Journal of Engineering and Applied Sciences 13 (Special Issue 14): 10888-10895, 2018
ISSN: 1816-949X

New Travelling Solitary Wave Solutions for an Evolution Equation by Three Schemes

Anwar Ja'afar Mohamad Jawad and Mahmood J. Abu-Al Shaeer
Al-Rafidain University College, 10014 Baghdad, Iraq

Abstract: In this study, we establish new travelling wave Solitons solutions by using three methods. These methods are Tan-Cot method, Extended Tanh-Coth method and modified simple equation method. These methods are used to obtain new solitary wave trigonometric and hyperbolic functions solutions for the generalized Schamel-Korteweg-de Vries (S-KdV) equation. These methods have been successfully applied to construct new solitary solutions as illustrated in figures. The three methods are efficient and reliable for solving great many nonlinear partial differential equations in Physics.

Key words: Partial, methods, reliable, solitary, hyperbolic, solutions

INTRODUCTION

The generalized Schamel-Korteweg-de Vries (S-KdV) equation which contains a root of degree n nonlinearity is considered as a valid model (Washimi and Taniuti, 1996). In the study of ion-acoustic waves in plasma and dusty plasma, the propagation of ion-acoustic wave in different types of plasma has been investigated extensively. The study of different methods for the solution of evolution equations has enjoyed from both theoretical and practical of powerful methods (Malfliet, 1992; Jawad, 2013; Wazwaz, 2005; El-Wakil and Abdou, 2007; Fan, 2000; Xia *et al*., 2001; Yusufoglu and Bekir, 2006; Inc and Ergut, 2005; Sheng, 2006; Feng, 2002; Ding and Li, 1996; Mitchell and Griffiths, 1980; Parkes and Duffy, 1996; Jawad *et al*., 2010).

In this study, three methods are applied. These methods are the Tan-Cot method, extended Tanh-Coth method and MSEM to solve the following (S-KdV) equation given by Al-Atawi (2017) and Yang and Tang (2015):

$$u_t+\left(\alpha+\delta u^{\frac{1}{n}}\right)u^{\frac{1}{n}}u_x+\gamma u_{xxx}=0,\ n\neq 0,\ -1,\ -2 \tag{1}$$

Where:

u(x, t) = The perturbed ion density in plasma with non-isothermal electrons

α, δ and γ = Real constants

Equation 1 reduces to the schamel KdV equation for n = 2, δ = 0 and the mKdV equation follows for n = 1/2, δ = 0.

MATERIALS AND METHODS

The traveling wave solution: Let consider the non-linear PDEs in the form:

$$F\left(u,\ u_{t,}\ u_{x,},u_{y,}\ u_{xy,},\ u_{tt,}\ u_{xx},\ u_{tx,},\ \ldots,\right)=0 \tag{2}$$

In Eq. 1, u(x, t) is the travelling solitary wave solution of non-linear PDE. We assume the transformation u(x, t) = fξ where, ξ = x-λt. Then, we use the following changes (Eq. 3):

$$\frac{\partial}{\partial t}(.)=-\lambda\frac{d}{d\xi}(.),\ \frac{\partial}{\partial x}(.)=\frac{d}{d\xi}(.),\ \frac{\partial^2}{\partial x^2}(.)=\frac{d^2}{d\xi^2}(.) \tag{3}$$

therefore Eq. 1 transforms to non-linear Ordinary Differential Equation ODE Eq. 4:

$$Q\left(f,\ f',\ f'',\ f''',\ \ldots,\right)=0 \tag{4}$$

Equation 4 is then integrated with zero constants. Then, the traveling wave solution for S-KdV Eq. 1 when we first use the wave variable ξ = x-λt is:

$$-\lambda u'+\left(\alpha u^{\frac{1}{n}}u'+\delta u^{\frac{2}{n}}u'\right)+\gamma u'''=0 \tag{5}$$

where, λ is a constant and u' = du/dξ. Integrating Eq. 5 once with zero constant of integration, we can find:

$$-\lambda u+\left(\frac{n\alpha}{1+n}u^{\frac{1+n}{n}}+\frac{n\delta}{2+n}u^{\frac{2+n}{n}}\right)+\gamma u''=0 \tag{6}$$

Corresponding Author: Anwar Ja'afar Mohamad Jawad, Al-Rafidain University College, 10014 Baghdad, Iraq

The Tan-Cot function method: Jawad (2012a, b) proposed in the first time that the solution of non-linear ODEs is of the form Eq. 7 and 8:

$$f(\xi) = A\tan^{\beta}(\mu\xi),\ |\xi|\leq\frac{\pi}{2\mu} \quad (7)$$

or Eq. 8:

$$f(\xi) = A\cot^{\beta}(\mu\xi),\ |\xi|\leq\frac{\pi}{2\mu} \quad (8)$$

where, A, μ, β are parameters to be calculated, μ and λ are the wave number and the wave speed, respectively. For Eq. 9:

$$\begin{aligned} &f(\xi) = A\tan^{\beta}(\mu\xi)\\ &f' = A\beta\mu\left[\tan^{(\beta-1)}(\mu\xi)+\tan^{(\beta+1)}(\mu\xi)\right]\\ &f'' = A\beta\mu^2\begin{bmatrix}(\beta\text{-}1)\tan^{(\beta-2)}(\mu\xi)+2\beta\tan^{\beta}(\mu\xi)+\\(\beta+1)\tan^{(\beta+2)}(\mu\xi)\end{bmatrix}\end{aligned} \quad (9)$$

and their derivatives or for Eq. 10:

$$\begin{aligned} &f(\xi) = A\cot^{\beta}(\mu\xi)\\ &f' = -A\beta\mu\left[\cot^{(\beta-1)}(\mu\xi)+\cot^{(\beta+1)}(\mu\xi)\right]\\ &f'' = A\beta\mu^2\begin{bmatrix}(\beta\text{-}1)\cot^{(\beta-2)}(\mu\xi)+2\beta\cot^{\beta}\\(\mu\xi)+(\beta+1)\cot^{(\beta+2)}(\mu\xi)\end{bmatrix}\end{aligned} \quad (10)$$

and so on. Now, to apply tan function substituting Eq. 9 into Eq. 6 yields:

$$\begin{aligned} &-\lambda\ A\tan^{\beta}(\mu\xi)+\frac{n\alpha}{1+n}A^{\frac{1+n}{n}}\tan^{\frac{1+n}{n}\beta}(\mu\xi)+\\ &\frac{n\delta}{2+n}A^{\frac{2+n}{n}}\tan^{\frac{2+n}{n}\beta}(\mu\xi)+\\ &\begin{bmatrix}\gamma\ A\beta\mu^2(\beta\text{-}1)\tan^{(\beta-2)}(\mu\xi)+2\gamma A\beta^2\mu^2\tan^{\beta}\\(\mu\xi)+\gamma\ A\beta\mu^2(\beta+1)\tan^{(\beta+2)}(\mu\xi)\end{bmatrix}=0\end{aligned} \quad (11)$$

When equate the coefficients of each pair of the tan functions, system of algebraic equation:

$$\begin{aligned} &(\beta+2) = \frac{2+n}{n}\beta\\ &(\beta+1)\neq 0\\ &-\lambda A = 2\gamma A\beta^2\mu^2\\ &\frac{n\delta}{2+n}A^{\frac{2+n}{n}} = \gamma A\beta\mu^2(\beta+1)\\ &\frac{n\alpha}{1+n}A^{\frac{1+n}{n}} = 0\\ &\gamma A\beta\mu^2(\beta\text{-}1) = 0\end{aligned} \quad (12)$$

on solving Eq. 12 yields:

Case 1:

$$\beta = n,\ \mu = \frac{1}{n}\left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}},\ A = \left(\gamma\mu^2\frac{(n+1)(n+2)}{\delta}\right)^{\frac{n}{2}},$$
$$\alpha = 0,\ n = 1$$

then, Eq. 1 reduced to Eq. 13:

$$u_t+\delta u^2u_x+\gamma u_{xxx} = 0 \quad (13)$$

with:

$$\beta = 1,\ \mu = \left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}},\ A = \left(-\frac{3\lambda}{\delta}\right)^{\frac{1}{2}}$$

therefore, Eq. 14:

$$u(x,t) = \left(-\frac{3\lambda}{\delta}\right)^{\frac{1}{2}}\tan\left(\left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x\text{-}\lambda t)\right) \quad (14)$$

where, λ<0, γ>0, δ>0. For λ = -2, γ = δ = 1, Eq. 14 becomes:

$$u(x,t) = (6)^{\frac{1}{2}}\tan(x+2t) \quad (15)$$

Figure 1 illustrates the solitary wave in equation in Eq. 15 for -10≤x≤10; 0≤t≤1.

Case 2: For λ>0:

$$u(x,t) = \left(\frac{3\lambda}{\delta}\right)^{\frac{1}{2}}\tanh\left(\left(\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x\text{-}\lambda t)\right) \quad (16)$$

where, γ>0, δ>0. For λ = 2, γ = δ = 1, Eq. 16 becomes:

$$u(x,t) = (6)^{\frac{1}{2}}\tanh(x-2t) \quad (17)$$

Figure 2 represents the solitary wave in Eq. 17 for -10≤x≤10; 0≤t≤1.

Case 3: For n = 2, α = 0. Equation reduces to the KdV:

$$u_t+\delta uu_x+\gamma u_{xxx} = 0 \quad (18)$$

and:

$$\beta = 2,\ \mu = \frac{1}{2}\left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}},\ A = \frac{-3\lambda}{2\delta}$$

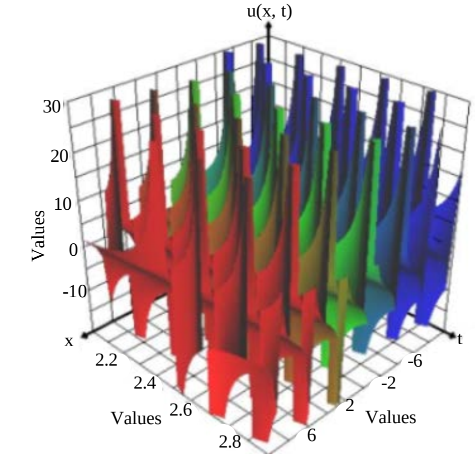

Fig. 1: The solitary wave in Eq. 15

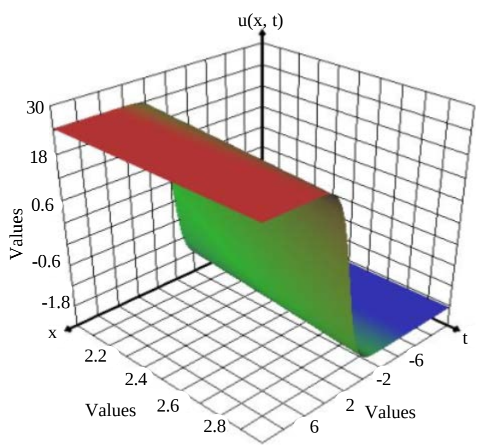

Fig. 2: The solitary wave in Eq. 17

then:

$$u(x,t)=\frac{-3\lambda}{2\delta}\tan^2\left(\frac{1}{2}\left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x-\lambda t)\right) \tag{19}$$

where, $\lambda<0$, $\gamma>0$, $\delta>0$. For $\lambda=-2$, $\gamma=\delta=1$, Eq. 19 becomes:

$$u(x,t)=3\tan^2\left(\frac{1}{2}(x+2t)\right) \tag{20}$$

Figure 3 represents the solitary wave in equation in Eq. 20, for $-10\leq x\leq 10;\ 0\leq t\leq 1$.

Case 4: For $\lambda>0$:

$$u(x,t)=\frac{3\lambda}{2\delta}\tanh^2\left(\frac{1}{2}\left(\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x-\lambda t)\right) \tag{21}$$

where, $\gamma<0$, $\delta>0$, for $\lambda=2$, $\gamma=\delta=1$, Eq. 21 becomes:

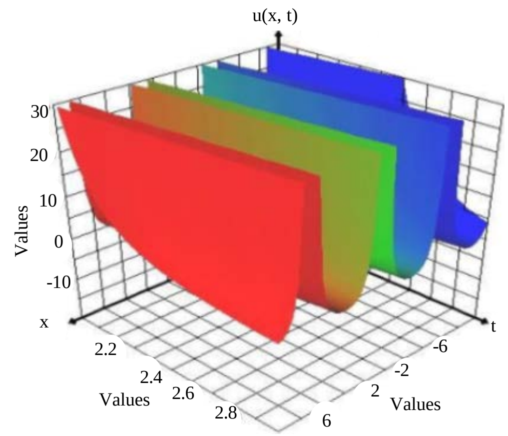

Fig. 3: The solitary wave in Eq. 20

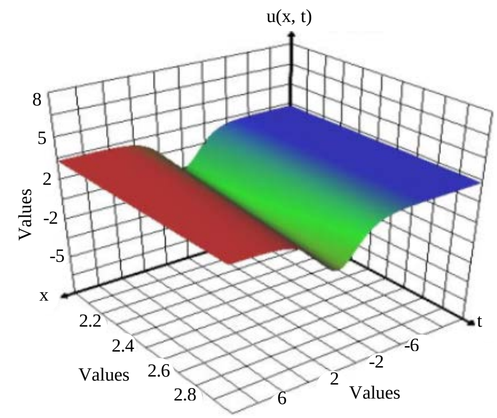

Fig. 4: The solitary wave in Eq. 22

$$u(x,t)=3\tanh^2\left(\frac{1}{2}(x-2t)\right) \tag{22}$$

Figure 4 represents the solitary wave in Eq. 2 for $-10\leq x\leq 10;\ 0\leq t\leq 1$.

Case 5: Now, to apply Cot function substituting Eq. 10 into Eq. 6 and if n = 2, $\alpha=0$ yields:

$$\begin{aligned}&-\lambda A\cot^{\beta}(\mu\xi)+\frac{\delta}{2}A^2\cot^{2\beta}(\mu\xi)+\\&\left[\begin{aligned}&\gamma A\beta\mu^2(\beta-1)\cot^{(\beta-2)}(\mu\xi)+2\gamma A\beta^2\mu^2\cot^{\beta}\\&(\mu\xi)+\gamma A\beta\mu^2(\beta+1)\cot^{(\beta+2)}(\mu\xi)\end{aligned}\right]=0\end{aligned} \tag{23}$$

Equating the coefficients of each pair of the Cot functions, we find:

$$\begin{aligned}&(\beta+2)=2\beta\\&\frac{\delta}{2}A^2=A\beta\mu^2(\beta+1)\\&2\gamma A\beta^2\mu^2=-\lambda A\end{aligned} \tag{24}$$

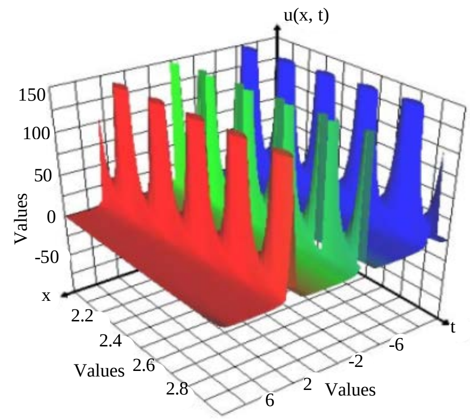

Fig. 5: The solitary wave in Eq. 26

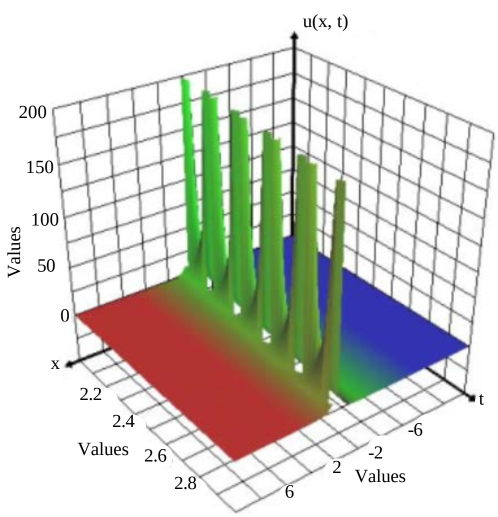

Fig. 6: The solitary wave in Eq. 28

Therefore, when solve system (Eq. 24), we get:

$$u(x,t) = \frac{-3\lambda}{16\gamma}\cot^2\left(\frac{1}{2}\left(-\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x-\lambda t)\right) \quad (25)$$

where $\lambda<0$, $\gamma>0$, $\delta>0$. For $\lambda = -2$, $\gamma = \delta = 1$, Eq. 25 becomes:

$$u(x,t) = \frac{3}{8}\cot^2\left(\frac{1}{2}(x+2t)\right) \quad (26)$$

Figure 5 represents the solitary wave in Eq. 26 for $-10\le x\le 10$; $0\le t\le 1$.

Case 6: For $\lambda>0$:

$$u(x,t) = \frac{3\lambda}{16\gamma}\coth^2\left(\frac{1}{2}\left(\frac{\lambda}{2\gamma}\right)^{\frac{1}{2}}(x-\lambda t)\right) \quad (27)$$

for $\lambda = 2$, $\gamma = \delta = 1$, Eq. 27 becomes:

$$u(x,t) = \frac{3}{8}\coth^2\left(\frac{1}{2}(x-2t)\right) \quad (28)$$

Figure 6 represents the solitary wave in Eq. 28 for $-10\le x\le 10$; $0\le t\le 1$.

RESULTS AND DISCUSSION

The extended Tanh-Coth method: The method consists of using the new independent variable (Jawad, 2012a, b; Jawad *et al*., 2017):

$$Y = \tanh(\xi) \quad (29)$$

that leads to the change of variables:

$$\frac{dU}{d\xi} = \left(1-Y^2\right)\frac{dU}{dY} \quad (30)$$

$$\frac{d^2U}{d\xi^2} = -2Y\left(1-Y^2\right)\frac{dU}{dY}+\left(1-Y^2\right)^2\frac{d^2U}{dY^2} \quad (31)$$

Now, the solution is expressed in the form Eq. 32:

$$U(\xi) = \sum_{i=0}^{m} a_i Y^i + \sum_{i=1}^{m} b_i Y^{-i} \quad (32)$$

We balance in Eq. 6, U2+n/n with (d^2U/dY^2) to obtain: (2+n/n) m = m+2, then, m = n.

Case 1: For n = 1. Equation 6 becomes:

$$-\lambda u+\frac{\alpha}{2}u^2+\frac{\delta}{3}u^3+\gamma u'' = 0 \quad (33)$$

The solution by the extended Tanh-Coth method takes the following finite expansion:

$$u = a_0+a_1Y+b_1Y^{-1} \quad (34)$$

$$\frac{du}{dY} = a_1-b_1Y^{-2} \quad (35)$$

a_0, a_1 and b_1 are to be calculated. Substituting Eq. 34 and 35 into Eq. 33 will get:

$$-\lambda\left(a_0+a_1Y+b_1Y^{-1}\right)+\frac{\alpha}{2}\begin{pmatrix}a_0^2+2a_1b_1+2a_0a_1Y+\\ a_1^2Y^2+2a_0b_1Y^{-1}+b_1^2Y^{-2}\end{pmatrix}+$$
$$\frac{\delta}{3}\begin{pmatrix}a_0^3+3a_0^2a_1Y+3a_0a_1^2Y^2+a_1^3Y^3+3\begin{pmatrix}a_0^2Y^{-1}+2a_0a_1+\\ a_1^2Y\end{pmatrix}\\ b_1+3\left(a_0Y^{-2}+a_1Y^{-1}\right)b_1^2+b_1^3Y^{-3}+\end{pmatrix}+$$
$$-2\gamma\left(a_1Y-a_1Y^3-b_1Y^{-1}+b_1Y\right)+2b_1\gamma\left(Y^{-3}-2Y^{-1}+Y\right)=0 \tag{36}$$

The expressions at Y^i, (i = -3, -2, -1, 0, 1, 2, 3) are equal to zero, then, we have the following system of algebraic equations:

$$\begin{aligned}
&Y^{-3}:\ \left(\delta b_1^2+6\gamma\right)b_1=0\\
&Y^{-2}:\ \left(\alpha+2\delta a_0\right)b_1^2=0\\
&Y^{-1}:\ \left(-\lambda+\alpha a_0+\delta\left(a_0^2+a_1b_1\right)-2\gamma\right)b_1=0\\
&Y^{0}:\ -\lambda a_0+\frac{\alpha}{2}\left(a_0^2+2a_1b_1\right)+\frac{\delta}{3}a_0\left(a_0^2+6a_1b_1\right)=0\\
&Y^{1}:\ \left(-\lambda+\alpha a_0+\delta\left(a_0^2+a_1b_1\right)-2\gamma\right)a_1=0\\
&Y^{2}:\ \left(\alpha+2\delta a_0\right)a_1^2=0\\
&Y^{3}:\ \left(\delta a_1^2+6\gamma\right)a_1=0
\end{aligned} \tag{37}$$

Solve system (Eq. 37), we get:

$$\gamma=-\frac{\alpha^2}{96\delta},\ =-\frac{\alpha^2}{6\delta},\ a_0=-\frac{\alpha}{2\delta},\ a_1=b_1=-\frac{\alpha}{4\delta} \tag{38}$$

Therefore:

$$u(x,t)=-\frac{\alpha}{4\delta}\left\{2+\left[\begin{matrix}\tanh\left(x+\frac{\alpha^2}{6\delta}t\right)+\\ \coth\left(x+\frac{\alpha^2}{6\delta}t\right)\end{matrix}\right]\right\} \tag{39}$$

for $\delta=\alpha=1$:

$$u(x,t)=-\frac{1}{4}\left\{2+\left[\tanh\left(x+\frac{1}{6}t\right)+\coth\left(x+\frac{1}{6}t\right)\right]\right\} \tag{40}$$

Figure 7 represents the solitary wave in Eq. 40 for $-10\le x\le 10$; $0\le t\le 1$.

Case 2: For n = 2, α = 0, Eq. 6 reduces to the KdV equation:

$$-\lambda\ u+\frac{\delta}{2}u^2+\gamma u''=0 \tag{41}$$

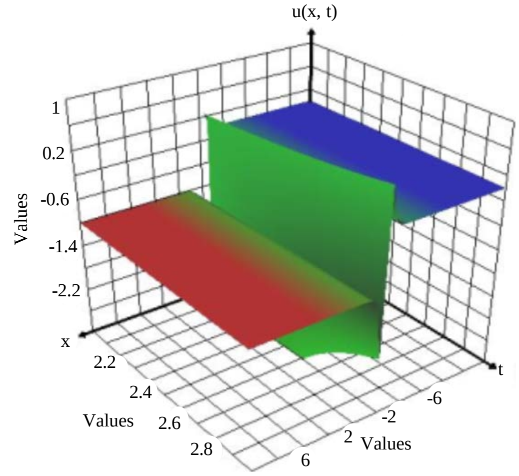

Fig. 7: The solitary wave in Eq. 40

Then, the extended Tanh-Coth method admits the use of the finite expansion for Eq. 42-44:

$$u=a_0+a_{001}Y+b_1Y^{-1}+a_2Y^2+b_2Y^{-2} \tag{42}$$

$$\frac{du}{dY}=a_1-b_1Y^{-2}+2a_2Y-2b_2Y^{-3} \tag{43}$$

$$\frac{d^2u}{dY^2}=2b_1Y^{-3}+2a_2+6b_2Y^{-4} \tag{44}$$

a_0, a_1, b_1, a_2, b_2 are to be determined. Substituting Eq. 42-44 into Eq. 41 will get:

$$-\lambda\left(a_0+a_1Y+b_1Y^{-1}+a_2Y^2+b_2Y^{-2}\right)+\frac{\delta}{2}$$
$$\begin{pmatrix}\left[a_0^2+2a_0a_1Y+a_1^2Y^2+\left(2a_0b_1Y^{-1}+2a_1b_1\right)+b_1^2Y^{-2}\right]+\\ \left[\begin{matrix}2a_0a_2Y^2+2a_0b_2Y^{-2}+2a_1a_2Y^3+2a_1b_2Y^{-1}+\\ 2a_2b_1Y+2b_1b_2Y^{-3}+a_2^2Y^4+2a_2b_2+b_2^2Y^{-4}\end{matrix}\right]+\end{pmatrix}$$
$$\gamma\begin{pmatrix}a_1\left(-2Y+2Y^3\right)-b_1\left(-2Y^{-1}+2Y\right)+2a_2\\ \left(-2Y^2+2Y^4\right)-2b_2\left(-2Y^{-2}+2\right)+2b_1\\ \left(Y^{-3}-2Y^{-1}+Y\right)+2a_2\\ \left(1-2Y^2+Y^4\right)+6b_2\left(Y^{-4}-2Y^{-2}+1\right)\end{pmatrix}=0 \tag{45}$$

Equating to zero all expressions at Y^i (i = -4, -3, -2, -1, 0, 1, 2, 3, 4), we get the following system of algebraic equation:

$$Y^{-4}:\left(\delta b_2+12\gamma\right)b_2=0$$
$$Y^{-3}:\left(\delta b_2+2\gamma\right)b_1=0$$

$$\begin{aligned}
&Y^{-2}: -2\lambda b_2+\delta\left(b_1^{\,2}+2a_0b_2\right)-16\gamma b_2=0\\
&Y^{-1}: -\lambda b_1+\delta\left(a_0b_1+a_1b_2\right)-2\gamma b_1=0\\
&Y^{0}: -\lambda a_0+\frac{\delta}{2}\left[a_0^{\,2}+2a_1b_1\right]+\delta a_2b_2+2\gamma\left(a_2+b_2\right)=0\\
&Y^{1}: -\lambda a_1+\delta\left(a_0a_1+a_2b_1\right)-2\gamma a_1=0\\
&Y^{2}: -2\lambda a_2+\delta\left(a_1^{\,2}+2a_0a_2\right)-16\gamma a_2=0\\
&Y^{3}: \left(\delta a_2+2\gamma\right)a_1=0\\
&Y^{4}: \left(\delta a_2+12\gamma\right)a_2=0
\end{aligned}\tag{46}$$

Solving the system of Eq. 46, we get:

Family 1:

$$\lambda=26\gamma,\ a_0=\frac{40\gamma}{\delta},\ a_1=b_1=\frac{12\gamma}{\delta},\ a_2=b_2=-\frac{12\gamma}{\delta}\tag{47}$$

$$u(x,t)=\frac{\gamma}{\delta}\left(40+12\left[\begin{array}{l}\tanh(x-26\gamma t)+\coth(x-26\gamma t)\\ -\tanh^2(x-26\gamma t)-\coth^2(x-26\gamma t)\end{array}\right]\right)\tag{48}$$

for $\gamma=\delta=-1$, Eq. 48 becomes:

$$u(x,\ t)=\left(40+12\left[\begin{array}{l}\tanh(x+26t)+\coth(x+26t)\\ -\tanh^2(x+26t)-\coth^2(x+26t)\end{array}\right]\right)\tag{49}$$

Figure 8 represents the solitary wave in Eq. 49 for $-1\le x\le 1;\ 0\le t\le 1$.

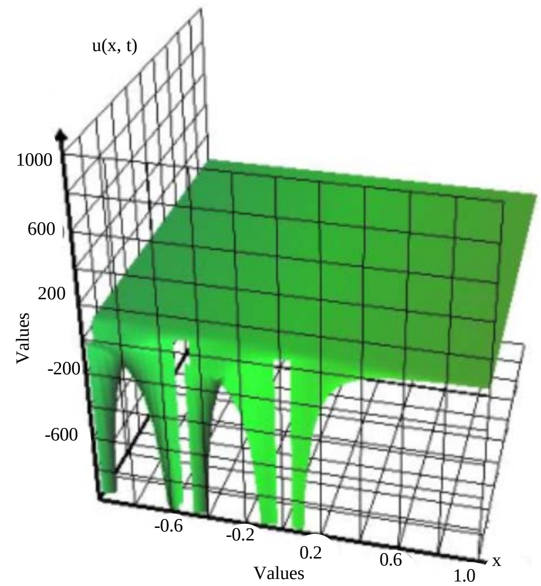

Fig. 8: The solitary wave in Eq. 49

Family 2:

$$\begin{aligned}
&\lambda=i2\sqrt{6}\gamma,\ a_0=\left(2+i\sqrt{6}\right)\frac{2\gamma}{\delta}\\
&a_1=b_1=\mp i\frac{4\gamma}{\delta},\ a_2=b_2=-\frac{2\gamma}{\delta}
\end{aligned}\tag{50}$$

Therefore, Eq. 51:

$$u(x,\ t)=\frac{\gamma}{\delta}\left(\begin{array}{l}2\left(2+i\sqrt{6}\right)+\\ \left[\begin{array}{l}4\left(\tan\left(x-i2\sqrt{6}\gamma t\right)+\cot\left(x-i2\sqrt{6}\gamma t\right)\right)+\\ 2\left(\tanh^2\left(x-i2\sqrt{6}\gamma t\right)+\coth^2(x-i2\sqrt{6}\gamma t)\right)\end{array}\right]\end{array}\right)\tag{51}$$

The Modified Simple Equation Method (MSEM): Finally, we apply MSEM proposed by Jawad *et al.* (2010). We also apply it on n = 2 the more general Eq. 1. The idea is to consider the function $u(\xi)$ in the following form, For $n=2$, $\alpha=0$, i.e., to the KdV equation in Eq. 41:

$$u(\xi)=A_0+A_1\frac{\psi_\xi}{\psi}+A_2\left(\frac{\psi_\xi}{\psi}\right)^2\tag{52}$$

where, Ψ is the unknown function and $\Psi\neq 0$. Then, the first and second derivative of $u(\xi)$ are given by:

$$u_\xi=\left(A_1+2A_2\frac{\psi_\xi}{\psi}\right)\left(\frac{\psi\psi_{\xi\xi}-\psi_\xi^{\,2}}{\psi^2}\right)\tag{53}$$

$$u_{\xi\xi}=A_1\left(\frac{\psi^2\psi_{\xi\xi\xi}-3\psi\psi_{\xi\xi}\psi_\xi+2\psi_\xi^{\,3}}{\psi^3}\right)+2A_2\left(\begin{array}{l}\dfrac{\psi^3\psi_\xi\psi_{\xi\xi\xi}-3\psi^2\psi_\xi^{\,2}\psi_{\xi\xi}+2^2\psi\psi_\xi^{\,4}}{\psi^5}+\\ \dfrac{\psi^2\psi_{\xi\xi}^{\,2}-2\psi\psi_{\xi\xi}\psi_\xi^{\,2}+\psi_\xi^{\,4}}{\psi^4}\end{array}\right)\tag{54}$$

substitute Eq. 52-54 in Eq. 41 yields:

$$-\lambda\left(A_0+A_1\frac{\psi_\xi}{\psi}+A_2\frac{\psi_\xi^{\,2}}{\psi^2}\right)+\frac{\delta}{2}\left(\begin{array}{l}A_0^{\,2}+2A_0A_1\dfrac{\psi_\xi}{\psi}+\left(A_1^{\,2}+2A_0A_2\right)\\ \dfrac{\psi_\xi^{\,2}}{\psi^2}+2A_1A_2\dfrac{\psi_\xi^{\,3}}{\psi^3}+A_2^{\,2}\dfrac{\psi_\xi^{\,4}}{4}\end{array}\right)+\gamma A_1$$

$$\left(\frac{\psi_{\xi\xi\xi}}{\psi}-3\frac{\psi_{\xi\xi}\psi_\xi}{\psi^2}+2\frac{\psi_\xi^{\ 3}}{\psi^3}\right)+2\gamma A_2 \left(\frac{\psi_{\xi\xi}^{\ 2}+\psi_\xi\psi_{\xi\xi\xi}}{\psi^2}-5\frac{\psi_\xi^{\ 2}\psi_{\xi\xi}}{\psi^3}+3\frac{\psi_\xi^{\ 4}}{\psi^4}\right)=0 \quad (55)$$

Equating expressions to zero at ψ^{-1}, ψ^{-2}, ψ^{-3} and ψ^{-4}, we have the following system of equations:

$$\begin{aligned}
&(\delta A_2+12\gamma)A_2=0\\
&\left[(\delta A_1A_2+2\gamma A_1)\psi_\xi-5\gamma A_2\psi_{\xi\xi}\right]\psi_\xi^{\ 2}=0\\
&\left[-\lambda A_2+\frac{\delta}{2}\left(A_1^{\ 2}+2A_0A_2\right)\right]\psi_\xi^{\ 2}-3\gamma A_1\psi_{\xi\xi}\psi_\xi+\\
&2\gamma A_2\psi_{\xi\xi}^{\ 2}+2\gamma A_2\psi_\xi\psi_{\xi\xi\xi}=0\\
&\left[(\delta A_0-\lambda)\psi_\xi+\gamma\psi_{\xi\xi\xi}\right]A_1=0\\
&(\delta A_0-2\lambda)A_0=0
\end{aligned} \quad (56)$$

We conclude that the system of Eq. 56 can be satisfied simultaneously for:

$$A_0=\frac{2\lambda}{\delta},\ A_1=\frac{3}{\delta}\sqrt{2\lambda\gamma},\ A_2=-\frac{12\gamma}{\delta} \quad (57)$$

$$\psi_{\xi\xi}-\sqrt{\frac{\lambda}{2\gamma}}\frac{1}{2}\psi_\xi=0 \quad (58)$$

$$\frac{\psi_{\xi\xi\xi}}{\psi_{\xi\xi}}-\sqrt{\frac{2\lambda}{\gamma}}=0 \quad (59)$$

Therefore, Eq. 60:

$$\psi=B+Ae^{\sqrt{\frac{2\lambda}{\gamma}}\xi} \quad (60)$$

and:

$$u(x,t)=\frac{2\lambda}{\delta}+\frac{3}{\delta}\sqrt{2\lambda\gamma}\left(\frac{Ae^{\sqrt{\frac{2\lambda}{\gamma}}\xi}}{B+Ae^{\sqrt{\frac{2\lambda}{\gamma}}\xi}}\right)-\frac{12\gamma}{d}\left(\frac{Ae^{\sqrt{\frac{2\lambda}{\gamma}}\xi}}{B+Ae^{\sqrt{\frac{2\lambda}{\gamma}}\xi}}\right)^2 \quad (61)$$

for Eq. 62:

$$u(x,t)=2+3\sqrt{2}\left(\frac{e^{\sqrt{2}(x-t)}}{1+e^{\sqrt{2}(x-t)}}\right)-12\left(\frac{e^{\sqrt{2}(x-t)}}{1+e^{\sqrt{2}(x-t)}}\right)^2 \quad (62)$$

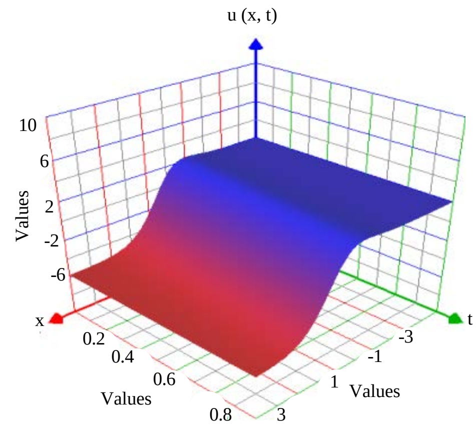

Fig. 9: The solitary wave in Eq. 62

Figure 9 represents the solitary wave in Eq. 62 for $-5\leq x\leq 5$; $0\leq t\leq 1$.

CONCLUSION

Three methods are applied to get new soliton solutions for S-KdV equation with a root of degree n nonlinearity. These methods are the Tan-Cot function method, extended Tanh-Coth method and MSEM. The Tan-Cot method provided many solutions depending on the specific parameter n, so that, many Soliton solutions are presented and illustrated by figures. While Tanh-Coth method presented many hyperbolic functions specified the Soliton solutions for different cases. Finally, for n = 2, α = 0 MSEM presented new Soliton solution. The three methods are efficient and reliable to be used to solve the problems of evolution equations (NLPDs).

ACKNOWLEDGEMENT

Researchers appreciate the helpful comments and constructive suggestions of reviewers and the organizing committee of ICMSS2018 conference. This study was supported by Al-Rafidain University, College RUC in Baghdad, Iraq.

REFERENCES

Al-Atawi, N.O., 2017. Exact solitary wave solutions for the generalized schamel Korteweg-de Vries equation. J. Math. Res., 9: 126-132.

Ding, T.R. and C.Z. Li, 1996. Ordinary Differential Equations. Peking University Press, Beijing, China,.

El-Wakil, S.A. and M.A. Abdou, 2007. New exact travelling wave solutions using modified extended tanh-function method. Chaos Solitons Fractals, 31: 840-852.

Fan, E.G., 2000. Extended tanh-function method and its applications to nonlinear equations. Phys. Lett. A., 277: 212-218.

Feng, Z., 2002. The first-integral method to study the Burgers-korteweg-de vries equation. J. Phys. A Math. Gen., 35: 343-349.

Inc, M. and M. Ergut, 2005. Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method. Appl. Math. E Notes Electron. Only, 5 5: 89-96.

Jawad, A.J.A.M., 2012a. Soliton solutions for nonlinear systems (2+1)-dimensional equations. IOSR J. Math., 96: 1-9.

Jawad, A.J.A.M., M.D. Petkovic and A. Biswas, 2010. Modified simple equation method for nonlinear evolution equations. Applied Math. Comput., 217: 869-877.

Jawad, A.J.M., 2012b. New exact solutions of nonlinear partial differential equations using Tan-Cot function method. Stud. Math. Sci., 5: 13-25.

Jawad, A.J.M., 2013. Soliton solutions for the Boussinesq equations. J. Math. Comput. Sci., 3: 254-265.

Jawad, A.J.M., M. Mirzazadeh, Q. Zhou and A. Biswas, 2017. Optical solitons with anti-cubic nonlinearity using three integration schemes. Superlattices Microstruct., 105: 1-10.

Malfliet, W., 1992. Solitary wave solutions of nonlinear wave equations. Am. J. Phys., 60: 650-654.

Mitchell, A.R. and D.F. Griffiths, 1980. The Finite Difference Method in Partial Differential Equations. John Wiley & Sons, Hoboken, New Jersey, USA., ISBN:9780471276418, Pages: 272.

Parkes, E.J. and B.R. Duffy, 1996. An automated tanh-function method for finding solitary wave solutions to non-linear evolution equations. Comput. Phys. Commun., 98: 288-300.

Sheng, Z., 2006. The periodic wave solutions for the (2+1)-dimensional Konopelchenko-Dubrovsky equations. Chaos Solitons Fractals, 30: 1213-1220.

Washimi, H. and T. Taniuti, 1996. Propagation of ion-acoustic solitary wave of small amplitude. Phys. Rev. Lett., 17: 996-998.

Wazwaz, A.M., 2005. The TANH method: Solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-dodd-bullough equations. Chaos Solitons Fractals, 25: 55-63.

Xia, T.C., B. Li and H.Q. Zhang, 2001. New explicit and exact solutions for the Nizhnik-Novikov-Vesselov equation. Appl. Math. E Notes, 1: 139-142.

Yang, J. and S.Q. Tang, 2015. Exact travelling wave solutions the Schamel-Korteweg-de Vries (Schamel-KdV) equation. J. Math. Sci. Adv. Appl., 31: 25-36.

Yusufoglu, E. and A. Bekir, 2006. Solitons and periodic solutions of coupled nonlinear evolution equations by using the sine-cosine method. Intl. J. Comput. Math., 83: 915-924.