u_1 and u_2 represent random variables with the U(0, 1) distribution. The first part of the velocity formula is called "inertia", the second one "the cognitive (personal) component", the third one is "the social (neighborhood) component". Position of particle i changes according to $x_i(t+1) = x_i(t)+v_i(t+1)$.

We are now in the position to give a general template for TS, integrating the elements we have seen so far. We suppose that we are trying to minimize a function f(S) over some domain and we apply the so-called best improvement version of TS, i.e., the version in which one chooses at each iteration the best available move (this is the most commonly used version of TS).

Notation:

- S, the current solution
- S*, the best-known solution
- f*, the value of S*
- N(S), the Neighborhood of S
- $\hat{N}(S)$, the admissible subset of N(S) (i.e., non-Tabu or allowed by aspiration)
- T is the Tabu list
- Choose (construct) an initial solution S_0. Set $S \leftarrow S_0$, $f^* \leftarrow f(S_0)$, $S^* \leftarrow S_0$, $T \leftarrow 0$
- Search while termination criterion not satisfied do select S in argmin $S \in \hat{N}(S)[f(S)]$
- if $f(S)<f^*$, then set $f^* \leftarrow f(S)$, $S^* \leftarrow S$; record Tabu for the current move in T (delete oldest entry if necessary)
- Finally, now we introduce basics of artificial BCA
- Simulates behaviour of real bees for solving multidimensional and multimodal optimization problems
- The colony of artificial bees consists of three groups of bees: employed bees, onlookers and scouts
- The first half of the colony consists of the employed artificial bees and the second half includes the onlookers
- The number of employed bees is equal to the number of food sources around the hive
- The employed bee whose food source has been exhausted by the bees becomes a scout

RESULTS AND DISCUSSION

Computational results: LSA was examined by coding than in MATLAB R2009b and work on a Pentium 4 at 2.00 GHz, 2.92 GB computer. The examined problem cases are created as follows:

For n = 10, 20, 30, 50, 100, 200, 500 and 1000 and integer p_i for $i \in N = \{1, 2, ..., n\}$ is created by randomly choosing integers from the interval [1, 10] in our tests, problem cases of 5-10 jobs were random. The processing times were created uniformly in the range [10, 30]. The fuzzy due dates were created randomly with the backing in the range of [1, W] where W was also randomly chosen among the values of {10, 20, 30, 40, 50}.

Table 1 shows the efficiency LSAs (BCA, PSO and TS) have been approached in terms of comparable rate of

Table 1: Compares values of LSAs

n	BCA	PSO	TS
10	251142.5	252506.4	252506.4
20	4096548.35	4159320.8	4160580.3
30	25577937.48	26032106.1	26039735
50	240816409	242882432.2	245228155.9
100	21203908156.55	21815973840.8	21684262088.1
200	438296149075.3	439594332194.1	432856396491
500	31518098290787.4	31265902758315.1	31575340340511.8
1000	2977583938712640	2984336740378730	2973108658464380

Table 2: Compares times of LSAs

n	BCA	PSO	TS
10	0.025910	0.023968	0.026644
20	0.027926	0.024980	0.030838
30	0.032509	0.024928	0.036990
50	0.043891	0.025358	0.049092
100	0.070475	0.026353	0.081282
200	0.128047	0.028194	0.154579
500	0.316112	0.033878	0.450929
1000	0.739297	0.041298	1.011098

value. BCA gives the best solution for the small jobs until 100 jobs then the TS comes in the second and PSO was worst.

Table 2 shows the efficiency LSAs(BCA, PSO and TS) have been neared in cods of similar rate of times. PSO grants the better times for all jobs and TS was worst.

CONCLUSION

We have advanced a new sample to subedit the case where jobs with fuzzy due dates are to be scheduled on a single machine. The LSAs applied to solve all the large problems the result shows the robustness and elasticity of LSAs.

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