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Using of Generalized Baye's Theorem to Evaluate the Reliability of Aircraft Systems

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Abstract: In this study, some concepts of graph theory have been introduced to evaluate the reliability of complex systems inside aircraft. For the purpose of establishing the structure function to find the reliability of complex system (bridge network) as well as we will use some concepts in the probability theory and apply it on the graph theory such as conditional probability approach that depend on it Baye's theorem method which will be use it in this study to evaluate the reliability of electrical aircraft system, all figures and equations are written by wolfram mathematica programming.

Key words: Reliability polynomials, aircraft designs, optimal design, graph theory, complex, probability

INTRODUCTION

Many researchers work to evaluate the reliability of complex systems like (Beichelt and Tittmann, 2012; Hassan *et al*., 2015; Yuan, 1987). The theory of complex systems considers as an essential part in a wide assortment of disciplines going from PC science, humanism, designing and material science to atomic and populace science. A system can be represented by methods for a chart whose components (both vertices and edges) are considered as binary objects described by a working or a fizzled condition. In a two fold system, the availability among any two vertices can be communicated as a Boolean function (Beichelt and Tittmann, 2012). We will consider a bridge network as a mixed graph (Amini *et al*., 2016; Diestel, 2000; Gardner, 1999).

Reliability has turned into a key factor in the plan and task of the present expansive, complex and costly mechanical systems. Several methods such as inclusion-exclusion, Decomposition, Boolean and matrix. These methods have been used as general techniques for finding the reliability for complex networks (Hassan and Balan, 2017; Jula and Costin, 2012). Generally, they are neither great nor sufficiently successful to infer a correct reliability for a perplexing system, for example, numerous systems specified previously (Gross *et al*., 2014; Brown and Colbourn, 1992).

A new way is proposed in this study which originated from the need of genuine building configuration venture. The approach uses the thought of the existing Baye's theorem method (pivotal decomposition method), applying the conditional probability theorem is then applied conditioned on all selected pivotal undirected edges.

Some important concepts: This section gives a few essentials information about graphs and reliability polynomials. It incorporates some definitions of a graph theory, different types of graphs such as undirected, directed and mixed graph as well as some operations on graphs such as contraction, deletion (Chang and Shrock, 2003; Gross *et al*., 2014) and it includes some definitions of the reliability polynomial (Kuo and Zuo, 2003; Udriste *et al*., 2017a) structural function and Baye's theorem method (Udriste *et al*., 2017b; Shiker and Amini 2018; Yuan, 1987).

MATERIALS AND METHODS

Some definitions of graph theory

Definition 2.1.1: A graph G = (V, E) is defined by an ordered pair (V, E) where V is a nonempty set whose elements are called vertices (nodes) and E is a set pairs of distinct elements of $V(G), \left(E \subseteq \binom{V}{2}\right)$. The elements of E are called edges or (lines, arcs) of the graph G.

Definition 2.1.2: An edge of a graph G that joins a vertex to itself is called a loop.

Definition 2.1.3: If two vertices of a graph G are joined by more than one edge then these edges are called multiple edges.

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Definition 2.1.4: A graph which has neither loops nor multiple edges is called a simple graph.

Definition 2.1.5: An edge associated to a u set $\{u, v\}\epsilon E$ where u and v are vertices is called undirected edge. In other words, the edges can be traversed in both directions.

Definition 2.1.6: An edge associated to an ordered pair $(u, v)\epsilon E$ and it considered to have direction from nodes u to v is called a directed edge.

Definition 2.1.7: If all edges of the graph G are directed, then G is called a directed graph.

Definition 2.1.8: If all edges of the graph G are undirected then G is called an undirected graph.

Definition 2.1.9: A graph G with both directed and undirected edges is called a mixed graph.

Definition 2.1.10: Let G be a graph and e is an edge of a graph G, the operation that allow to delete e from G denote by G-e is called deletion.

Definition 2.1.11: Let G be a given graph if there exists a path between each pair of vertices then G called connected graph.

Basics of reliability polynomial

The structure function: Suppose a network can be divided into n edges (subnetwork). we assume that the network and each edges may only be in one of two possible states, working or failed. Thus, the state of each edges can be denoted by a random variable, X_i that takes on the value $X_i = 1$ if the edge is working for the desired time and $X_i = 0$ if the edge fails during this time (Amini *et al*., 2016; Aven and Jensen, 1999; Kuo and Zuo, 2003). In general, then, X_i is a binary random variable defined by:

$$X_i = \begin{cases} 1, \text{ if edge } i \text{ is work} \\ 0, \text{ if edge } i \text{ is failed} \end{cases} \quad (1)$$

Then, vector $X = (X_1, X_2, ..., X_n)$ represents the states of all edge and is called the edge state vector. The condition of the system is estimated by the binary random variable $\phi(X)$ where:

$$\phi(x) = \begin{cases} 1, \text{ if network is work} \\ 0, \text{ if network is failed} \end{cases} \quad (2)$$

where, ϕ is called the structure function.

Definition 2.2.1: A function ϕ of a network with n edges is a mapping $\phi: \{0, 1\}^n \rightarrow \{0, 1\}$, n is called order of the network.

Definition 2.2.2: If X and the X_i being random variables then there exist probabilities R_s and R_i such that:

$$R_s = Pr(\phi(X) = 1) = IE(\phi(X)),$$
$$R_i = Pr(X_i = 1) = IE(X_i)$$

Where

R_s = The network Reliability

R_i = The Reliability of edge i

The network reliability $R_s = Pr(\phi(X) = 1)$ is the expected value of ϕ:

$$R_s = IE(\phi(X)) \quad (3)$$

Thus, if the $X_1, X_2, ..., X_n$ are independent random variables, then R_s is a function of the reliabilities of the edges $R_1, R_2, ..., R_n$. In this case, letting $R_s = R_1, R_2, ..., R_n$, Eq. 3 can be written as:

$$R_s = \phi(R) \text{ if } R = R_1 = R_2, ..., R_n \quad (4)$$

If $(R_1, R_2, R_3, \ldots, R_n)$ are independent identical. We called Eq. 4 reliability polynomial (Brown and Colbourn, 1999) and if $R_s = (R_1, R_2, ..., R_n)$ are independent not identical then Eq. 2 called multivariate reliability polynomial.

Definition 2.2.3: Element X_i is called irrelevant if $\phi(0_i, X) = \phi(1_i, X)$ for all $X\epsilon V_n$. Otherwise, X_i is called relevant $I = 1,2, ..., n$.

Definition 2.2.4: A binary network N with structural function $\phi(X)$, $X\epsilon V_n$ is called coherent if every component of the network is relevant and $\phi(0_i, X)\leq\phi(1_i, X)$ for all $X\epsilon V_n$, i.e.; $\phi(X)$ is increasing function.

Baye's theorem method: The Baye's theorem method may be used in derivation of the structure function of a network (Udriste *et al*., 2017b; Hassan *et al*., 2015). This method relies on the enumeration of the states of a selected edge. For any i such that $1\leq i\leq n$, the following equation may be used for any network structure (Hassan and Balan, 2017; Shiker and Amini, 2018).

$$\phi(X) = X_i\phi(1_i, X)+(1-X_i)\phi(0_i, X) \quad (5)$$

where, $(a_i, X) = (X_1, X_2, ..., X_{i-1}, a, X_{i+1}, ..., X_n)$, $a\in\{0, 1\}$. Edge i is called a pivotal edge. Principally, every undirected edge can be selected as the pivotal edge (keystone edge). However, one will select the pivotal edge as undirected edge in such a way that the Boolean functions $\phi(1_i, x)$ become "as simple as possible". Then applying the pivotal decomposition Eq. 5. Of course, Eq. 2 can also be applied to $\phi(1_i, X)$. So, for example, we apply 2.5 to these Boolean functions with pivotal edges j (Diestel, 2000; Yuan, 1987).

$$\phi\big((1_i, X)\big) = X_j\phi\big((1_i, 1_j, X)\big)+(1-X_j)\phi\big((1_i, 0_j, X)\big)$$

$$\phi\big((0_i, X)\big) = X_j\phi\big((0_i, 1_j, X)\big)+(1-X_j)\phi((0_i, 0_j, X))$$

Inserting these decompositions into Eq. 5 yields:

$$\phi(X) = X_iX_j\phi((1_i, 1_j, X))+X_i(1-X_j)\phi\big((1_i,0_j,X)\big)+ (1-X_i)X_j\phi(0_i,1_j,X)+(1-X_i)(1-X_j)\phi(0_i,0_j,X) \quad (6)$$

The last formula of $\phi(X)$; all terms are disjoint. Proceeding in this way till all the variables X_i in X are fixed by 0 or 1. By applying Eq. 3 on both sides of Eq. 6 obtain a decomposition formula with pivotal edge i for the system reliability:

$$R_s = R_ih(1_i,R)+(1-R_i)h(0_i,R), i = 1, 2, ..., n$$

With:

$$h(1_i, R) = E(\phi(a_iX)) \text{ for } a\in\{0, 1\}$$

From Eq. 6, we get:

$$R_s = R_iR_jh(1_i, 1_j, R))+R_i(1-R_j)h\big((1_i,0_j,R)\big)+ (1-R_i)R_jh(0_i,1_j,R)+(1-R_i)(1-R_j)h(0_i,0_j,R) \quad (7)$$

RESULTS AND DISCUSSION

Bridge structure: In some engineering systems (network), units may be connected in a bridge configuration as shown in Fig. 1 which represent a bridge network (complex system) in aircraft (Hassan *et al*., 2016; Jula and Costin 2012).

Consider a bridge network with the Reliability Block Diagram (RBD) given by Fig. 1 which represent a simple, connected and mixed graph, all edges are directed unless C and d are undirected edges with $X = (X_a, X_b, X_c, X_d, X_e, X_f, X_g, X_h)$. In order to find the structure function, we apply Eq 6 with pivotal edges c and d.

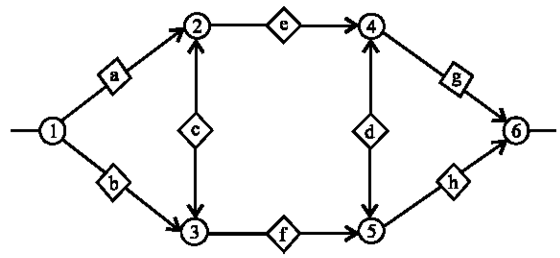

Fig. 1: A bridge network

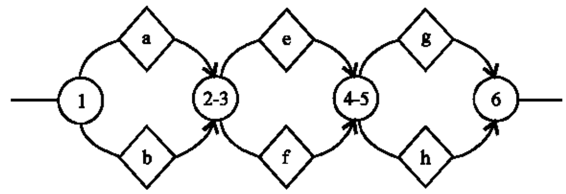

Fig. 2: A bridge subnetwork G with nodes 2 = 3 and 4 = 5

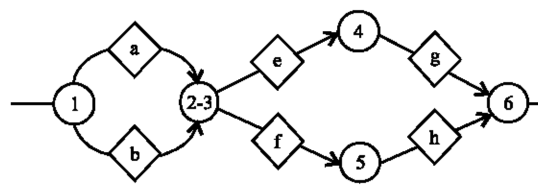

Fig.3: A bridge subnetwork G-{D} with nodes 2 = 3

The state vector $(1_c, 1_d, X)$ implies that there is c always a connection between nodes 2 and 3 and d connection between nodes 4 and 5 in the bridge structure. Hence, the RBD belonging to $\phi(1_c, 1_d, X)$ arises from the bridge structure by fusing nodes 2 and 3 to the node 2-3 and fusing nodes 4 and 5 to the node 4-5. When doing this, edges c and d becomes a loop which will be deleted Fig. 2.

It is becoming a subnetwork, so, the structure function is a parallel-series network given by:

$$\phi(1_c,1_d,X) = [1-(1-X_a)(1-X_b)][1-(1-X_e)(1-X_f)][1-(1-X_g)(1-X_h)]$$

State vector $(1_c, 1_d, X)$ implies that edge c is always a connection between nodes 2 and 3 and d not available. Hence, the RBD belonging to $\phi(1_c, 1_d, X)$ arises from the bridge structure by fusing nodes 2 and 3 to the node 2-3, c becomes a loop which will be deleted with d Fig. 3.

It is becoming a subnetwork, so, the structure function is a parallel-series network given by:

$$\phi(1_c,0_d,X) = [1-(1-X_a)(1-X_b)]\big[1-(1-X_eX_g)(1-X_fX_h)\big]$$

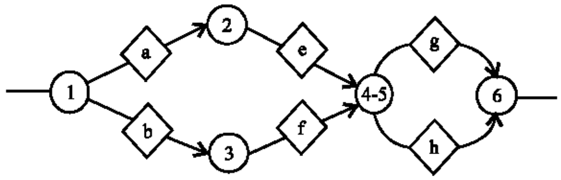

Fig. 4: A bridge subnetwork G-{c} with nodes 4 = 5

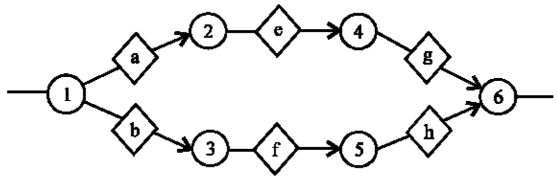

Fig. 5: A bridge subnetwork G-{c, d}

State vector (1_c, 1_d, X) implies that edge c not available and d is always a connection between nodes 4 and 5. Hence, the RBD belonging to $\phi(1_c, 1_d, X)$ arises from the bridge structure by deleting edge c and fusing nodes 4 and 5 to the node 4-5, d becomes a loop which will be deleted Fig. 4.

It is becoming a subnetwork, so, the structure function is a parallel-series network given by:

$$\phi(0_c, 1_E, X) = [1\text{-}(1\text{-}R_1R_5)(1\text{-}R_2R_6)][1\text{-}(1\text{-}R_7)(1\text{-}R_8)]$$

State vector (1_c, 1_d, X) implies that edges c and d not available. Hence, the RBD belonging to ϕ (1_c, 1_d, X) arises from the bridge structure by deleting edges c and d Fig. 5.

It is becoming a subnetwork, so, the structure function is a series-parallel network given by:

$$\phi(1_c, 0_d, X) = [1\text{-}(1\text{-}X_aX_eX_g)(1\text{-}X_bX_fX_h)$$

Hence, by Eq. 2, we can find the structure function of bridge network:

$$\phi(x) = X_cX_d[1\text{-}(1\text{-}X_a)(1\text{-}X_b)][1\text{-}(1\text{-}X_e)(1\text{-}X_f)] [1\text{-}(1\text{-}X_g)(1\text{-}X_h)]+X_c(1\text{-}X_d)[1\text{-}(1\text{-}X_a)(1\text{-}X_b)] [1\text{-}(1\text{-}X_eX_g)(1\text{-}X_fX_h)]+(1\text{-}X_c)X_d [1\text{-}(1\text{-}R_1R_5)(1\text{-}R_2R_6)][1\text{-}(1\text{-}R_7)(1\text{-}R_8)]+(1\text{-}X_c) (1\text{-}X_d)[1\text{-}(1\text{-}X_aX_eX_g)(1\text{-}X_bX_fX_h)]$$

Equation 2 with structure function of bridge network, we get the reliability network which represented the multivariate reliability polynomial.

$$R_s = R_aR_eR_g+R_bR_cR_eR_g\text{-}R_aR_bR_cR_eR_g+ R_bR_dR_fR_g+R_aR_cR_dR_fR_g\text{-}R_aR_bR_cR_dR_fR_g\text{-} R_aR_bR_dR_eR_fR_g\text{-}R_aR_cR_dR_eR_fR_g\text{-} R_bR_cR_dR_eR_fR_g+2R_aR_bR_cR_dR_eR_fR_g+ R_aR_dR_eR_h+R_bR_cR_dR_eR_h\text{-}R_aR_bR_cR_dR_eR_h+ R_bR_fR_h+R_aR_cR_fR_h\text{-}R_aR_bR_cR_fR_h\text{-} R_aR_bR_dR_eR_fR_h\text{-}R_aR_cR_dR_eR_fR_h\text{-} R_bR_cR_dR_eR_fR_h+2R_aR_bR_cR_dR_eR_fR_h\text{-} R_aR_dR_eR_gR_h\text{-}R_bR_cR_dR_eR_gR_h+ R_aR_bR_cR_dR_eR_gR_h\text{-}R_bR_dR_fR_gR_h\text{-} R_aR_cR_dR_fR_gR_h+R_aR_bR_cR_dR_fR_gR_h\text{-} R_aR_bR_eR_fR_gR_h\text{-}R_aR_cR_eR_fR_gR_h\text{-} 2R_aR_bR_dR_eR_fR_gR_h+2R_aR_cR_dR_eR_fR_gR_h+ 2R_bR_cR_dR_eR_fR_gR_h\text{-}4R_aR_bR_cR_dR_eR_fR_gR_h$$

If (R_a, R_b, ..., R_h) are independent identical, get the reliability polynomial.

$$R_s = 2R^3+4R^4\text{-}2R^5\text{-}13R^6+14R^7\text{-}4R^8$$

CONCLUSION

We can apply this technique to evaluate the reliability of all multi-bridge systems. This study can be considered as one of graph theory applications on probability theory, i.e., two pivotal undirected edges are selected and conditional probability principle is then applied in order to compute the reliability for a given system.

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