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Thermal Induced Vibration of non Homogeneous Tapered Square Plate

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Abstract: A mathematical model is presented to study the effect of circular variation in density (as a non-homogeneity parameter) and linear variation in thickness to natural vibration of square plate on clamped (C-C-C-C) boundary condition under temperature enviornment. It is considered that the temperature variation along both the axes are parabolic. To obtain frequency modes, Rayleigh Ritz technique has been applied. All the results are presented in the form of tables. Comparison of the result with existing result (in literature) is also presented in the form of figures.

Key words: Tapered square plate, thermal induced, vibration, circular variation, homogeneity, temperature variation, Rayleigh Ritz

INTRODUCTION

Non homogeneous tapered plates are extensively used in structural components of engineering. To avoid the failure of structures or to increase the reliability of structures, it is required to have better understanding of vibration characteristic of plate. For modal analysis of structures, it is essential to calculate natural frequencies and mode shapes. These parameters play an essential role in the designing of structures and determine the dynamic characteristics of existing structures.

Leissa (1969) provided vibration of different structures (plates of different shape) on different boundary (clamped, simply supported and free) conditions in his excellent monograph. Liew *et al*. (1993) studied the transverse vibration of thick rectangular plate. Kalita and Haldar (2015) discussed parametric study on vibration of thick plate using FSDT. Thermal effect on vibration properties of double layered nanoplates at small scale has been studied by Wang *et al*. (2011). Leissa and Nartia (1980) studied natural frequency of simply supported circular plate. Tariverdilo *et al*. (2013) derived natural frequencies for clamped circular plate with incompressible fluid. Khanna *et al*. (2015) discussed the effect of temperature on free vibration of non homogeneous and non uniform square plate. Zhou (2002) provided the vibrations of point supported rectangular plate with variable thickness using a set of static tapered beam functions. Sharma *et al*. (2016a, b) studied the vibration of isotropic and orthotropic non homogeneous rectangular plate with two dimensional temperature effects. Sharma and Sharma (2016) provided a mathematical modeling on vibration of parallelogram plate with non-homogeneity effect. Sharma *et al*. (2016a, b) studied the vibration of square plate with thermal effect and circular variation in density. Hosseini-Hashemi *et al*. (2013) provided a mathematical model to study free vibration of stepped circular and annular FG plates. Khanna and Kaur (2016a, b) studied the vibration of non homogeneous of rectangular plate with exponential variation in non-homogeneity parameter with temperature effect. Kazerouni *et al*. (2010) presented an exact solution for thin functionally graded simply supported (two opposite edges) rectangular plate. To obtained stability equation they used the principle of minimum total potential energy. Buckling behavior of moderately thick functionally graded rectangular plates resting on elastic foundation subjected to linearly varying in plane loading has been investigated by Bodaghi and Saidi (2011). They used first-order shear deformation plate theory and the neutral surface concept to obtained the equilibrium and stability equations. Alibakhshi (2012) studied the effect of anisotropy on the free vibration of laminated rectangular (simply supported) plate supporting a localized patch mass. The equation of motion is derived by calculus of variation. Baferani *et al*. (2012) investigated buckling analysis of functionally graded annular thin and moderately thick plates under mechanical and thermal loads by using classical and first order shear deformation plate theory. An Analytic approach to free vibration and buckling analysis of functionally graded beams with edge cracks using four engineering beam theories have been provided by Sherafatnia *et al*. (2013).

In available literature, authors/researchers focused on linear, parabolic and exponential variation in non-homogeneity (density) but none of them considered circular variation. Here, researcher studied the effect of circular variation in density. Here, researcher also studied the effect of tapering parameter and thermal gradient on vibrational frequency modes. The first two vibrational modes are calculated for different values of plate's parameters. To validate the result of present study, research compared frequency modes with (Khanna *et al*., 2015) corresponding to taper constant β and non-homogeneity m.

Equation of motion: The equation of motion for non-homogeneous tapered isotropic plate is:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x \partial y}+\frac{\partial^2 M_y}{\partial y^2}=\rho g\frac{\partial^2 \phi}{\partial t_2} \tag{1}$$

Where:

$$\left.\begin{aligned} M_x &= -\square D_1\left(\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right)\\ M_y &= -\square D_1\left(\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right)\\ M_{xy} &= -\square D_1(1\text{-}\nu)\frac{\partial^2\phi}{\partial x\partial y}\end{aligned}\right\} \tag{2}$$

Substitute Eq. 2 in Eq. 1, we get:

$$\square\left[\begin{aligned}&D_1\left(\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}\right)+\\ &2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\phi}{\partial y^3}+\frac{\partial^3\phi}{\partial y\partial x^2}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right)+\\ &\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right)+2(1\text{-}\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\phi}{\partial x\partial y}\end{aligned}\right]+\rho g\frac{\partial^2\phi}{\partial t^2}=0 \tag{3}$$

For solution of Eq. 3, we can take:

$$\phi(x,y,t)=\Phi(x,y)*T(t) \tag{4}$$

Substitute Eq. 4 in Eq. 3, we get:

$$\frac{\left[\begin{aligned}&D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}\right)+\\ &2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}\right)+\\ &2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\partial x^2}\right)+\\ &\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right)+\\ &\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right)+\\ &2(1\text{-}\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\end{aligned}\right]}{\rho g\Phi}=-\frac{\partial^2 T/\partial t^2}{\square T} \tag{5}$$

Equating positive constant k^2 both sides of Eq. 5, we get:

$$\left[\begin{aligned}&D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}\right)+\\ &2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}\right)+\\ &2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\partial x^2}\right)\\ &+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right)+\\ &\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right)+\\ &2(1\text{-}\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\end{aligned}\right]\text{-}\rho k^2 g\Phi=0 \tag{6}$$

And:

$$\frac{\partial^2 T}{\partial t^2}+k^2\square T=0 \tag{7}$$

Equation 6 and 7 represents equation of motion and time function of tape0red non homogeneous isotropic plate and $D = Yg^3/12(1\text{-}\nu^2)$ is called flexural rigidity of the plate.

MATERIALS AND METHODS

Assumptions: Vibration of plate is very vast area. It is not possible to study vibration at once therefore, the present study requires some limitations in the form of assumptions. We consider the temperature variation on the plate is parabolic, therefore:

$$\tau=\tau_0\left(1\text{-}\frac{x^2}{a^2}\right)\left(1\text{-}\frac{y^2}{a^2}\right) \tag{8}$$

where, τ and τ are known as temperature above the mention temperature at any point on the plate and at origin, i.e., x = y = 0. For engineering material, the modulus of elasticity is:

$$Y=Y_0(1\text{-}\gamma\tau) \tag{9}$$

Where:
Y_0 = The young's modulus at $\tau = 0$
γ = Known as slope of variation

Substitute Eq. 8 and 9, we get:

$$Y=Y_0\left\{1\text{-}\alpha\left(1\text{-}\frac{x^2}{a^2}\right)\left(1\text{-}\frac{y^2}{a^2}\right)\right\} \tag{10}$$

where, α ($0\leq\alpha<1$) is known as temperature gradient which is the product of temperature at origin and slope of variation, i.e., $\alpha = \gamma\tau_0$. The thickness of the plate is assumed to be linear in one direction as:

$$g = g_0\left(1+\beta\frac{x}{a}\right) \quad (11)$$

where, β ($0\leq\beta\leq1$) is known as tapering parameter of the plate and $g = g_0$ at $x = 0$. For non-homogeneity in the plate's material, density the plate varies circularly in one direction as:

$$\rho = \rho_0\left\{1\text{-}m\left(1\text{-}\sqrt{1\text{-}\frac{x^2}{a^2}}\right)\right\} \quad (12)$$

where, m ($0\leq m\leq1$) is known as non-homogeneity constant. Using Eq. 10 and 11, the flexural rigidity of the plate becomes:

$$D_1 = \frac{Y_0 g_0^{\,3}\left[1\text{-}\alpha\left\{1\text{-}\frac{x^2}{a^2}\right\}\left\{1\text{-}\frac{y^2}{a^2}\right\}\right]\left\{1+\beta\frac{x}{a}\right\}^3}{12\left(1\text{-}\nu^2\right)} \quad (13)$$

Here, we computing vibrational frequency on clamped plate, i.e. (C-C-C-C). Therefore, boundary conditions of the plate are:

$$\Phi = \frac{\partial\Phi}{\partial x} = 0, x = 0, a \text{ and } \Phi = \frac{\partial\Phi}{\partial y} = 0, y = 0, a \quad (14)$$

The two term deflection function which satisfy Eq. 14 could be:

$$\Phi = \left[\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1\text{-}\frac{x}{a}\right)\left(1\text{-}\frac{y}{a}\right)\right]^2 \left[B_1 + B_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1\text{-}\frac{x}{a}\right)\left(1\text{-}\frac{y}{a}\right)\right] \quad (15)$$

where, B_1 and B_2 represents arbitrary constants.

Solution for frequency equation: We are using Rayleigh Ritz technique (i.e., maximum kinetic energy T_S must equal to maximum strain energy V_S) to solve equation of motion, therefore, we have:

$$\delta\left(V_s\text{-}T_s\right) = 0 \quad (16)$$

Where:

$$T_s = \frac{1}{2}k^2\int_0^a\int_0^a \rho g F^2\, dydx \quad (17)$$

$$V_s = \frac{1}{2}\int_0^a\int_0^a D_1\left\{\left(\frac{\partial^2\Phi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2} + 2\left(1\text{-}\nu\right)\left(\frac{\partial^2\Phi}{\partial x\partial y}\right)^2\right\}dydx \quad (18)$$

Now, converting x and y into non-dimensional variable X and Y as:

$$X = \frac{x}{a}, Y = \frac{y}{a} \quad (19)$$

Using Eq. 17-19 becomes:

$$T_s = \frac{1}{2}k^2\int_0^1\int_0^1 \rho g\Phi^2\, dYdX \quad (20)$$

$$V_s = \frac{1}{2}\int_0^1\int_0^1 D_1\left\{\left(\frac{\partial^2\Phi}{\partial X^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial Y^2}\right)^2 + 2\nu\frac{\partial^2\Phi}{\partial X^2}\frac{\partial^2\Phi}{\partial Y^2} + 2\left(1\text{-}\nu\right)\left(\frac{\partial^2\Phi}{\partial X\partial Y}\right)^2\right\}dYdX \quad (21)$$

Using Eq. 20 and 21, Eq. 16 becomes:

$$\left(V_s^*\text{-}\lambda^2 T_s^*\right) = 0 \quad (22)$$

Where:

$$V_s^* = \int_0^1\int_0^1\left[1\text{-}\alpha\left\{1\text{-}X^2\right\}\left\{1\text{-}Y^2\right\}\right]\left\{1+\beta X\right\}^3\left\{\left(\frac{\partial^2\Phi}{\partial X^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial Y^2}\right)^2 + 2\nu\frac{\partial^2\Phi}{\partial X^2}\frac{\partial^2\Phi}{\partial Y^2} + 2\left(1\text{-}\nu\right)\left(\frac{\partial^2\Phi}{\partial X\partial Y}\right)^2\right\}dYdX$$

$$T_S^* = \int_0^1\int_0^1\left\{1\text{-}m\left(1\text{-}\sqrt{1\text{-}X^2}\right)\right\}\left\{1+\beta X\right\}\Phi^2\, dYdX$$

and λ^2 is known as frequency parameter. Equation 22 consists of two unknowns constants, i.e., B_1, B_2 because of substitution of deflection function ϕ.These constants can be determined by:

$$\frac{\partial\left(V_s^*\text{-}\lambda^2 T_s^*\right)}{\partial B_i} = 0, i = 1, 2 \quad (23)$$

After simplifying Eq. 23, we get homogeneous system of equation as:

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (24)$$

where, d_{11} and d_{12} involve parametric constant and frequency parameter. To get non trivial solution, the determinant of the coefficient matrix of Eq. 24 must be zero. So, we get equation of frequency as:

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} = 0 \quad (25)$$

Equation 25 is quadratic equation from which we get two roots as λ_1 (1st mode) and λ_2 (2nd mode).

RESULTS AND DISCUSSION

The following parameters are used for calculating vibration frequency modes for different values of plate's parameter (non-homogeneity, taper constant and temperature gradient).

$$\rho_0 = 2.80*10^3 \text{kg/m}^3, \nu = 0.345, g_0 = 0.01\text{ m}$$

Table 1 provides the frequency modes corresponding to tapering parameter for fixed value of non-homogeneity m = 0.2 in plate's material and for three different values of temperature gradient α = 0, 0.4 and α = 0.8. From Table 1, we conclude that frequency modes increases when tapering parameter increase from 0 to 1 for all the three values of thermal gradient. On the other hand frequency mode decreases when the temperature on the plate increases from 0-0.8.

Table 2 gives the frequency modes corresponding to non-homogeneity in plate's material for three different values of thermal gradient α = 0, 0.4, 0.8 and tapering parameter β = 0, 0.4, 0.8. From Table 2, we enlighten the fact that when non-homogeneity in the plate's material increases from 0-1, the frequency modes decreases with less rate of decrement (due to circular variation in density). When combined value of thermal gradient and tapering parameter increases from 0-0.8, the frequency mode increases.

Table 3 represents the modes of frequency corresponding to temperature variation on the plate for three different values of non-homogeneity constant m = 0, 0.4, 0.8 and tapering parameter of the plate β = 0, 0.4, 0.8. From Table 3, one can easily get that frequency modes decreases when the temperature gradient on the plate increases from 0-0.8. Also, the mode of vibrations increases when the combined value of non-homogeneity and thickness parameter increases from 0-0.8.

Comparison of result: The findings of the present study (frequency modes) is compared with Khanna *et al.* (2015) with the help of figures. Figure 1 gives the comparison for frequency (first mode) of present study with Khanna *et al.* (2015) corresponding to tapering parameter for fixed value of non-homogeneity constant and thermal gradient, i.e., m = 0.25, α = 0. It is clear from the figure that frequency and frequency variation in present study is less than the frequency and frequency variation by Khanna *et al.* (2015). Figure 2 provides the comparison for frequency (first mode) of present study with (Khanna *et al.*, 2015) corresponding to non-homogeneity in the plate's material for fixed value of tapering parameter and thermal gradient, i.e., $\alpha = \beta = 0.4$. In Fig. 2, frequency mode is less than (Khanna *et al.*, 2015) when non-homogeneity varies from 0-0.6. But when the value of non homogeneity is 0.8, the frequency of Khanna *et al.* (2015) is minutely less than the frequency of present study.

Table 1: Thickness (tapering parameter β) variation in plate vs. vibrational frequency (λ) for m = 0.2

	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.8$	
β	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	35.46	138.48	32.04	125.17	28.17	110.26
0.2	39.13	152.80	35.59	139.05	31.60	123.79
0.4	43.04	168.00	39.34	153.74	35.19	138.03
0.6	47.11	183.88	43.24	169.04	38.89	152.80
0.8	51.30	200.27	47.24	184.81	42.67	167.97
1.0	55.59	217.05	51.32	200.93	46.51	183.43

Table 2: Non-homogeneity (m) variation in plate's material vs. vibrational frequency (λ)

	$\alpha = \beta = 0$		$\alpha = \beta = 0.4$		$\alpha = \beta = 0.8$	
m	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	35.99	140.88	39.96	156.56	43.34	171.18
0.2	35.46	138.48	39.34	153.74	42.67	167.97
0.4	34.94	136.21	38.76	151.07	42.02	164.92
0.6	34.45	134.04	38.20	148.54	41.41	162.05
0.8	33.98	131.98	37.66	146.13	40.82	159.32
1.0	33.53	130.01	37.15	143.85	40.26	156.74

Table 3: Temperature (α) variation on plate vs. vibrational frequency (λ)

	$m = \beta = 0$		$m = \beta = 0.4$		$m = \beta = 0.8$	
α	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	35.99	140.88	42.20	165.09	49.07	190.02
0.2	34.31	134.28	40.62	158.23	47.18	182.82
0.4	32.52	127.34	38.76	151.07	45.19	175.33
0.6	30.63	119.99	36.78	143.55	43.08	167.51
0.8	28.60	112.18	34.66	135.63	40.82	159.32

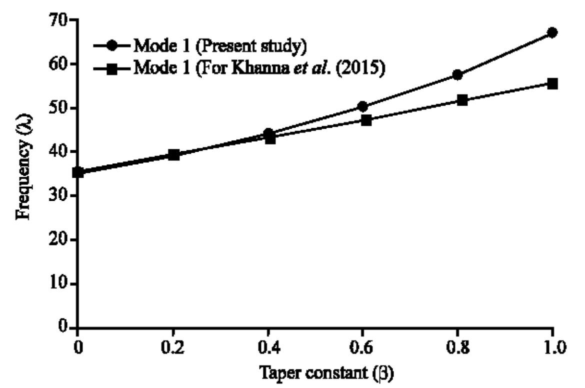

Fig. 1: Vibrational frequency (first mode) corresponding to tapering parameter for present paper and Khanna *et al.* (2015)

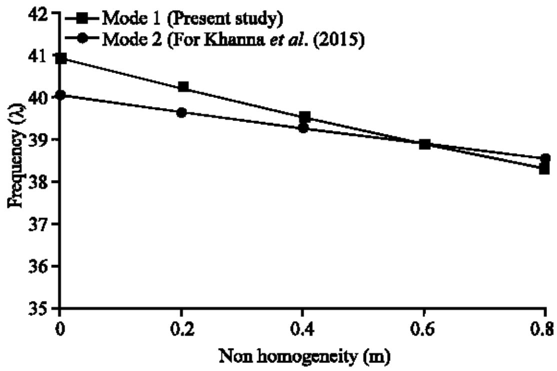

Fig. 2: Vibrational frequency (first mode) corresponding to non-homogeneity for present paper and Khanna *et al.* (2015)

CONCLUSION

Based on results discussion and graphical comparison, researchetr conclude the following points:

- The plate's parameter directly affects the frequency and frequency variation
- Due to linear variation in thickness in present study, frequency and variation in frequency is less when compared to exponential variation in thickness as by Khanna *et al.* (2015)
- Circular variation in density as in preset study provides less frequency when compared to parabolic variation in density as by Khanna *et al.* (2015)
- The present model provides a fine numerical data for frequency modes
- Frequency can be optimize by taking appropriate variation in parameters of plate

SYMBOLS

a	= Length of plate
x, y	= Coordinates axes of plate
M_x, M_y	= Bending moment intensities in x and y direction
M_{xy}	= Twisting moment intensity
Y	= Young's modulus
v	= Poisson's ratio
$\tilde{D}$	= Visco elastic operator
D_1	= Flexural rigidity
ρ	= Density per unit volume of the plate's material
t	= Time
ϕ (x, y, t)	= Deflection of plate
Φ	= Deflection function
T (t)	= Time function
g	= Thickness of plate at x, y
β	= Tapering parameter
m	= Non-homogeneity of the material
α	= Temperature gradient

REFERENCES

Alibakhshi, R., 2012. The effect of anisotropy on free vibration of rectangular composite plates with patch mass. Intl. J. Eng., 25: 223-232.

Baferani, A.H., A.R. Saidi and A. Naderi, 2012. On symmetric and asymmetric buckling modes of functionally graded annular plates under mechanical and thermal loads. Intl. J. Eng. Trans. A Basics, 26: 433-446.

Bodaghi, M. and A.R. Saidi, 2011. Buckling analysis of functionally graded Mindlin plates subjected to linearly varying in-plane loading using power series method of Frobenius. Intl. J. Eng. Trans. A Basics, 25: 89-109.

Hosseini-Hashemi, S., M. Derakhshani and M. Fadaee, 2013. An accurate mathematical study on the free vibration of stepped thickness circular/annular Mindlin functionally graded plates. Appl. Math. Modell., 37: 4147-4164.

Kalita, K. and S. Haldar, 2015. Parametric study on thick plate vibration using FSDT. Mech. Mech. Eng., 19: 81-90.

Kazerouni, S.M., A.R. Saidi and M. Mohammadi, 2010. Buckling analysis of thin functionally graded rectangulare plates with two opposite edges simply supported. Intl. J. Eng. Trans. B Appl., 23: 179-192.

Khanna, A. and N. Kaur, 2016b. Effect of structural parameter on vibration of non homogeneous visco elastic rectangular plate. J. Vibr. Eng. Technol., 4: 459-466.

Khanna, A. and N. Kaur, 2016a. Effect of thermal gradient on vibration of non-uniform visco-elastic rectangular plate. J. Inst. Eng. India. Ser. C., 97: 141-148.

Khanna, A., R. Deep and D. Kumar, 2015. Effect of thermal gradient on vibration of non-homogeneous square plate with exponentially varying thickness. J. Solid Mech., 7: 477-484.

Leissa, A.W. and Y. Narita, 1980. Natural frequencies of simply supported circular plates. J. Sound Vibr., 70: 221-229.

Leissa, A.W., 1969. Vibration of Plate. NASA, Washington, DC., USA., Pages: 353.

Liew, K.M., Y. Xiang and S. Kitipornchai, 1993. Transverse vibration of thick rectangular plates-I: Comprehensive sets of boundary conditions. Comput. Struct., 49: 1-29.

Sharma, A. and A.K. Sharma, 2016. Mathematical modeling of vibration on parallelogram plate with non homogeneity effect. Rom. J. Acoust. Vibr., 13: 53-57.

Sharma, A., A.K. Raghav, A.K. Sharma and V. Kuamr, 2016. A modeling on frequency of rectangular plate. Intl. J. Control Theory Appl., 9: 277-282.

Sharma, A., A.K. Sharma, A.K. Raghav and V. Kumar, 2016a. Effect of vibration on orthotropic visco-elastic rectangular plate with two dimensional temperature and thickness variation. Indian J. Sci. Technol., 9: 1-7.

Sharma, A., A.K. Sharma, A.K. Raghav and V. Kumar, 2016b. Vibrational study of square plate with thermal effect and circular variation in density. Rom. J. Acoust. Vibr., 13: 146-152.

Sherafatnia, K., G. Farrahi and S.A. Faghidian, 2013. Analytic approach to free vibration and buckling analysis of functionally graded beams with edge cracks using four engineering beam theories. Intl. J. Eng. Trans. C Aspects, 27: 979-990.

Tariverdilo, S., M. Shahmardani, J. Mirzapour and R. Shabani, 2013. Asymmetric free vibration of circular plate in contact with incompressible fluid. Applied Mathe. Mod., 37: 228-239.

Wang, Y.Z., F.M. Li and K. Kishimoto, 2011. Thermal effects on vibration properties of double-layered nanoplates at small scales. Compos. Part B. Eng., 42: 1311-1317.

Zhou, D., 2002. Vibrations of point-supported rectangular plates with variable thickness using a set of static tapered beam functions. Intl. J. Mech. Sci., 44: 149-164.