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# Some Properties of Total Frame Domination in Graphs

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**Abstract:** The principle of domination in graph theory has been applied in various fields, epically in computer science. Several definitions of domination have been presented during last decades. In this research we introduce a new concept of domination in graph G = (V, E) named "total frame domination". Also, we define the inverse total frame domination. In addition, we determine the total frame domination number and total frame dominating set through this research. Furthermore, we prove some important properties of total frame domination number. Finally, we study the effects of removing vertex or edge from the given graph. According to the result of this research, we conclude that the total frame domination is more effective in real life applications, especially in social media field.

**Key words:** Domination, frame domination, total frame domination, mathematical subject, graph, social media

## INTRODUCTION

Recently, huge computer science applications have been presented in various fields. Graph theory is a modern mathematical branch that has many applications in the real world. Social media is one of the most important of these applications.

Throughout this research, we deal with finite, simple and undirected graphs. For a graph, G = (V, E) the vertex and edge setsare denoted by V(G) and E(G), respectively. The complement G of a simple graph G with vertex set V(G) is the graph in which two vertices are adjacent if and only if they are not adjacent in. G. G [D] is a subgraph of G induced by the vertices in D. Thesubgraphs obtained from G by deleting the vertex v or edge e are denoted by G-v and G-e, respectively. While G+e is the graph obtained from G by adding the edge e where $e\in\bar{G}$. The contraction of an edge e = uv is the operation that induced by removing the edge e from the graph and merges the two vertices u and v in one vertex. This operation is denoted by G/e.

Harary (1969) introduced a theoretic terminology for graph in details. The concept of domination number γ(G) in graph theory was introduced by Ore (1962). Haynes *et al*. (1998a, b) studied several topics in domination. Haynes *et al*. (1998) provides advance topics in domination. The inverse domination number $(\gamma^{-1})(G)$ was introduced by Kulli and Sigarkanti (1991). Several studies dealt with the domination number parameters can found by Kiunisala (2016), Omran and Harere (2016), Sampathkumar and Neeralagi (1986), Kral *et al*. (2012) and Walikar *et al*. (1979).

In this research, we introduce a new concept of domination in graph, named "total frame domination". We also propose total frame dominating and an inverse total frame dominating sets. In addition, we determine the total frame domination number and inverse total frame domination number through this research. Furthermore, we provesome important properties of total frame dominating and inverse total frame dominating sets. Finally, we study the effects of removing vertex or edge from the given graph.

## THE CONCEPT OF TOTAL FRAME DOMINATION

In this study, we need to refer to twodefinitions related to the frame domination on graphs that were used in our previous research (Omran and Harere, 2016). For more clarify, we found it's better to rewrite these definitions because the related work under publish (accepted) as follows.

**Definition 2.1:** "Let G = (V, E) be a graph and F⊆V(G). Then, the set F is called a "frame dominating set" if for every v∈V-F there is a cycle contains V and a vertex in F. The frame domination number $\gamma_f(G)$ is the minimum cardinality of a frame dominating set of Omran and Harere (2016).

**Proposition 2.2:** "If G has a frame domination number, γf(G) then γf(G)≤γ(G) and equality hold if and only if every cycle in G is a triangle" (Omran and Harere, 2016).

Subsequently, we introduce a new definition of domination in graphs, named total frame domination as well as total frame dominating set and total frame domination number. Furthermore, we determine and prove some properties about total frame dominating set andtotal frame domination number.

**Definition 2.2:** Let $G = (V, E)$ be a graph and $T \subseteq V(G)$. The set T is called a "total frame dominating set" if G[T] has no an isolated vertex. The total frame domination number $\gamma tf(G)$ is the minimum cardinality of a total frame dominating set of G.

**Definition 2.3:** Let $G = (V, E)$ be a graph and $T \subseteq V(G)$ is a frame dominating set. The set T' is called an inverse total frame dominating set with respect toT, if V-T contains a total frame dominating set. The minimum cardinality of an inverse total framedominating set of G is called the inverse total frame domination number denoted by $\gamma_{tf}^{-1}(G)$. From the definition of totalframe domination, one can easily calculate the next observation.

**Observation 2.4:** $\gamma_{tf}(C_n) = \gamma_{tf}(K_n) = \gamma_{tf}(W_n) = \gamma t_f(K_{m,n)} = 2$.

**Proposition 2.5:** Let, G be a graph of order n if G has a total frame domination numbert hen $\gamma_{tf}(G)$ then $2 \leq \gamma_{tf}(G) \leq 2 \lfloor n/3 \rfloor$.

**Proof:** Since, the total frame dominating set has no isolated vertex, then the minimum number of total frame domination is two and this case happens when a vertex is common to all cycles in the graph or there are two adjacent vertices such that each cycle in the graph is adjacent to one of the two vertices. The upper bound occurs when G have components and each component is a triangle.

**Theorem 2.6:** If a graph G of order n has a total frame domination number $\gamma_{tf}(G)$ then:

- G is not necessarily having an inverse total frame domination
- $\gamma_{tf}(G) \leq \gamma_{tf}^{-1}(G)$
- $4 \leq \gamma_{tf}(G) + \gamma_{tf}(G) \leq n$

**Proof:** If a disjoint graph G has total frame domination and one of its components is, $K_3$ then any total frame dominating set must contains two vertices from this component. Therefore, the remaining vertex in only one (isolated vertex). In this case, it's obvious that G has no inverse total frame dominating set.

It is obvious from the definitions of total frame and inverse total frame domination. By the definition of total frame domination, the minimum cardinality of all total frame dominating sets are two as well for inverse total frame dominating set. Thus, the lower bound is achieved. The upper bound occurs when every component isisomorphic to $C_4$ in this case every vertex must belongs to theminimum total frame dominating set T or to the inverse total frame dominating set with respect to T.

## COMPARISON BETWEEN VARIOUS PARAMETERS OF DOMINATION

In this study, we introduce acomparison between various dominationsparameters, namely, domination, frame domination and total frame domination. According to definitions of frame domination and total frame domination mentioned above, we would like to refer that every vertex must belong to a cycle.

**Theorem 3.1:** If G has domination, frame domination and total frame domination number, then:

- $\gamma_f(G) \leq \gamma(G)$
- $\gamma_f(G) \leq \gamma_{tf}(G)$
- The comparison between $\gamma(G)$ and $\gamma_{tf}(G)$ holds all possibilities

**Proof:**

- The result is straightforward by proposition 2.5
- The result is hold by definitions of frame domination and total frame domination number
- There are three cases as follows
- Case 1; if each componentisa cycle of order three, then it is clear that in this case $\gamma(G) < \gamma_{tf}(G)$
- Case 2; if each componentisa cycle of order four, then it is clear that in this case $\gamma(G) = \gamma_{tf}(G)$
- Case 3; if each components isa cycle of order greater than four in this case $\gamma(G) > \gamma_{tf}(G)$

There are other possibilities to achieve the above cases in general, so, all the possibilities are open and thus we get the result. Before, we state and prove the next theorem, we prefer to remember the reader by the following definition.

**Definition 3.2:** "The Triangular snake $T_n$ is the graph obtained from the path $P_n$ having the vertices $v_1, v_2, ..., v_n$ by adding new vertices $w_1$, w2, ..., $w_{n-1}$ and connecting $w_i$ to the vertices $v_i$, $v_{i+1}$ for each i" (Fig. 1) (Selvi, 2015).

**Theorem 3.3:** Let be a triangular snake, then $\gamma_{tf}(G)$:

$$\gamma_{tf}(G) = \left\{ \begin{array}{l} 2\left\lceil \frac{n-1}{3} \right\rceil, \text{if } n \equiv 0, 1\,(\text{mod}3) \\ 2\left\lfloor \frac{n-1}{3} \right\rfloor + 1\text{if } n \equiv 2\ \text{mod}3) \end{array} \right\}$$

**Proof:** There are two cases depending on the number of triangular induced subgraphs as follows. If, n = 0, 1 (mod 3) then let:

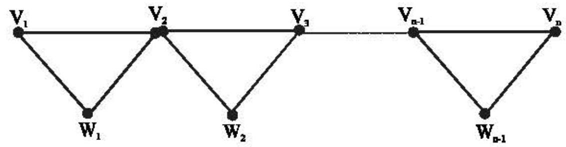


Fig. 1: Triangular snake $T_n$

$$T = \left\{ V_{2+3i}, V_{3+3,i}\ i = 1, 2, ..., \left\lceil \frac{N-1}{3} \right\rceil \right\}$$

It is clear that the set T is a total frame dominating set in sit in $T_n$, since, these vertices represent the common vertices between the path and the triangles with the following number:

$$\left\{ 2+3_i, i = 0, 1, ..., \left\lceil \frac{n-2}{3} \right\rceil \right\}$$

Also, the number of these vertices is the minimum total frame domination number. If we delete one vertex from T then the set T-v must contains an isolated vertex. Therefore, the other vertices cannot dominate all triangles. In this case one can determine that $|T| = 2\lceil n-1/3 \rceil$ so, we get the result.

If, n = 2 (mod 3), then let, T = $\{v_{2+3i}, v_{3+3i}, i = 1, 2, .., \lfloor n-1/3 \rfloor$. By the same manner in case (i) it is clear that the set $\{v_{2+3i}, v_{3+3i}, i = 1, 2, ..., \lfloor n-1/3 \rfloor$ is the minimum total frame dominating set in $T_n$ except the two vertices $\{v_n, w_{n-1}\}$. Therefore, in order to dominate these vertices, we can add the vertex $\{v_{n-1},\}$ to T. This vertex is adjacent to the last vertex in the set T and common with two vertices $\{v_n, w_{n-1})$ by a cycle. Thus, T is the minimum total frame dominating set in $T_n$ It easy to calculate that:

$$|T| = 2\left\lfloor \frac{n-1}{3} \right\rfloor + 1$$

**Definition 3.4:** The square snake $Sq_n$ is the graph obtained from the path $P_n$ having the vertices $v_1, v_2, ..., vn$ where n is odd and adding the new vertices $w_1, w_2, ..., w\ \lfloor n/2 \rfloor$ such that each $w_i$ is adjacent to the vertices $v_{2i-1}$ and $v_{2i+1}$ for each i (Fig. 2).

**Theorem:** Let, G be a square snakegraph, $Sq_n$ then $Y_{tf}(G) = \lfloor n/2 \rfloor$.

**Proof:** To prove this theorem we have to process the following three cases depending on the number of squarescontained in G.

If $\lfloor n/2 \rfloor = 0$ (mod 3) then let, T = $\{v_{3+6i}, v_{4+6i}, i = 0, 1, .., \lfloor n/2/3-1 \rfloor\}$. It is clear that the number of the squares induced subgraphs ($C_4$) is $\lfloor n/2 \rfloor$. While every vertex

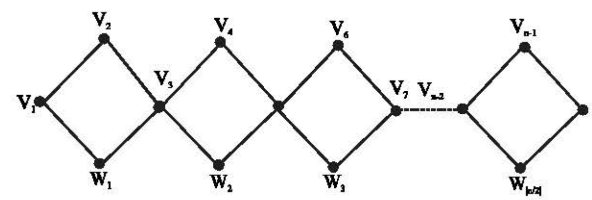


Fig. 2: Square snake $Sq_n$

wi, i= 1, 2, ..., $\lfloor n/2 \rfloor$ is common with exactly one square, then T is a total frame dominating set, since, every three consecutive vertices in T are dominating the corresponding three squares in $Sq_n$. It is obvious that T has a minimum cardinality of all other total frame dominating sets, since, if we delete any vertex from T, we loss the domination or the total domination. In this case the cardinality number of T is equal to $\lfloor n/2 \rfloor$.

If $\lfloor n/2 \rfloor = 2$ (mod 3), then let, T = $\{\{v_{3+6i,} v_{4+6i}, i = 0, 1, ..., \lfloor n/2 \rfloor -2\} \cup \{v_{n-2}, v_{n-1}\}\}$ in the same manner of previous case the set $\{\{v_{3+6i,} v_{4+6i}, i = 0, 1, ..., \lfloor n/2 \rfloor -2\}$ is the minimum dominating set to all vertices in the squares which contain the vertices $\{w_i, I = 1, 2, ..., \lfloor n/2 \rfloor -2\}$. Thus, this set isnot total frame dominating set, since, it contains the last two squares which have the vertices $\{w\lfloor n/2 \rfloor -1,^w\ \lfloor n/2\}$. Therefore, the set $\{v_{n-2}, v_{n-1}\}$ is a total frame dominating set with minimum cardinality. Thus, $|T| = \lfloor n/2 \rfloor$ so, we get the result for this case. If $\lfloor n/2 \rfloor = 1$ (mode 3) then let:

$$T = \left\{ v_{3+6i}, v_{4+6i}, v_{5+6i}, i = 0, 1, ..., \left\lfloor \frac{\frac{n}{2}}{3} \right\rfloor - 4 \right\} \cup \{v_{n-6}, v_{n-5}, v_{n-2}, v_{n-1}\}$$

Again in the same manner of the previous two cases, the set $\{v_{3+6i}, v_{4+6i}, i = 0, 1, ..., |n/2|-4$ is the minimum dominating set to all vertices in the squares which contains the vertices $\{w_i, i = 1, 2, ..., |n/2|-4$. The remaining vertices that are not dominated by this set are contained in the last four squares. But thelast four squares containthe vertices {w |n/2-3, w |n/2|-2, W |n/2|-1, w|n/2|. Therefore, it is obvious that the set $\{v_{n-6}, v_{n-5}, v_{n-2}, v_{n-1}$ is the total frame dominating set with minimum cardinality. Thus, in this case $|T| = \lfloor n/2 \rfloor$ and we achieve the result.

**Changing and unchanging:** In this study, we investigate the affection of total frame domination parameter when a graph is modified by deleting or adding or contracting an edge or deleting a vertex. As in the first section, we need to refer to the following theorems related to the frame domination that were used in our previous research (Omran and Harere, 2016).

**Theorem 4.1:** "If a graph G has a frame domination number $\gamma_f(G)$ then G-v then has either no frame domination number or" $\gamma_f(G\text{-}v) \geq \gamma_f(G)$" (Omran and Harere, 2017).

**Theorem 4.2:** "If G is a graph which has frame domination number $\gamma_f(G)$ and $e\epsilon E$, then G-e has either no frame domination number or $\gamma_f(G\text{-}e) = \gamma_f(G)$" (Omran and Harere, 2016).

**Theorem 4.3:** "If G is a graph has frame domination number $\gamma_f(G)$ and $e\epsilon E$ then" $\gamma_f(G+e) \leq \gamma_f(G)$ (Omran and Harere, 2016).

**Theorem 4.4:** "If G is a graph has a frame domination number and $e\epsilon E$, then G/e has no frame domination number or $\gamma_f(G/e) \leq \gamma_f(G)$" (Omran and Harere, 2016).

For save time, we can briefly say that all changes occurred in above theorems in this study as a result of deletion, addition or contraction in frame domination will remain hold in total frame domination and this applies to the edges that connect the two sets T and V-T. The difference appears in only three cases as follows.

**Case 1:** When we remove avertex connects with a vertex of degree 2. In this case the remaining vertex will be isolated. Therefore, we lose the total frame domination.

**Case 2:** When we remove an edge such that at least one of the end vertices become isolated in this case we lose the total frame domination.

**Case 3:** When we get an isolated vertex caused by contraction an edge, again in this case, we lose the total frame domination.

## CONCLUSION

In this research, we introduced a new definition for the concept of domination in graphs, calledtotal frame domination. We, also defined the inverse total frame domination as well as the total frame dominating set. In addition, we defined a total frame domination number. We have discussed some properties of total frame domination number and compared with counterparts in other dominations. In this resaerch, we also provide analysis tovarious theorems related to the total frame domination. According to the result of this resaerch we can conclude that the total frame domination is stronger than other dominations. Therefore, the total frame domination is more effective in real life applications, especially in social media field.

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