Journal of Engineering and Applied Sciences 13 (8): 1963-1967, 2018
ISSN: 1816-949X

Calculation of Friction Resistance of End Plates Affecting Flat Jet Fading

G. Toleuov, M. Isataev, Sh. Ospanova, Zh. Seydulla, A. Nurmukhanova, D. Babasheva,
A. Musatayev, T. Akhanserina, G. Paizolla, G. Musabayeva, S. Ermaganbetova,
A. Nurseytova, M. Nurmagambetov and D. Tamabek
Scientific Research Institute of Experimental and Theoretical Physics,
Al-Farabi Kazakh National University, Almaty, Kazakhstan

Abstract: This research shows results of calculation of friction resistance of end plates affecting tendencies of free flat jet behavior. Flow diagram of jet flowing among end surfaces is built up. Estimate of friction at turbulent boundary layer is fulfilled. Calculation formula for the first study which describes variation of maximum jet velocity at a first approximation is acquired. These calculation data are compared with the experimental data. Also, a theoretical calculation of the second study was conducted. For this study calculation formula about the maximum velocity variation was received.

Key words: Plates, affecting, flow diagram, calculation formula, experimental data, theoretical, calculation

INTRODUCTION

During recent decades dynamic and pulsation characteristics of free three-dimensional jet flowing out of the nozzle with rectangular outlet section within the major and partially the initial sections of friction have been studied in details (Abramovich *et al*., 1984; Quinn, 1992; Faghani *et al*., 2010). During recent time also vortex structure has been studied and its effect on development of turbulent and averaged flow parameters within the initial, transitional and major sections of free jet stream have been focused. At studying flat jet in experimental plants as a rule in order to exclude interference of finiteness of rectangular nozzle height the flow field is confined by end plates installed in parallel with the flow direction as continuation of end walls of outlet section of the rectangular nozzle. Here as we can see by virtue of influence of end walls, we have a flat jet confined by these side walls instead of three dimensional jet. We might as well say that new obtained experimental and theoretical data provide wide information on effect of end walls and large scale coherent vortexes on development of turbulent jets flowing out of the rectangular nozzle. For instance in the research (Isataev *et al*., 2015) friction resistance of end plates affecting tendencies of free flat jet behavior is studied by experiments. During recent time coherent flow structures of walls jets are also under focus of attention (Namgyal and Hall, 2013). This investigation area is an important subject for studies. Also, it's important to carry on studies of dynamic flow parameters. This research study described as continuation of experimental studies shown in the research study (Isataev *et al*., 2015) contains theoretical calculation of friction resistance of end plates affecting tendencies of free flat jet behavior.

CALCULATION OF RESISTANCE INFLUENCE ALONG END WALLS

To build up calculation of resistance influence of end walls on flat jet fading let's consider the following diagram of jet flowing among end surfaces. Figure 1 shows diagrams of a jet stream confined by flat end walls in planes xoy and xoz. In the plane xoy, the jet just as in the ordinary free jet has initial section (index "I"), transitional section (index "t") and major section as well as side free shifting borders, the nozzle width along the axis oy equals 2b. In the plane xoz, the jet flowing out of the nozzle with height 2 h on sides along the axis oz is confined by end plates. Within the first section of the jet after leaving the nozzle along the end walls laminar or turbulent boundary layers are developing with homogenous profile along the axis z between the limits boundary layers. Development of boundary layers is analogical to the boundary layer typical for homogeneous flow flowing along the plate. At the end of the 1st section boundary layers join on the jet axis and the 2nd jet section begins in which in the plane oz the flow is analogical to current flow in a flat channel. Accordingly development of the boundary layer and the current within the 1st section are analogical to homogeneous flow flowing along the plate within the 2nd section-analogical to current flow in a flat channel.

Corresponding Author: G. Toleuov, Scientific Research Institute of Experimental and Theoretical Physics, Al-Farabi Kazakh National University, Almaty, Kazakhstan

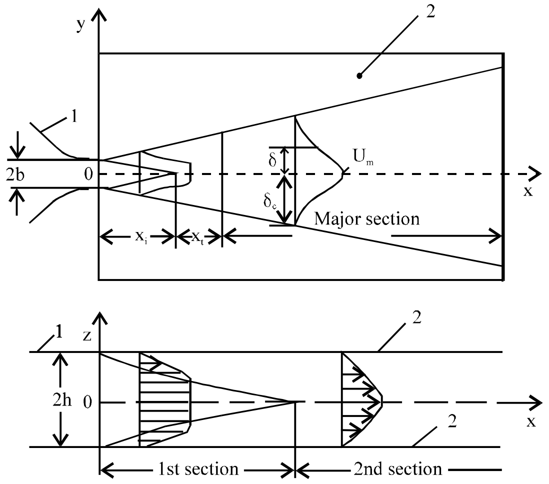

Fig. 1: Diagram of a flat jet confined by end walls; 1-nozzle; 2-end plates. x, y, z-rectangular Descartes coordinates; δ-the current value of boundary layer thickness; δ_c-thickness of the boundary layer of free jet a long the axis y. U_m-velocity on the jet axis

Geometrical parameter $\lambda = 2h/2b$ characterizes relative elongation of the exhaust nozzle area. Subject to the foregoing within the 1st jet section let's assume variation of boundary layer thickness along z on the end plates in the form of the flowing dependencies:

$$\delta_z = \frac{5.0 \cdot x}{\sqrt{\dfrac{U_m x}{v}}} \tag{1}$$

For laminar boundary layer:

$$\delta_z = \frac{0.37x}{\left(\dfrac{U_m x}{v}\right)^{\frac{1}{5}}} \tag{2}$$

For turbulent boundary layer: Here x-longitudinal coordinate, U_m-velocity along the jet axis, v-kinematical viscosity and $U_m x/v$-let's assume it as Reynolds number $Re_{mx} = U_m x/v$. Length of the first section is defined based on the condition $x = x_1$ at $\delta_z = h$. Accordingly to calculate wall resistance we can use the formulas from the research study (Isataev *et al.*, 2015):

$$C_f = \frac{0.664}{\sqrt{Re_{mx}}} \text{ or } C_f = \frac{0.0576}{\left(\dfrac{U_m x}{v}\right)^{0.2}}$$

After joining the boundary layers resistance law in a flat channel is applicable for the second flow section with hydraulic resistance coefficient ξ for the laminary flow:

$$\xi = \frac{16}{Re} \text{ where} Re = \frac{U_m 2h}{v} \tag{3}$$

For turbulent flow:

$$\xi = \frac{0.3164}{Re^{\frac{1}{4}}}, Re = \frac{U_m d_r}{v} \tag{4}$$

where, $d_r = 4F/\vartheta$-hydraulic diameter defined as ratio of quadruplicated section area of channel F to its perimeter ϑ.

There is an approximated calculation of variation of total impulse of the jet affected by resistance of end walls for both sections under question under turbulent conditions of jet flow provided below.

CALCULATION OF RESISTANCE AT TURBULENT BOUNDARY LAYER

In presence of resistance of end walls the total jet impulse is not preserved and will reduce along the jet length:

$$\frac{dK}{dx} = -2\int_{-\delta_c}^{\delta_c} \tau_w dy \tag{5}$$

Where:

K = Current total impulse in an optional jet section
τ_w = Friction stress at the wall at distance y from the symmetry plane
δ_c = Total jet halfwidth equaling to the distance from the axis to the outside boundary at U = 0

As it's shown in the Fig. 1, there are boundary layers on end walls with δ_z thickness in the first section of the jet stream and the central section with constant velocity U_m in the section y = 0. Let's assume that the jet width along the axis y is not changing along the axis z and equals to δ_c. Let's take velocity distribution across the section in the form of polynom (offered by G. Schlichting) in which U_m in the central part along the axis z won't change. Let's take velocity change in the wall-wise area in the form of power law as follows:

$$\frac{U}{U_1} = \left(1 - \frac{z}{\delta_z}\right)^{\frac{1}{n}} \tag{6}$$

Where:

$$\frac{U_1}{U_m} = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4 \tag{7}$$

here U-longitudinal component of velocity, U_1-velocity at the edge of the wall-wise boundary layer at the distance δ_z of the wall with corresponding distances $\eta = y/\delta_c$ from the plane zox in the given section (δ_c-total jet halfwidth equaling to the distance from the external limit at U = 0 and related to the conditional width $\delta_c = 2.59\ \delta$) for numbers Re = $U_m 2h/v < 10_5$, n = 7 and <U> = 0.817 U_m. Inserting Eq. 7 in 6 we will acquire velocity distribution in the wall-wise boundary layer:

$$\frac{U}{U_m} = \left(1 - \frac{z}{\delta_z}\right)^{\frac{1}{7}} \left(1 - 6\eta^2 + 8\eta^3 - 3\eta^4\right) \tag{8}$$

Then total jet impulse in the section being at the distance x away from the nozzle will equal:

$$K = \int \rho U^2 dydz = 4\int_0^{\delta_z}\int_0^{\delta_c} \rho U^2 dydz + 2\left(h - \delta_z\right)\int_0^{\delta_c} \rho U^2 dy \tag{9}$$

where, ρ-density of liquid (gas). Taking into account (Eq. 7 and 8):

$$K = 4\rho U_m^2 \delta_z \delta_c \int_0^1 \left(1 - \frac{z}{\delta_z}\right)^{\frac{2}{7}} d\left(\frac{z}{\delta_z}\right) \int_0^1 \left(1 - 6\eta^2 + 8\eta^3 - 3\eta^4\right)^2 d\eta + 2\left(h - \delta_z\right)\rho U_m^2 \int_0^1 \left(1 - 6\eta^2 + 8\eta^3 - 3\eta^4\right) d\eta \tag{10}$$

After computing the values of integrals, on rearrangement we have:

$$K = \rho U_m^2 \delta_c \left(\frac{20}{63}\delta_z + \frac{4}{7}h\right) \tag{11}$$

As the measurements show (Isataev *et al*., 2015), distributions of friction stress on the end walls in the coordinates $\tau/\tau_w = f(y/\delta)$ are similar to the velocity profile in the major section and summarized friction stress applied on the section with dimensions $4\delta_c$ dx on both end walls will be equal:

$$2\int_{-\delta_c}^{\delta_c} \tau_w dydx = 4\tau_{wm}\delta_c dx \int_0^1 \left(1 - 6\eta^2 + 8\eta^3 - 3\eta^4\right) d\eta = 4.15\delta\tau_{wm} dx \tag{12}$$

Where:

τ_w = Friction stress on the wall at the distance y from the symmetry plane

τ_{wm} = Maximum friction stress on the wall at y = 0

Inserting the values δ_c, δ, δ_z, K, τ_{wm} to Eq. 5, we obtain:

$$\frac{d}{d\left(\frac{x}{b}\right)}\left\{0.0280\rho U_0^2 b \frac{\left(\frac{U_m}{U_0}\frac{x}{b}\right)^{1.8}}{Re_0^{0.2}} + 0.136\rho U_0^2 b \frac{h}{b}\frac{x}{b}\left(\frac{U_m}{U_0}\right)^2\right\} = -0.0110\frac{b\rho U_0^2}{Re_0^{0.2}}\left(\frac{U_m}{U_0}\right)^{1.8}\left(\frac{x}{b}\right)^{0.8} \tag{13}$$

where, $Re_0 = U_0 2b/v$. Here U_0-initial outlet velocity. Taking the derivative by x on the left and rearranging it, we acquire:

$$\frac{d\left(\frac{U_m}{U_0}\right)}{\frac{U_m}{U_0}} = -\frac{\frac{1}{2}\left[\left(\frac{d\left(\frac{x}{b}\right)}{\frac{x}{b}} + \frac{0.4517}{\lambda Re_0^{0.2}}\frac{d\left(\frac{x}{b}\right)}{\left(\frac{U_m}{U_0}\frac{x}{b}\right)^{0.2}}\right)\right]}{1 + \frac{0.1854}{\lambda Re_0^{0.2}}\frac{\left(\frac{x}{b}\right)^{0.8}}{\left(\frac{U_m}{U_0}\right)^{0.2}}} \tag{14}$$

Given that in the denominator (Eq. 14) the second member one order less than unity let's change them into numerator using the method of expansion procedure the following values $1/1+x \approx 1-x+x^2-x^3+$, ... keeping the first four members of series unchanged. At zero-order approximation let's insert to the right side of Eq. 14 the value:

$$\frac{U_m}{U_0} = \frac{N}{\sqrt{\frac{x}{b}}}$$

and integrate the equation within limits by x from the end of the initial section x_H up to the optional distance x. As a result we obtain a solution for changing maximum velocity at a first approximation:

$$\frac{U_m}{U_0} = \frac{N}{\sqrt{\frac{x}{b} + \frac{x_0}{b}}}\exp\left\{-\frac{0.1481}{A}\left(\frac{x}{b}\right)^{0.9} + \frac{0.01372}{A^2}\left(\frac{x}{b}\right)^{1.8} - \frac{0.00288}{A^3}\left(\frac{x}{b}\right)^{0.27}\right\} \tag{15}$$

where, A = $\lambda\ Re_0^{\ 0.2}\ N^{0.2}$, $\lambda = 2h/2b$, $Re_0 = U_0 2b/v$, x_0-polar distance. Calculations made according this equation show that towards the end of the 1st section velocity decrement correction is up to 35%.

However, length of the 1st section as the value λ grows is abruptly increasing (Fig. 2) and with λ>10 at the distances up to x/b<200 effect of resistance will not exceed 10%.

Comparison of the calculation results by the Eq. 15 with experimental data (Isataev *et al*., 2015) is shown in the Fig. 3 at λ = 3 and U_0 = 4.3 and 63.8 m/sec.

Here, with it should be appreciated that in Fig. 3 the value of measured maximum velocity corresponds to the jet axial line. Values of maximum velocity averaged by the axis z along the overall jet height are computed in theoretical calculations. Therefore, the experimental values of maximum velocity shall be somewhat higher than the theoretically computed values.

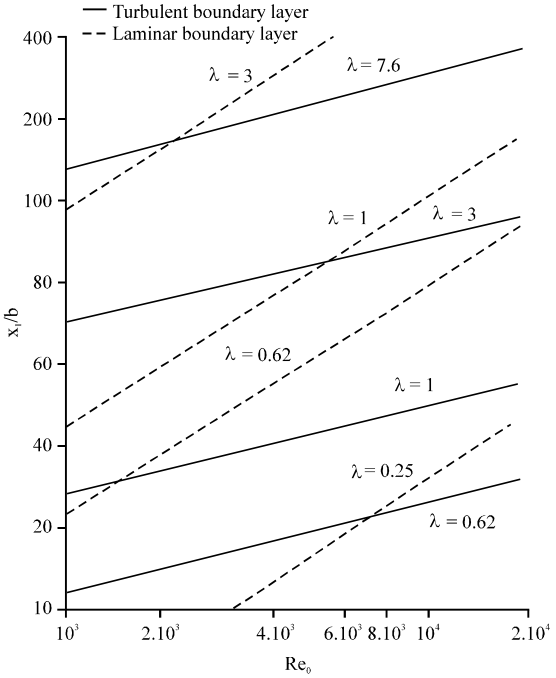

Fig. 2: Dependence of length of the 1st jet section with end plates on λ and Re_0

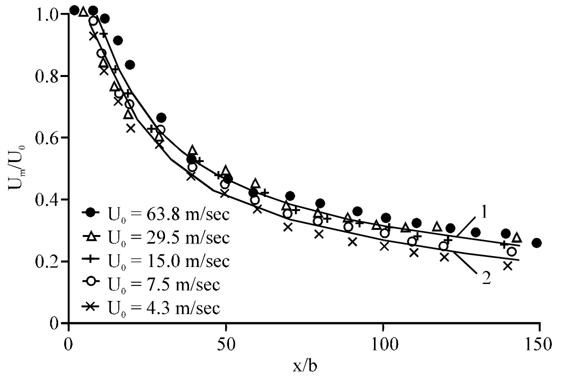

Fig. 3: Variation of jet maximum velocity at λ = 3 and $Re_0 = U_0 2b/v$

Figure 4 shows values of turbulence level along the jet axis referred to maximum velocity for λ = 3. As you can see for all values of velocity U_0 from 4.3-30 m/sec turbulence levels vary the same way as for a jet at λ>3.

In the 2nd section of the jet boundary layers at the wall reach the middle of the current and liquid will flow like liquid flowing in a flat channel with width 2 h. In this case velocity profiles shall be described by the Eq. 8 and the jet impulse equals:

$$K = 4\int_0^h\int_0^{\delta_c}\rho U^2 dydz = 4\rho U_m^2\delta_c h\int_0^1\int_0^1\left(\frac{U}{U_m}\right)^2 d\left(\frac{y}{\delta_c}\right)d\left(\frac{z}{h}\right) =$$

$$4\rho U_m^2\delta_c h\int_0^1\left(1-\frac{z}{h}\right)^{\frac{2}{7}} d\left(\frac{z}{h}\right)\int_0^1\left(1-6\eta^2+8\eta^3-3\eta^4\right)^2 d\eta = \frac{8}{9}\rho U_m^2\delta_c h \tag{16}$$

Resistance force of end walls in the section with dimension $4\delta_c dx$ shall be also defined by the Eq. 12. Inserting them into Eq. 5, we have:

$$\frac{d}{dx}\left\{\frac{8}{9}\rho U_m^2\delta_c h\right\} = -4.15\tau_w\delta \tag{17}$$

Inserting the values:

$$\delta = 0.092x, \delta_c = 0,238x, \tau_w = \frac{0.3164}{\left(\frac{<U>4h}{\nu}\right)^{\frac{1}{4}}}\frac{\rho<U^2>}{8} =$$

$$\frac{0.01963}{\left(\frac{U_m b}{\nu}\right)^{\frac{1}{4}}\left(\frac{h}{b}\right)^{\frac{1}{4}}}\cdot\rho U_m^2$$

into Eq. 17, after rearrangement, we obtain:

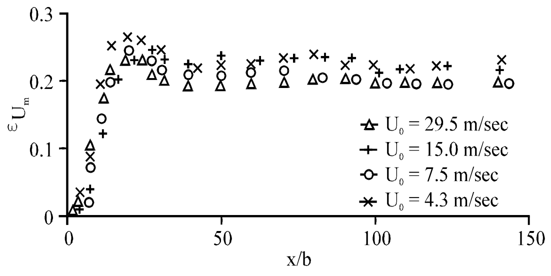

Fig. 4: Turbulence level along the jet axis referred to maximum velocity for λ = 3 and $Re_0 = U_0 2b/v$

$$\int_{\frac{U_{m1}}{U_0}}^{\frac{U_m}{U_0}} \frac{d\left(\frac{U_m}{U_0}\right)}{\frac{U_m}{U_0}} = -\int_{\frac{x_1}{b}}^{\frac{x}{b}} \left[\frac{1}{2} + \frac{0.01771\frac{x}{b}}{\left(\frac{h}{b}\right)^{1.25} Re_0^{0.25}\left(\frac{U_m}{U_0}\right)^{0.25}}\right] \frac{d\left(\frac{x}{b}\right)}{\frac{x}{b}} \qquad (18)$$

At zero-order approximation assuming:

$$\frac{U_m}{U_0} = \frac{N}{\sqrt{\frac{x}{b}}} \qquad (19)$$

and inserting it to the right part Eq. 18, we obtain a solution at a first approximation. Integrating by x shall be carried out from the end of the first section and so on. Here with the velocity value Um_1/U_0 shall be calculated by the Eq. 15 with value $x = x_1$. Then after integrating, we have:

$$\frac{U_m}{U_0} = \frac{U_{m1}}{U_0}\sqrt{\frac{\frac{x_1}{b}+\frac{x_0}{b}}{\frac{x}{b}+\frac{x_0}{b}}}\exp\left[-\frac{0.01575\left[\left(\frac{x}{b}\right)^{1.125}-\left(\frac{x_1}{b}\right)^{1.125}\right]}{\left(\frac{h}{b}\right)^{1.25}\left(Re_0 N\right)^{0.25}}\right] \qquad (20)$$

This equation shall be used at values of parameter $\lambda \leq 3$, since, for values $\lambda > 3$ the value $x_1/b \geq 100$ and reaches at $\lambda = 25$ and $U_0 = 30$ m/sec up to $x_1/b = 850$ which is beyond the measuring range limits and area of applying jet streams.

CONCLUSION

Jet flow scheme between the restrictive end plates has been built. In a xoy plane the jet is distributed as a free jet. In a xoz plane in first section formed boundary layer on the end walls similar to the boundary layer in a uniform flow around the plates in the second section when the boundary layers are closed on the jet axis flow is similar for the flow in a flat channel. This research shows results of calculation of friction resistance of end plates affecting tendencies of free flat jet behavior. Estimate of friction at turbulent boundary layer is fulfilled. Calculation formula for the first section which describes variation of maximum jet velocity at a first approximation is acquired. These calculation data are compared with the experimental data. Also a theoretical calculation of the second section was conducted. For this study calculation formula about the maximum velocity variation was received.

REFERENCES

Abramovich, G.N., T.A. Girshovich, S.I. Krasheninnikov, A.N. Sekundov and I.P. Smirnova, 1984. The Theory of Turbulent Jets. The MIT Press, Cambridge, Massachusetts, Pages: 684.

Faghani, E., R. Maddahian, P. Faghani, and B. Farhanieh, 2010. Numerical investigation of turbulent free jet flows issuing from rectangular nozzles: The influence of small aspect ratio. Arch. Appl. Mech., 80: 727-745.

Isataev, S., G. Toleuov, M. Isataev, S. Ospanova and S. Bolysbekova, 2015. Impact of frictional resistance of end plates on flat jet attenuation. J. Eng. Appl. Sci., 10: 76-79.

Namgyal, L. and J.W. Hall, 2013. Coherent streamwise vortex structures in the near-field of the three-dimensional wall jet. J. Fluids Eng., 135: 061204-061211.

Quinn, W.R., 1992. Turbulent free jet flows issuing from sharp-edged rectangular slots: The influence of slot aspect ratio. Exp. Therm. Fluid Sci., 5: 203-215.