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Nuclear Structure of N_p = 10 Bosons in the Frame Work of IBA-2

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Abstract: In this research, the nuclear structure of Zr (Z = 40, N = 60) and Kr (Z = 36, N = 40) have been studied by using the IBA-2 Version of the Interacting Boson Approximation. Energy levels of the two nuclei were calculated and from the calculation, a decay scheme created. From the matrix elements, the electromagnetic properties produced. In this publication, the mixing ratios δ (E2/M1) of transitions linking the gamma, beta and ground state bands have been examined. The ratios X (E0/E2) for ΔI = 0 transitions have been estimated the predicted theoretical calculations were compared with the experimental results in respective figures and tables. It was seen that the predicted results are a good with the experimental data.

Key words: Interacting Boson Approximation (IBA-2), evenÂeven (^{100}Zr, ^{76}Kr), quadruple moments, mixing ratios, energy levels, N_p = 10 bosons

INTRODUCTION

The transitional nuclear regions 28≤Z≤50 where the nuclei $^{100}Zr_{40}$ and $^{76}_{36}Kr$ are part of, always represent a challenge to the experimental and the theoretical nuclear physicists. This is, appears from asymmetry property occurs in the decay schemes certain negions. The interacting boson approximation (Armia and Iachello, 1976, 1978; Iachillo and Arima, 1987; Pfeifer, 1988) should provide a suitable description for nuclear structure of nuclei in this region. In the current papers the starting point is choosing the nuclei which have a certain relation, like an equal boson number outside the closed shell at N = Z = 28 or 50. Both of the chosen isotopes have a total of 10 bosons outside the closed shell (N_π+N_ν = 10). The series of isotopes shown in Fig. 1 which these two nuclei belong to were amatter of a many theoretical studies using different approximations (Lalkovski and Isacker, 2009; Dejbakhsh *et al*., 1995; Jakob *et al*., 1999; Werner *et al*., 2002; Garcia-Ramos *et al*., 2004; Turkan *et al*., 2006; Gorgen *et al*., 2005). These two nuclei have a property that the lying closed to the ground state, presented the third excited state and decays by two kinds of transitions as shown in Fig. 2. The case of Zr isotope A = 100 is interested because it displays the transition from the subshell closer at N = 58 to the filling of the $h_{11/2}$ subshell and extensively deformed. The aims of the present publication listed as follows:

- To carry out systematic calculations for nuclei in this region
- Choose the best parameters to keep the calculated values very closed to experimental data and project IBM-1 calculations to IBA-2
- To describe the electromagnetic main future properties of these isotopes

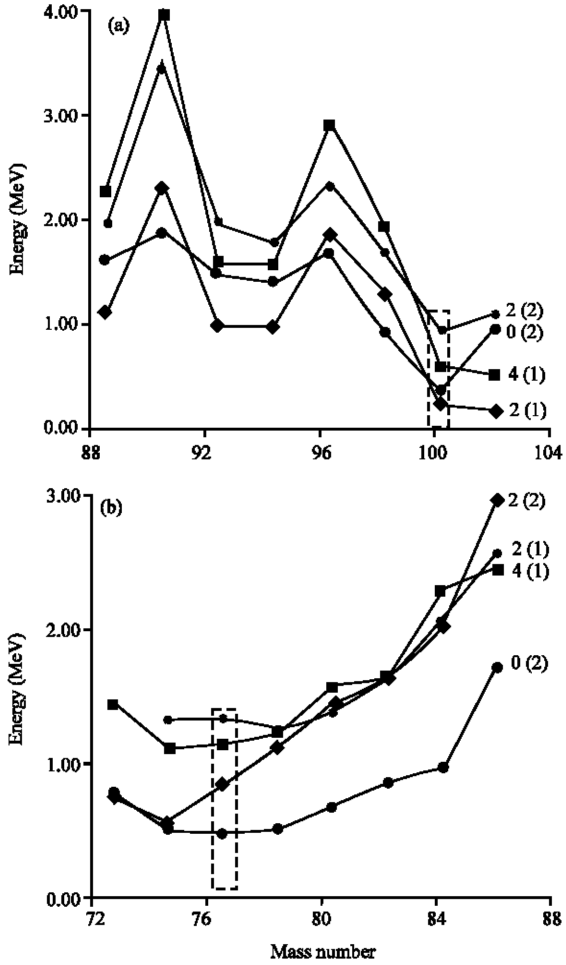

Fig. 1: a, b) Systematic of experimental energy levels of Zr and Kr isotopes where 2 (1), 4 (1), 0 (0) and is the energy leves, showing the area of study as a doted lines in both figs

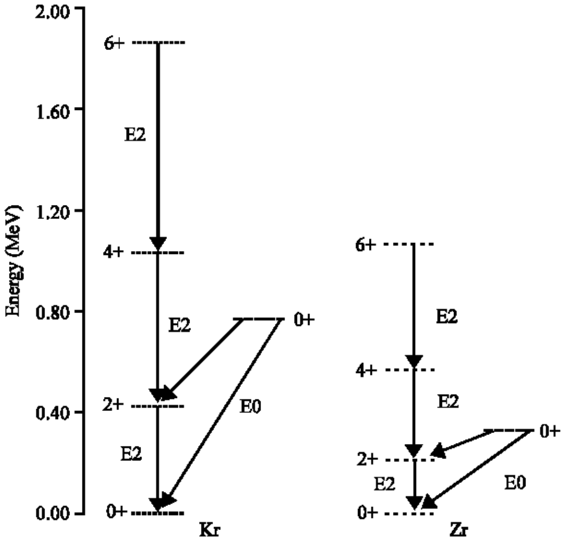

Fig. 2: The experimental decay of 0^+ states in the two nuclei where red lines and blue lines is energy levels

MATERIALS AND METHODS

The model: In the IBA-2 Hamiltonian the S and D pairs of valance nucleons have angular momentum J = 0 and J = 2, respectively. These pairs correspond intuitively to the s and d bosons, respectively. The building block of the IBA-2 is the proton bosons s_π and d_π and s_v and d_v neutron. The Hamiltonian written as (Armia and Iachello, 1976):

$$H = \varepsilon_d(n_{dv}+n_{d\pi})+\kappa(Q_v.Q_\pi)+V_{vv}+V_{\pi\pi}+M_{v\pi} \tag{1}$$

where, the dot denoted the scalar product. The first term represents the single-boson energies for neutron and protons, ε_d is the energy difference between s-and d-boson and $n_{d\rho}$ is the number of d-bosons where ρ correspond to π (proton) or v (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e. the quadrupole-quadrupole interaction between neutron and proton bosons with the strength κ. The quadrupole operator is:

$$Q_\rho = [d_\rho{}^+s_\rho+s_\rho{}^+d_\rho]^{(2)}+\chi_\rho[d_\rho{}^+d_\rho]^{(2)} \tag{2}$$

where, χ_ρ determines the structure of the quadruple operator and is determined empirically. The square bracket in Eq. 2 denotes angular momentum coupling. The terms $V_{\pi\pi}$ and V_{vv} in Eq. 1 which correspond to interaction between like-boson are sometimes included in order to improve the fit to experimental energy spectra. They are of the form:

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_L{}^\rho([d^+{}_\rho d^+{}_\rho]^{(L)}.[d_\rho d_\rho]^{(L)}) \tag{3}$$

For the simplicity sake, one can halve the number of the free parameters C_L^ρ by requiring $C_L^\pi = C_L^v = C_L$. The Majorana term, $M_{v\rho}$ which contains three parameters ξ_1-ξ_3 may be written as:

$$M_{v\pi} = \frac{1}{2}\xi_2([s_v{}^+d_\pi{}^+-d_v{}^+s_\pi{}^+]^{(2)}.[s_vd_\pi-d_vs_\pi]^{(2)})- \sum_{k=1,3}\xi_k([d_v{}^+d_\pi{}^+]^{(k)}.[d_vd_\pi]^{(k)}) \tag{4}$$

The B(E2) were calculated by using the operator:

$$T^{(E2)} = e_\pi Q_\pi+e_vQ_v \tag{5}$$

Where:

Q_ρ = The same as in Eq. 2

e_π and e_v = Boson effective charges depending on the boson Number N_ρ

They can take any value to fit the experimental results. The M1 operator is:

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2}(g_\pi L_\pi{}^{(1)}+g_vL_v{}^{(1)}) \tag{6}$$

where, g_π, g_v are the boson g-factors in units of μN and $L^{(1)} = \sqrt{10}(d^+\times\tilde{d})^{(1)}$. The numerical diagonalization has been carried out by using an improved version of the computer program NPBOS .

RESULTS AND DISCUSSION

Parameters: To locate the Zr and Kr on the IBA-2 parameters map, one has to examine a few characteristics features of these nuclei. Numbers of nucleons outside the magic closed shell at 50 are 10 proton holes (≡ 5 bosons) in Zr nuclie and 8 (≡ 4 bosons) in Krnuclie while neutron particles are 10 (≡ 5 bosons) and 12 (≡ 6 bosons) for $^{100}Zr_{40}$ and $^{76}_{36}Kr$, respectively. So, the full boson number in each isotope is N = 10. We have two nuclei with different number of valance neutrons and protons but experimental have found that $R_1 = E4_1/E2_1$ for Kr equal to 2.44 and for Zr = 2.66, $R_2 = E6_1/E2_1$ for Kr equal to 4.38 and for Zr = 4.98 and $R_3 = E0_2/E2_1$ for Kr equal to 1.82 and for Zr = 1.56. All these properties give indication that these nuclei have the same characteristics. A list of the IBA-2 parameters arearranged in Table 1. It is found that the parameters $C_{0,\ 2,\ 4}$ have a great impact on the energy of 0_2 which is closed to the 2_1 in both nuclei. For the nucleus $^{76}_{36}Kr$, the

Table 1: The IBA-2 parameters

Nucleus	N_π	N_ν	N	ε	κ	x_π	x_ν	C0, 2, 4	$\xi_1 = \xi_3, \xi_2$
$^{76}_{36}$Kr	4	6	10	0.95	-0.08	-0.8	-1.2	-1.0, 0.2, 0.05	0.27,0.055
$^{100}_{40}$Kr	5	5	10	0.81	-0.21	-0.9	-0.92	0.75,0.05,0.05	-0.225,-0.02

Table 2: Experimental and theoretical values of energy levels for ^{76}Kr and ^{100}Zr

Nucleus/Values	2_1	4_1	6_1	8_1	0_2	2_2	4_2
^{76}Kr							
Exp.	0.424	1.035	1.859	2.878	0.770	1.222	1.957
IBA-2	0.426	1.033	1.808	2.732	0.706	1.211	1.926
^{100}Zr							
Exp.	0.213	0.566	1.069	1.689	0.331	0.879	1.414
IBA-2	0.212	0.667	1.291	2.035	0.381	0.552	0.923

parameters of Dejbakhsh *et al.* (1995) were used with slight refinements to get better agreement. For $^{100}_{40}$Zr, the the parameters changed according to the energy of the 2_1 and it was hard to excellent results. However, it is found that the slight difference in the value of energy has a minimum effect on the transition matrix elements of high energy states.

Excitation energies: Calculation results of energy levels are demonstrated in Table 2. It is clear from the table that the fit for the Kr is much better than Zr. However, the approximation able to calculate the energy level 0_2 very well. The maximum deviation from experimental data is 0.345 MeV for the of 8^+ of Zr which is almost acceptable for high energy states. However, the over all results is fairly acceptable. In order to check how much collectivity in the levels scheme of both nuclei one has to calculate the ratios including the band head of the β_1, γ_1 (Bonatsos *et al.*, 2004; Maras *et al.*, 2010):

$$R_{2,0,\beta,g} = \frac{E(2^+_\beta)\text{-}E(0^+_\beta)}{E(2^+_1)},$$

$$R_{4,2,\beta,g} = \frac{E(4^+_\beta)\text{-}E(2^+_\beta)}{E(4^+_1)\text{-}E(2^+_1)},$$

$$R_{4,2,\gamma,g} = \frac{E(2^+_\beta)\text{-}E(0^+_\beta)}{E(4^+_1)\text{-}E(2^+_1)}$$

the result are listed in Table 3.

Electromagnetic transitions probabilities and moments: The matrix element of E2 operator of Eq. 5 have been estimated by using the effective charges for both nuclei, namely $e_\pi = 0.075$ eb and $e_\nu = 0.090$ eb. Some significant reduced E2 transitions explained in Table 4.

The effective charges were chosen, so that, a reasonable agreement with the transition from the first excited state obtained throughout the investigated region. The overall agreements with previous experimental data are very good.

Table 3: Comparison between the ratios $R_1 = R_{2,0,\beta,g}$, $R_2 = R_{4,2,\beta,g}$ and $R_3 = R_{4,2,\gamma,g}$

Nucleus/Values	R_1	R_2	R_3
Kr			
Experimental	1.066	1.203	0.728
Theoretical	1.185	1.178	0.831
Zr			
Experimental	2.573	1.515	1.552

Table 4: Experimental and theoretical values of electric quadruple reduced transitionprobability of ^{76}Kr and ^{100}Zr nuclei

	^{76}Kr		^{100}Zr	
Transition	Exp.	Theo.	Exp.	Theo.
B (E2; 2_1-0_1)	0.164(6)	0.164	0.21(2)	0.224
B (E2; 4_1-2_1)	0.198(20)	0.270	0.28(3)	0.294
B (E2; 6_1-4_1)	0.177(15)	0.310	0.38(6)	0.240
B (E2; 2_1-0_1)	0.009	0.005	-	0.004
B (E2; 2_2-0_1)	0.004	0.051	-	0.005
B (E2; 2_2-2_1)	-	0.175	0.184	0.004
B (E2; 0_1-2_1)	-	0.002	-	0.012
B (E2; 2_3-0_1)	-	0.030	-	0.000
B (E2; 3_1-2_1)	-	0.000	-	0.004
B (E2; 1_1-0_1)	-	0.000	-	0.000
B (E2; 3_1-1_1)	-	0.000	-	0.142
B (E2; 4_2-4_1)	0.001	0.023	-	0.003
B (E2; 4_2-2_2)	0.086(30)	0.180		0.013

Table 5: The values of electric qaudrupole moments of the isotones N = 88

Nucleus	$[\text{Q efm}^2]_{\text{exp.}}$	$[\text{Q efm}^2]_{\text{theor.}}$
$^{76}\text{Kr}_{36}$	0.82(3)	-0.99
$^{100}\text{Zr}_{40}$	0.93(9)	-1.26

Electric quadrupole moment: The 2^+_1 electric quadrupole moments were predicted. Since, it is not available, one can calculate it by using, the relation associated with the deformed nuclei written (Robinson *et al.*, 2006).

$$B(E2;0,...,2) = \frac{5}{16\pi} Q_0^2$$
$$Q(2^+_1) = -\frac{2}{7} Q_o \quad (7)$$

where, Q_o is the static quadrupole moment. The results shown in Table 5. The predicted values were always greater than one and they were having the same negative sign of experimental value.

Table 6: A comparison between available experimental of and IBA-2 prediction

Nucleus/Values	δ (E2/M1; 2_2-2_1)	δ (E2/M1; 2_3-2_1)	δ (E2/M1; 4_2-4_1)
Kr			
Experimental	+0.2(1)	-	-0.84(5)
Theoretical	-4.033	+37.52	-2.96
Zr			
Experimental	+1.0(3)	-	-
Theoretical	+0.17	-3.44	-34.56

Table 7: A comparison between available experimental data and approximation prediction of X (E_0/E_2) ratio.

Nucleus/Values	X (E0/E2;2_2-2_1)	X (E2/E2;0_2-0_1)	X (E0/E2;0_3-0_1)
Kr			
Experimental	-	-	-
Theoretical	0.035	0.211	3.56
Zr			
Experimental	-	0.060(11)	-
Theoretical	0.044	0.113	5.68

The M1 transition and mixing ratio δ (E2/M1): That predicts the reduced M1 matrix elements, using Eq. 6, one should estimate the values of g_π, g_v. Sambataro and Dieperink (1981) showed that the total experimental, g factor, of the 2_1 level have a simple linear relationship to g_π, g_v as the relation:

$$g = g_\pi \frac{N_\pi}{N_\pi+N_v} + g_v \frac{N_v}{N_\pi+N_v} \qquad (8)$$

The magnetic moment of Zr nucleus is $\mu = 0.56\,(7)$ nm and from $\mu = 2$ g, one get the relation $g_\pi+g_v = 0.56$. The estimated value of $g_\pi = 0.19$ and $g_v = 0.37$, these values used to calculate the M1 matrix elements by using Eq. 6. The M1 matrix and E2 matrix elements were linked in the calculation of the mixing ratio δ(E2/M1) using the relation:

$$\delta(E2/M1) = 0.835 E_\gamma(\mathrm{MeV}) \times \frac{T(E2)}{T(M1)} \qquad (9)$$

From Table 5, one can noticethe disagreement between experimental data and the calculated ones, especially, when δ(E2/M1; 2_2-$2_)$). However, the small experimental value, 0.2 (1), indicates a small value of E2 matrix elements while in the literatures the E2 reduced transitions bribability is 2.0 w.u and M1 is 0.024 w.u which gives the mixing ratio much >1 (like the approximation prediction). The same argument for the transition 4_2Â-4_1 which has B (E2) = 11 (4) w.u and B (M1) = 0.009 (35) w.u.. While experimental mixing ratio is -0.84 (5) as shown on Table 6.

The X(E0/E2) ratio: The E0 transition happens between two states that have similar spin and parity by transferring the energy and zero unit of angular momentum and it has no competing gamma ray. When there is a surface change in the nucleus, the E0 transition exists. For example, in the nuclear approximations where the surface is said to be fixed E0 transitions are strictly forbidden like the shell and IBM-1 approximations. Electric monopole transitions can be produced not only in 0^+--0^+ transition but also in competition with gamma multipole transition and depending on transition selection rules may compete in any ΔI = 0 decay such as a 2^+---2^+ or any $I_i = I_f$ states in the scheme. At transition energy greater than 2 $m_\circ c^2$, monopole pair production is also possible. The E0 reduced transitions probability written as (Church and Weneser, 1956):

$$B(E0; I_i\text{-}I_f) = e^2R^4\rho^2(E0) \quad I_i = I_f \qquad (10)$$

Where

e = The electronic effective charge
R = The nuclear Radius
ρ(E0) = The transition matrix element

Nevertheless, there are only limited cases where, ρ (E0) can be measured directly. In major cases we have to determine the intensity ratio of E0 to the competing E2 transition calling this as X (E0/E2) value (Sambataro and Dieperink, 1981) where they can be written as:

$$X\left({}^{E0}\!/\!_{E2}\right) = \frac{B(E0; I_i\text{-}I_f)}{B(E2; I_i\text{-}I_{f'})} \qquad (11)$$

Where:

I_f = $I_{f'}$, for $I_i \neq 0$
I_f = 0, $I_f = 0$, $I_{f'} = 2$ For $I_i = 0$

The T $^{(E0)}$ operator can be found by setting l = 0 on the IBA-2 operator (Church and Weneser, 1956):

$$\rho_{if}(E0) = \frac{Z}{R_0^{\ 2}} \sum \tilde{\beta}_{0\rho} \left\langle f \middle| d^+{}_\rho x d_\rho \middle| i \right\rangle \qquad (12)$$

where, $R_0 = 1.2A^{1/3}$ fm and ρ (E0) is a dimensionless quantity. The two parameters $\tilde{\beta}_{0\pi}, \tilde{\beta}_{0v}$ in Eq. 12 may be predestined by fitting in isotope shift, a difference in the square radius $\delta\langle r^2\rangle$ between neighboring isotopes in their ground state (Rasmussen, 1960), i.e:

$$\delta\left\langle r^2\right\rangle = \left\langle 2_1^{\ +} \middle| T_0 \middle| 2_1^{\ +}\right\rangle - \left\langle 0_1^{\ +} \middle| T_0 \middle| 0_1^{\ +}\right\rangle \qquad (13)$$

The results of calculation are presented in Table 7.

CONCLUSION

The nuclear structure of two nuclei equal in the number of bosons calculated from different closed shell, have been studied. This investigation increases the theoretical knowledge of both isotopes for energy levels and reduced transitions probabilities and their mixing. Results of IBA-2 have a satisfactory agreement with the available experimental results. It is concluded that more experimental results were required to fully investigate the level structure of these nuclei.

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