

The Comparative Study for Statistical Process Control for Software Reliability Property Utilizing Inverse Rayleigh and Rayleigh Distribution

Hee-Cheul Kim

Department of Industrial and Management Engineering, Namseoul University,
31020 Cheonan-si, Korea

Abstract: Software progress package enhancement has assisted in windup about trustworthy software produce. At this point, SPC (Statistical Process Control) can be special program of procedure running about performance of numerical algorithm. Henceforth, the value of mean value functions of inverse Rayleigh and Rayleigh distribution in order to develop proper mean control chart were projected. In this study, a control tool was established on time among failures observations using inverse Rayleigh and Rayleigh distribution property was planned using Non Homogeneous Poisson Process (NHPP). It is significant to use the finest chart using the specified statistics, state and requirement. The failure control chart has revealed using control signals, i.e., less the lower control limit. Consequently, the planned mean value chart signs out of control state at previous prompt than the condition in failure time switch chart. By taking reasonable measures from the initial discovery of software failure time will progress the software reliability. When the failure time among failures is less than lower control limit, it can be investigated using assignable sources from characteristic main process. In this object, from management quality side, Rayleigh distribution model is determined than inverse Rayleigh distribution more efficient model.

Key words: Statistical process control, inverse rayleigh and rayleigh distribution, non homogeneous process, control limits, failure control chart, efficient model

INTRODUCTION

An inspection of software reliability technique is main movement for enhancement software quality (Komuro, 2006). Until recently, numerous scholars have suggested the practice of SPC (Statistical Process Control) about software process judgment (Smitha *et al.*, 2015; Prasad *et al.*, 2011). Over the decades, SPC has ascended to be lengthily recycled in engineering industries for the self-control of monitoring and judgment criteria (Sargut and Demirors, 2006; Xie *et al.*, 2002).

In these research areas, SPC (Statistical Process Control) is a scheme of procedure organization done usage of numerical study which contains and comprises the significant, monitoring and refining of the progressions (Smitha *et al.*, 2015). The control chart in computing software reliability can be used as well-organized and proper SPC tools (Prasad *et al.*, 2011). In this study, a regulator device was formed on time between failures reflections by means of inverse Rayleigh and Rayleigh distribution property was proposed which was shaped on Non Homogeneous Poisson Process (NHPP). The offered control device contains estimation value for the mean value function of finite failure model in order to develop appropriate mean control chart was reflected.

MATERIALS AND METHODS

NHPP procedure using finite failure property: The special feature about mean value and intensity property aimed at Non-Homogeneous Poisson Process (NHPP) Model are specified next shapes (Kim and Kim, 2014a, b).

$$m(t) = \int_0^t \lambda(s) ds, \frac{dm(t)}{dt} = \lambda(t) \quad (1)$$

So, $N(t)$ was acknowledged poisson Probability Mass Function (PMF) following the parameter $m(t)$. Considering this situation, it was can summarized by way of following feature (Goel and Okumoto, 1979).

$$p(N(t) = n) = \frac{[m(t)]^n}{n!} e^{-m(t)}, n = 0, 1, \dots, \infty \quad (2)$$

In summary, the failure period reflect representations using the NHPP property was shaped by the probability of occurrence of failure can be specified. As for the characteristics of this model if the failure intensity feature $\lambda(t)$ can be specified differently (Dubey, 1973), similarly mean value feature $m(t)$ will be stated different form.

The failure period reflect representations can be considered the finite failure NHPP features and infinite

failure feature (Kuo and Yang, 1996). Let θ indicates the expected value of faults that can be revealed from finite failure NHPP Model, property of the mean value feature about the finite failure NHPP Models were detailed using following relation from (Kuo and Yang, 1996).

$$m(t) = \theta F(t) \tag{3}$$

In Eq. 3, $F(t)$ represents Cumulative Distribution Function (CDF). Using Eq. 3, characteristic form of the intensity function $\lambda(t)$ for the finite failure NHPP Models were specified following relation form.

$$\lambda(t) = \theta F'(t) = \theta f(t) \tag{4}$$

If $\{t_n, n = 1, 2, \dots\}$ mean continuous software failure interval time, t_n specifies the failure period among $(n-1)^{th}$ and n^{th} failure time. If x_n expresses n^{th} failure time, the x_n can be specified next relation form.

$$x_n = \sum_{k=1}^n t_k \quad (k = 1, 2, \dots, n; 0 \leq x_1 \leq x_2 \leq \dots \leq x_n) \tag{5}$$

The characteristic form of likelihood function of x_1, x_2, \dots, x_n can be specified next relation form (Gokhale and Trivedi, 1999).

$$f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) = e^{-m(x_n)} \prod_{i=1}^n \lambda(x_i) \tag{6}$$

Software reliability model based on finite NHPP using inverse Rayleigh and Rayleigh distribution

Inverse Rayleigh distribution: The inverse Rayleigh distribution has numerous applications in the reliability department. A probability density function and the distribution feature were known next form (Rao *et al.*, 2013; Voda, 1972).

$$f(t) = \frac{2b}{t^3} \exp\left(-\frac{b}{t^2}\right), F(t) = \exp\left(-\frac{b}{t^2}\right) \quad (b > 0) \tag{7}$$

where $b > 0, t \in (0, \infty]$, b is the scale parameter. As a result, the characteristic form of the finite failure NHPP intensity feature and the mean value feature can be stated through Eq. 3 and 4 from the next relation form (Gokhale and Trivedi, 1999).

$$\lambda(t) = \theta f(t) = \theta \frac{2b}{t^3} \exp\left(-\frac{b}{t^2}\right), m(t) = \theta F(t) = \theta \exp\left(-\frac{b}{t^2}\right) \tag{8}$$

The characteristic form of likelihood function using Eq. 6 and 8 can be stated next relation (Gokhale and Trivedi, 1999).

$$L_{NHPP}(\Theta | \underline{x}) = \left[\prod_{i=1}^n \theta \frac{2b}{x_i^3} e^{-\frac{b}{x_i^2}} \right] \exp\left[-\theta e^{-\frac{b}{x_n^2}}\right] \tag{9}$$

where $\underline{x} = (x_1, x_2, x_3, \dots, x_n)$, Θ is parameter space. From log-likelihood function through Eq. 9, $\hat{\theta}_{MLE}$ and \hat{b}_{MLE} can be stated using the solutions of the following relation forms.

$$\hat{\theta} = \frac{n}{\frac{b}{x_n^2}} \tag{10}$$

$$\frac{n}{b} = -\sum_{i=1}^n \ln\left(\frac{1}{x_i^3}\right) + \sum_{i=1}^n \left(\frac{1}{x_i^2}\right) - \frac{\hat{\theta}}{x_n^2} e^{-\frac{\hat{b}}{x_n^2}} \tag{11}$$

Rayleigh distribution: The probability density feature and the distribution feature of Rayleigh distribution are recognized as following pattern formulae (Shin and Kim, 2014; Tadikamalla, 1980).

$$f(t) = 2bt \exp(-bt^2), F(t) = 1 - \exp(-bt^2) \tag{12}$$

where $b > 0, t \in (0, \infty]$. Accordingly, the characteristic form of finite failure NHPP intensity feature and the mean value feature can be specified next relation form using Eq. 3 and 4 as the following relation (Gokhale and Trivedi, 1999).

$$\lambda(t) = \theta f(t) = 2\theta b t \exp(-bt^2) \tag{13}$$

$$m(t) = \theta F(t) = \theta \left[1 - \exp(-bt^2)\right] \tag{14}$$

where that b states to the shaping parameter. The characteristic form of likelihood function using Eq. 6, 13 and 14 can be specified next relation form.

$$L_{NHPP}(\Theta | \underline{x}) = \left[\prod_{i=1}^n 2\theta b x_i e^{-bx_i^2} \right] \exp\left[-\theta(1 - e^{-bx_n^2})\right] \tag{15}$$

where $\underline{x} = (x_1, x_2, x_3, \dots, x_n)$, Θ states parameter space. From the characteristic form of the log-likelihood formula using Eq. 15, $\hat{\theta}_{MLE}$ and \hat{b}_{MLE} can be estimated as the solutions of the following relation forms.

$$\hat{\theta} = \frac{n}{1 - e^{-bx_n^2}}, \frac{n}{b} = \sum_{i=1}^n x_i^2 + \hat{\theta} x_n^2 e^{-\hat{b} x_n^2} \tag{16}$$

Approach for the statistical process control based on finite failure: In this study, inverse Rayleigh distribution and Rayleigh distribution for the analysis of time among failures and watching of reliability were useful. Using distribution feature (Rao *et al.*, 2011), UCL (Upper Control Limit) (t_U), LCL (Lower Control Limit) (t_L) and CL (Center Line) (t_C) for software reliability models can be established using 6-sigma probability (0.99865, 0.00135 and 0.5). The calculating for the upper control limit based on finite NHPP of inverse Rayleigh distribution model from Eq. 7 can be specified using next relation form.

$$F(t) = \exp\left(-\frac{b}{t^2}\right) = 0.99865 \quad (17)$$

From the characteristic Eq. 17, the upper control limit can be caused following relation.

$$t = \sqrt{\frac{b}{-\ln(0.99865)}} = t_U \quad (18)$$

In the same way, the lower control limit (t_L) and centerline (t_C) limit can be caused following relation property.

$$t = \sqrt{\frac{b}{-\ln(0.00135)}} = t_L, t = \sqrt{\frac{b}{-\ln(0.5)}} = t_C \quad (19)$$

From sequential difference using $m(t)$, the upper control limit ($m(t_U)$) lower control limit ($m(t_L)$) and centerline ($m(t_C)$) were can be specified next shapes (Prasad *et al.*, 2011).

$$m(t_U) = \theta F(t_U) = \theta \exp\left(-\frac{b}{t_U^2}\right) \quad (20)$$

$$m(t_L) = \theta F(t_L) = \theta \exp\left(-\frac{b}{t_L^2}\right) \quad (21)$$

$$m(t_C) = \theta F(t_C) = \theta \exp\left(-\frac{b}{t_C^2}\right) \quad (22)$$

From the characteristic point of Rayleigh distribution (Voda, 1972) from Eq. 12 in the same way, the upper control limit ($m(t_U)$), lower control limit ($m(t_L)$) and centerline ($m(t_C)$) can be specified next property from sequential difference of the $m(t)$.

$$m(t_U) = \theta F(t_U) = \theta \left[1 - \exp\left(-bt_U^2\right)\right] \quad (23)$$

$$m(t_L) = \theta F(t_L) = \theta \left[1 - \exp\left(-bt_L^2\right)\right] \quad (24)$$

$$m(t_C) = \theta F(t_C) = \theta \left[1 - \exp\left(-bt_C^2\right)\right] \quad (25)$$

$$t_U = \sqrt{\frac{\ln(0.00135)}{-b}}, t_L = \sqrt{\frac{\ln(0.99865)}{-b}}, \\ = t_C \sqrt{\frac{\ln(0.5)}{-b}}$$

As a result, it has the following characteristic properties. The first, the control limit is such that the point estimation above the $m(t_U)$ (UCL) is an alarm indicator. The second, the point estimation under the $m(t_L)$ (LCL) is a signal of bad quality of the software product because the time between failures time is short period. The third, the point estimation within the control limits specifies unchanging procedure. Also, $m(t_C)$ (CL) specifies central line (Rachna Soni *et al.*, 2011; Prasad *et al.*, 2011).

RESULTS AND DISCUSSION

Process control analysis using software failure time material: In this study, the failure time material (Hayakawa and Telfar, 2000) was used to analyze characteristic points about SPC. The failure time material was listed in Table 1.

Using the maximum likelihood estimation, the parameter estimation about projected model was performed. The consequence of parameter approximation was registered in Table 2. Additionally, the estimation value of $m(t_L)$, $m(t_U)$ and $m(t_C)$ were assessed in Table 3.

Table 1: Failure time data

Failure No.	Failure time (h)	Failure No.	Failure time (h)
1	0.479	16	10.771
2	0.745	17	10.906
3	1.022	18	11.183
4	1.576	19	11.779
5	2.610	20	12.536
6	3.559	21	12.973
7	4.252	22	15.203
8	4.849	23	15.640
9	4.966	24	15.980
10	5.136	25	16.385
11	5.253	26	16.960
12	6.527	27	17.237
13	6.996	28	17.600
14	8.170	29	18.122
15	8.863	30	18.735

Table 2: Maximum likelihood estimation

Model	Maximum Likelihood Estimation (MLE)
Inverse Rayleigh	$\hat{b}_{MLE} = 1.652 \times 10^{-4}$ $\hat{\theta}_{MLE} = 30.01$
Rayleigh	$\hat{\theta}_{MLE} = 36.05$ $b_{MLE} = 5.10 \times 10^{-3}$

Table 3: Control limits estimation

Model	Control limits		
	$m(t_L)$	$m(t_C)$	$m(t_U)$
Inverse Rayleigh	29.969	15.005	0.0405
Rayleigh	36.001	18.025	0.0487

Table 4: Sequential difference of mean value function

Failure No.	Failure time (i)	m (i) I = 1, 2, ..., 30		m = (i)-m (i-1) sequential difference of mean value function	
		Inverse rayleigh	Rayleigh	Inverse Rayleigh	Rayleigh
1	0.479	14.6073	0.0422	7.6771	0.0597
2	0.745	22.2844	0.1019	3.3354	0.0896
3	1.022	25.6198	0.1915	2.4591	0.2623
4	1.576	28.0789	0.4538	1.2121	0.7772
5	2.610	29.2910	1.2309	0.3302	1.0242
6	3.559	29.6211	2.2552	0.1159	0.9202
7	4.252	29.7370	3.1754	0.0629	0.8984
8	4.849	29.7999	4.0738	0.0098	0.1867
9	4.966	29.8096	4.2605	0.0130	0.2772
10	5.136	29.8226	4.5377	0.0082	0.1947
11	5.253	29.8309	4.7325	0.0630	2.3076
12	6.527	29.8939	7.0401	0.0150	0.9233
13	6.996	29.9089	7.9634	0.0269	2.4380
14	8.170	29.9358	10.4014	0.0111	1.4985
15	8.863	29.9470	11.8999	0.0203	4.2000
16	10.771	29.9673	16.0999	0.0010	0.2955
17	10.906	29.9683	16.3955	0.0020	0.6039
18	11.183	29.9704	16.9993	0.0039	1.2843
19	11.779	29.9743	18.2836	0.0042	1.5919
20	12.536	29.9785	19.8755	0.0021	0.8939
21	12.973	29.9806	20.7694	0.0080	4.1895
22	15.203	29.9886	24.9590	0.0012	0.7368
23	15.640	29.9897	25.6957	0.0009	0.5524
24	15.980	29.9906	26.2482	0.0009	0.6338
25	16.385	29.9915	26.8820	0.0012	0.8540
26	16.960	29.9928	27.7361	0.0005	0.3921
27	17.237	29.9933	28.1282	0.0007	0.4948
28	17.600	29.9940	28.6229	0.0009	0.6738
29	18.122	29.9949	29.2967	0.0010	0.7350
30	18.735	29.9959	30.0317		

From applied the nth failure time statistics, the estimation values of $m(t)$ about t_L, t_C, t_U can be reflected. Therefore, the sequential difference estimation value of the $m(t)$ can be calculated (n-1) values. In Table 4, displays the failures time, the corresponding $m(t)$ and sequential difference estimation among $m(t)$ using inverse Rayleigh distribution (Voda, 1972) and Rayleigh distribution (Tadikamalla, 1980).

Using $m(t_L), m(t_C)$ and $m(t_U)$, the failure control charts for inverse Rayleigh distribution and Rayleigh distribution NHPP Model are screening forms (Fig. 1 and 2).

In Fig. 1, the situation of failure control chart using inverse Rayleigh distribution is out of control from almost all the failure number, because the matching sequential difference estimation of $m(t)$ decreasing under the LCL. This case results in out-of-control for the software product quality. In such cases, should investigate the possible causes about this environment and make improvements manner (Soni *et al.*, 2011; Prasad *et al.*, 2011).

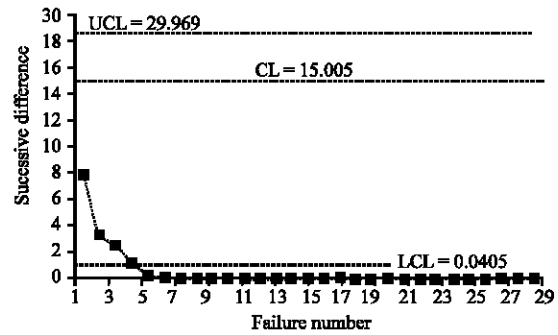


Fig. 1: Result of mean value chart using inverse Rayleigh distribution

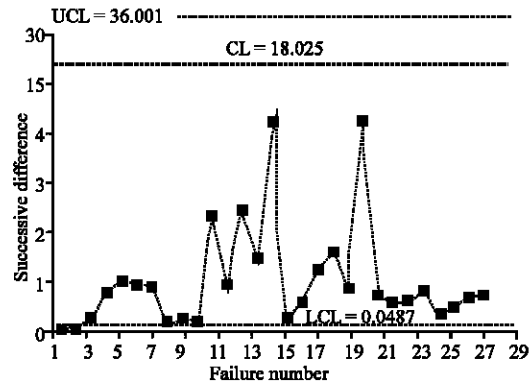


Fig. 2: Result of mean value chart using Rayleigh distribution

On the other hand, using Rayleigh distribution in Fig. 2, for situation of failure control chart, matching sequential difference estimation of $m(t)$ decreasing under not the LCL. Hence, the situation of failure control charts using Rayleigh distribution is in-of-control for the product feature. Furthermore, because Rayleigh distribution model than inverse Rayleigh distribution seems in-of-control situation of sequential difference between $m(t)$, the Rayleigh distribution NHPP Model than inverse Rayleigh distribution NHPP Model can be said that the failure time between failures is long period. Ultimately, the Rayleigh distribution model is determined inverse Rayleigh distribution model more efficient mode 1.

CONCLUSION

There are many charts using statistical analysis method. Software progress package enhancement has assisted in windup about trustworthy software produce. At this point, SPC (Statistical Process Control) can be special program of procedure running about performance of numerical algorithm. In this study, the value of mean value functions of inverse Rayleigh and Rayleigh

distribution in order to develop proper mean control chart were projected. The situation of failure control chart using inverse Rayleigh distribution is out of control from almost all the failure number because the matching sequential difference estimation of mean value decreasing under the lower control limit line. This case results in out-of-control for the software product quality. In such cases, should investigate the possible causes about this environment and make improvements manner, Using Rayleigh distribution, situation of failure control chart, matching sequential difference estimation of mean value decreasing under not the lower control limit. Hence, the situation of failure control charts using Rayleigh distribution is in-of-control for the product feature. Furthermore, because Rayleigh distribution model than inverse Rayleigh distribution seems in-of-control situation of sequential difference between mean value, the Rayleigh distribution NHPP Model than inverse Rayleigh distribution NHPP Model can be said that the failure time between failures is long period. Ultimately, the Rayleigh distribution model is determined than inverse Rayleigh distribution model more efficient model.

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