Journal of Engineering and Applied Sciences 13 (9): 2614-2621, 2018
ISSN: 1816-949X

Modeling and Simulation of a Pipeline Transportation Process

[1]Javier Jimenez Cabas and [2]Jose David Ruiz Ariza
[1]Department of Mechanical Engineering, Universidad del Norte, km 5 via. Puerto Colombia, Barranquilla, Colombia
[2]Department of Computer Science and Electronics, Corporacion Universidad de la Costa (CUC), Barranquilla, Colombia

Abstract: Two kind of pipeline models are considered, the first is based on Reynolds-Averaged Navier-Stokes (RANS) equations and the second is a transfer function based model obtained from the first one model linearization. The two models were implemented taking into account a real pipeline network parameters and models responses were compared with real data. A well-fitting data between real process and model response was observed in both cases.

Key words: Reynolds-averaged, linearization, implemented, parameters, observed, cases

INTRODUCTION

Transporting oil products through pipelines has been a common method to take these products to its final place for the last decades. Provided it is widely used, the final cost of the transported fluid is strongly dependent on how effective this transportation process has been carried out. Consequently, optimizing scheduling activities and process conditions, has been a main concern of study for at least 30 years. Pipelines can be constructed to accomplish a simple task such as taking one type of product from one source to one destination. On the other hand, complexity level can be higher where pipelines with multiple destinations can be mentioned and finally the more realistic pipelines systems that are totally capable of handling several destinations and multiple fluids, these are mainly used in refineries transporting fluids such as kerosene, naphtha and gas oil (Herran *et al*., 2010).

Pipeline configuration depends on topography, production and consumption rate, season production and region requirements. There can be several configurations of a transport pipeline, as function of distance between stations for storage, distribution or pumping; depending on these parameters, several models can be developed (Herran *et al*., 2010; Neuroth *et al*., 2000; Zhu *et al*., 2001; Matko *et al*., 2000; Lurie, 2008; Blazic *et al*., 2004; Jimenez *et al*., 2017). At current study, two models are considered (Matko *et al*., 2000; Blazic *et al*., 2004) the first is based on Reynolds-Averaged Navier-Stokes (RANS) equations and the second is a transfer function based model obtained of the first one model linearization.

The two models were implemented taking into account a real pipeline network parameters and models responses were compared with real data. A well-fitting data between real process and model response was observed in both cases.

MATERIALS AND METHODS

Pipeline model: Referring to pipeline governing equations, there have been developed several models (Herran *et al*., 2010; Matko *et al*., 2000). Thus, at current study next set of differential equations (obtained using continuity, momentum and energy equations) is used to represent the dynamical behavior of the fluid transportation along the pipeline.

A pipeline of length L_p and constant radius R as that shows in Fig. 1 is considered. The assumptions made for mathematical model derivation of the flow through pipelines are (Blazic *et al*., 2004):

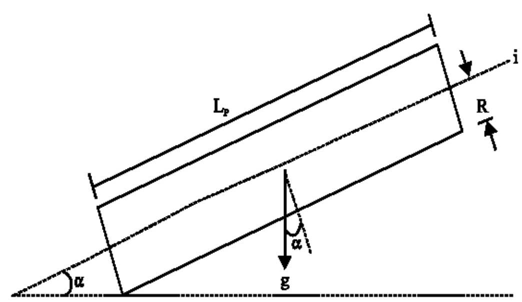

Fig. 1: Pipeline section diaram

Corresponding Author: Javier Jimenez Cabas, Department of Mechanical Engineering, Universidad del Norte, km 5 via. Puerto Colombia, Barranquilla, Colombia

- Compressible fluid, resulting in an unsteady flow
- Viscous flow, viscosity causes shear stresses in a moving fluid
- Adiabatic flow, no transfer of energy between fluid and pipeline will be considered
- Isothermal flow, temperature changes due to pressure changes and friction effects can be neglected, consequently, the temperature along the pipeline is considered constant
- One-dimensional flow, pipeline characteristics (as velocity and pressure) depend only on the z-axis laid along the pipeline
- Flow profile hydraulically developed. Which imply $\partial v_z/\partial z = 0$ and $v_z = v_z(r)$

First, continuity equation in cylindrical coordinates is stated (Bird *et al*., 2002):

$$\frac{\partial\rho}{\partial t}+\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)}_{5}+\frac{\partial}{\partial z}(\rho v_z)=0 \quad (1)$$

As it is considered to be flowing mainly in one direction (assumption 5) the continuity equation is then reduced to the expression:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial z}(\rho v_z)=0 \quad (2)$$

Considering that total derivative can be expressed as (Anderson, 1995):

$$\frac{d}{dt}=\frac{D}{Dt}=\frac{\partial}{\partial t}+V.\nabla$$

Where $V = [v_r\ v_\theta\ v]$is the velocity vector and :

$$\nabla=\frac{\partial}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}+\frac{\partial}{\partial z}\hat{z}$$

Then, for $\rho = \rho(r, \theta, z, t)$ we have:

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+v_r\frac{\partial\rho}{\partial r}+\frac{v_\theta}{r}\frac{\partial\rho}{\partial\theta}+v_z\frac{\partial\rho}{\partial z}$$

But due to assumption number 5 in our case $\rho = \rho(z, t)$ thus:

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+v_z\frac{\partial\rho}{\partial z}$$

By replacing Eq. 3 and 2:

$$\frac{d\rho}{dt}+\rho\frac{dv_z}{dz}=0 \quad (4)$$

Where $\partial(pv)/\partial z = p\ \partial v_z/\partial z+v_z\partial p/\partial z$ has been taken into account in Eq. 2. Second expression is given by the equation of motion (Bird *et al*., 2002):

$$\rho\left(\frac{\partial v_z}{\partial t}+\underbrace{v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}}_{5}+v_z\frac{\partial v_z}{\partial z}\right)= -\frac{\partial\rho}{\partial z}-\left[\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\frac{1}{r}\frac{\partial}{\partial\theta}\tau_{\theta z}}_{5}+\frac{\partial}{\partial z}\tau_{zz}\right]+\rho g_z \quad (5)$$

Newton's law of viscosity (Bird *et al*., 2002):

$$\tau_{rz}=\tau_{zr}=-\mu\left[\underbrace{\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}}_{5}\right]$$

$$\tau_{\theta z}=\tau_{z\theta}=-\mu\left[\underbrace{\frac{1}{r}\frac{\partial v_z}{\partial\theta}+\frac{\partial v_\theta}{\partial z}}_{5}\right]$$

$$\tau_{zz}=\tau_{zz}=-\mu\left[2\frac{\partial v_z}{\partial z}+\left(\frac{3}{2}\mu-\kappa\right)(\nabla.V)\right]$$

$$=-\mu\left[2\frac{\partial v_z}{\partial z}+\left(\frac{3}{2}\mu-\kappa\right)\left(\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(rv_r)}_{5}+\frac{1}{r}\underbrace{\frac{\partial v_\theta}{\partial\theta}}_{5}+\frac{\partial v_z}{\partial z}\right)\right]$$

$$=-\mu\left[2\frac{\partial v_z}{\partial z}+\left(\frac{3}{2}\mu-\kappa\right)\left(\frac{\partial v_z}{\partial z}\right)\right]$$

Bearing in mind assumptions Eq. 5 and 6 the Eq. 6 of motion is then reduced to:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial\rho}{\partial z}-\frac{\partial\tau_{zz}}{\partial z}-\rho g\sin\alpha \quad (6)$$

Where $g_z = g\ \sin\ \alpha$ is the gravitational effect along z-axis (Fig. 1) and it has been taken into account that:

$$\frac{dv_z}{dt}=\frac{\partial v_z}{\partial t}+\underbrace{v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}}_{5}+v_z\frac{\partial v_z}{\partial z}=\frac{\partial v_z}{\partial t}+ v_z\frac{\partial v_z}{\partial z}+\rho\frac{dv_z}{dt}=-\frac{\partial\rho}{\partial z}-\frac{\partial\tau_{zz}}{\partial z}-\rho g\sin\alpha \quad (7)$$

It is important to mention Eq. 7 is stated in term of the change of the normal and shear stresses respect to every direction, physically this term (second term in the right side of) represents loses of energy due to friction. However, in order to eliminate derivatives in r-axis, an empirical approach made by Darcy and Weisbach, Eq. 8 can be used to determine these loses:

$$\frac{\partial \tau_{zz}}{\partial z} = -\rho \frac{\lambda v|v|}{2d}$$

Where, λ represents a dimensionless friction coefficient and d is the pipeline diameter. Then, combining Eq. 8 and 6 using total derivatives, the equation of motion can be rewritten as:

$$\frac{dv}{dt} + \frac{1}{\rho}\frac{dp}{dz} + g\sin\alpha + \frac{\lambda v|v|}{2d} = 0 \tag{9}$$

Finally, this set of equations is closed by taking into account density changes due to pressure changes which can be stated as seen at Eq. 10:

$$\frac{dp}{dt} - a^2\frac{d\rho}{dt} = 0 \tag{10}$$

Where α is fluid speed of sound. Now, taking as boundary conditions the pipe line input pressure, P_0 and the output flow, Q_L and using finite differences, Eq. 4 and 9 can be discretized in n section as follows:

$$\begin{aligned} &\frac{d\rho}{dt} + \frac{\rho}{A}\frac{q_{i+1}-q_i}{\Delta z_i} = 0 \; \frac{1}{A}\frac{dq}{dt} + \\ &\frac{1}{\rho}\frac{p_{i+1}-p_i}{\Delta z_i} + g\sin\alpha + \frac{\lambda q|q|}{2dA^2} = 0 \end{aligned} \tag{11}$$

Where A is the cross-sectional area of the pipe, $q = Av$ is the flow rate, $i = 1, 2, ..., n$, $q_i = q_{in} = q_0$ and $q_{i+1} = q_{out} = q_L$. Thus, Eq. 10 and 11 constitute the first pipeline model to be implemented at current study.

Alternatively, due complexity to solve the set of equations composed by Eq. 4, 9 and 10 (actually, to date, there is no general closed-form solution) it has been proposed several linearized versions of this set (Matko *et al*., 2000; Blazic *et al*., 2004). In order to get a linearized model the procedure presented in Blazic *et al*., (2004) is followed and it start by replacing total derivatives with partial ones in Eq. 9 and 10; multiplying by Eq. 9 and p combining Eq. 4 and 10:

$$\begin{aligned} &\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g\sin\alpha + \frac{\lambda v|v|}{2d} = 0 \\ &\frac{\partial p}{\partial t} + v_z\frac{\partial p}{\partial z} + a^2\rho\frac{\partial v_z}{\partial z} = 0 \end{aligned} \tag{12}$$

Neglecting the usually small convective derivatives:

$$\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial z} \approx \frac{\partial v_z}{\partial t} \quad \frac{\partial p}{\partial t} + v_z\frac{\partial p}{\partial z} \approx \frac{\partial p}{\partial t}$$

And using q = Av, Eq. 12 can be rewritten as follows:

$$\begin{aligned} &\frac{\rho}{A}\frac{\partial q}{\partial t} + \frac{\partial p}{\partial z} + \rho g\sin\alpha + \frac{\lambda q|q|}{2dA^2}\rho = 0 \\ &\frac{\partial p}{\partial t} + \frac{a^2\rho}{A}\frac{\partial q}{\partial z} = 0 \end{aligned} \tag{13}$$

Defining the deviation variables around a operation point $(\bar{\rho}, \bar{q}, \bar{p})$:

$$\begin{aligned} D(z,t) &= \rho(z,t) - \bar{\rho}(z) \\ Q(z,t) &= q(z,t) - \bar{q}(z) \\ P(z,t) &= p(z,t) - \bar{p}(z) \end{aligned} \tag{14}$$

Linearizing:

$$\begin{aligned} &\frac{\bar{\rho}}{A}\frac{\partial Q}{\partial t} + \frac{\partial P}{\partial z} + Dg\sin\alpha + \frac{\lambda\bar{q}|\bar{q}|}{2dA^2}D + \frac{\lambda\bar{\rho}|\bar{q}|}{2dA^2}Q = 0 \\ &\frac{\partial P}{\partial t} + \frac{a^2\bar{\rho}}{A}\frac{\partial Q}{\partial z} + \frac{a^2}{A}\frac{\partial\bar{q}}{\partial z}D = 0 \end{aligned} \tag{15}$$

Since, $\partial\bar{q}/\partial z$ is typically very small (Blazic *et al*., 2004) and doing $C = A/a^2\bar{\rho}$ $L = \bar{\rho}/A$ $T = g\sin\alpha + \lambda\bar{q}|\bar{q}|/2dA^2$ and $R = \lambda\bar{\rho}|\bar{q}|/2dA^2$ Eq. 15 can be rewritten as:

$$L\frac{\partial Q}{\partial t} + TD + RQ = -\frac{\partial P}{\partial z} \quad C\frac{\partial P}{\partial t} = -\frac{\partial Q}{\partial z} \tag{16}$$

To get an analytical solution of , Eq. 16 llplace transform is used:

$$\begin{aligned} (Ls+R)Q(z,s) + TD(z,s) &= -\frac{dP(z,s)}{dz} + LQ(z,0) \\ CsP(z,s) &= -\frac{dQ(x,s)}{dz} + CsP(z,0) \end{aligned} \tag{17}$$

And since, transfer function definition implies null stationary initial condition:

$$
\begin{aligned}
&(Ls{+}R)Q(z,s)+TD(z,s) = -\frac{dP(z,s)}{dz}\\
&CsP(z,s) = -\frac{dQ(z,s)}{dz}
\end{aligned} \tag{18}
$$

Taking derivative with respect to z:

$$
\begin{aligned}
&-(Ls+R)\frac{dQ(z,s)}{dz}-T\frac{dD(x,s)}{dz} = \frac{\partial^2 P(z,s)}{\partial z^2}\\
&-Cs\frac{dP(z,s)}{dz} = \frac{d^2Q(z,s)}{dz^2}
\end{aligned} \tag{19}
$$

Combining Eq. 18 and 19:

$$
\begin{aligned}
&(Ls+R)CsP(z,s)-T\frac{dD(x,s)}{dz} = \frac{d^2P(z,s)}{dz^2}\\
&(Ls+R)CsQ(z,s)+TCsD(z,s) = \frac{d^2Q(z,s)}{dz^2}
\end{aligned} \tag{20}
$$

Let now to replace total derivatives with partial ones in Eq. 4 and to linearize:

$$
\begin{aligned}
&\frac{\partial D(z,t)}{\partial t}+\frac{1}{A}\frac{d\overline{\rho}(z)}{dz}Q(z,t)+\frac{\overline{q}}{A}\frac{\partial D(z,t)}{\partial z}\\
&+\frac{1}{A}\frac{\partial \overline{q}(z)}{\partial z}D(z,t) = 0
\end{aligned} \tag{21}
$$

By taking the Laplace transform:

$$
\begin{aligned}
&sD(z,s)-D(z,0)+\frac{1}{A}\frac{d\overline{\rho}(z)}{dz}Q(z,s)+\\
&\frac{\overline{q}}{A}\frac{\partial D(z,s)}{\partial z}+\frac{1}{A}\frac{\partial \overline{q}(z)}{\partial z}D(z,s) = 0
\end{aligned} \tag{22}
$$

Assuming null stationary initial condition:

$$
\frac{1}{A}\frac{\partial \overline{q}(z)}{\partial z} = \Delta q_z
$$

and doing Eq. 22 can be rewritten as:

$$
\frac{dD(z,s)}{dz}+\frac{(s{+}\Delta q_z)A}{\overline{q}}D(z,s) = -\frac{1}{\overline{q}}\frac{d\overline{\rho}(z)}{dz}Q(z,s) \tag{23}
$$

The homogeneous part of has as solution:

$$
D(z,s) = D_0(s)e^{-kx} \tag{24}
$$

Where D_0 (s) = D (0, s) and k = $(s+\Delta q_z)$ $A/\overline{q}$. It is possible to show that Eq. 24 can be taken as a solution of Eq. 23 (Blazic *et al*., 2004). Now, considering Eq. 23 and 20 can be transformed to:

$$
\begin{aligned}
&(Ls{+}R)CsP(z,s) = \frac{d^2P(z,s)}{dz^2}-TD_0(s)ke^{-kx}\\
&(Ls{+}R)CsQ(z,s) = \frac{d^2Q(z,s)}{dz^2}-CsTD_0(s)ke^{-kx}
\end{aligned} \tag{25}
$$

Which are known as wave equations and their solutions are:

$$
\begin{aligned}
&P(z,s) = C_1(s)e^{-nz}+C_2(s)e^{nz}-\frac{TD_0(s)k}{n^2-k^2}e^{-kz}\\
&Q(z,s) = C_3(s)e^{-nz}+C_4(s)e^{nz}-\frac{CsTD_0(s)}{n^2-k^2}e^{-kz}
\end{aligned} \tag{26}
$$

Where n^2 = (LS+R)Cs. By combining second Eq. 26 and first Eq. 18 in and by differentiating first Eq. 26 in with respect to z:

$$
\begin{aligned}
&(Ls{+}R)\left[C_3(s)e^{-nz}+C_4(s)e^{nz}-\frac{CsTD_0(s)}{n^2-k^2}e^{-kz}\right]\\
&= nC_1(s)e^{-nz}-nC_2(s)e^{nz}-\frac{TD_0(s)k^2}{n^2-k^2}e^{-kz}-TD_0(s)e^{-kz}
\end{aligned} \tag{27}
$$

Taking into account that:

$$
TD_0(s)e^{-kx}\left[\frac{(Ls{+}R)Cs}{n^2-k^2}-1-\frac{k^2}{n^2-k^2}\right] = 0
$$

The following relations are obtained from Eq. 27:

$$
\begin{aligned}
&\frac{C_1(s)}{C_3(s)} = \frac{Ls+R}{n} = \sqrt{\frac{Ls+R}{Cs}} = Z_K\\
&\frac{C_2(s)}{C_4(s)} = -\frac{Ls+R}{n} = \sqrt{Ls+RCs} = -Z_K
\end{aligned} \tag{28}
$$

Using the boundary condition for z = 0:

$$
\begin{aligned}
P(0,s) &= P_0(s) = C_1(s)+C_2(s)-\frac{TD_0(s)k}{n^2-k^2}\\
&= Z_K\left[C_3(s)-C_4(s)\right]-\frac{TD_0(s)k}{n^2-k^2}\\
Q(0,s) &= Q_0(s) = C_3(s)+C_4(s)-\frac{CsTD_0(s)}{n^2-k^2}
\end{aligned} \tag{29}
$$

It is obtained:

$$
\begin{aligned}
&C_3(s) = \frac{1}{2}\left[Q_0(s)+\frac{1}{Z_K}P_0(s)+\frac{TD_0(s)}{Z_K(n-k)}\right]\\
&C_4(s) = \frac{1}{2}\left[Q_0(s)-\frac{1}{Z_K}P_0(s)+\frac{TD_0(s)}{Z_K(n+k)}\right]
\end{aligned} \tag{30}
$$

Using the boundary condition for z = Lp in Eq. 26:

$$P(L_p,s)=P_L(s)=Z_K\left[C_3(s)e^{-nL_p}-C_4(s)e^{nL_p}\right]-\frac{TD_0(s)k}{n^2-k^2}e^{-kL_p}\quad Q(L_p,s)=Q_L(s)=C_3(s)e^{-nL_p}+C_4(s)e^{nL_p}-\frac{CsTD_0(s)}{n^2-k^2}e^{-kL_p} \tag{31}$$

By replacing Eq. 30 and 31 it is obtained:

$$\begin{aligned}P_L(s)&=-Z_KQ_0(s)\sinh(nL_p)+P_0(s)\cosh(nL_p)+\frac{TkD_0(s)}{n^2-k^2}\left[\cosh(nL_p)-\frac{n}{k}\sinh(nL_p)-e^{-kL_p}\right]\\Q_L(s)&=Q_0(s)\cosh(nL_p)-\frac{1}{Z_K}P_0(s)\sinh(nL_p)+\frac{TkD_0(s)}{Z_K(n^2-k)^2}\left[\frac{n}{k}\cosh(nL_p)-\sinh(nL_p)-\frac{n}{k}e^{-kL_p}\right]\end{aligned} \tag{32}$$

Taking into account $k=(s+\Delta q_z)A/\bar{q}$ $C=A/a^2\bar{\rho}L=\bar{\rho}/AR=\lambda\bar{\rho}|\bar{q}|/2dA^2$ and $n^2=(Ls+R)Csn/k$ can be written as follows:

$$\frac{n}{k}=\frac{\sqrt{(Ls+R)Cs}}{(s+\Delta q_z)A/\bar{q}}=\frac{\sqrt{(\bar{\rho}s+\bar{\rho}\lambda|\bar{q}|/dA)/a^2\bar{\rho}s}}{(s+\Delta q_z)A/\bar{q}}=\frac{\bar{q}}{aA}\sqrt{\frac{s^2+s\lambda|\bar{q}|/d}{(s+\Delta q_z)^2}} \tag{33}$$

Since, $a>>\bar{q}/A=\bar{v}|n/k|<<1$ and the terns with n/k in can be neglected. Additionally, considering that:

$$\frac{k}{n^2-k^2}=\frac{1}{k\left[(n/k)^2-1\right]}\approx -m^{-1}=\frac{\bar{q}}{(s+\Delta q_z)A}n/Z_K=Cs \tag{34}$$

Eq. 32 can be rewritten as follows:

$$P_L(s)=-Z_KQ_0(s)\sinh(nL_p)+P_0(s)\cosh(nL_p)+\frac{T\bar{q}D_0(s)}{(s+\Delta q_z)A}\left[\cosh(nL_p)-e^{-kL_p}\right]Q_L(s)=Q_0(s)\cosh(nL_p)-\frac{1}{Z_K}P_0(s)\sinh(nL_p)+\frac{T\bar{q}D_0(s)}{Z_K(s+\Delta q_z)A}\left[-\sinh(nL_p)\right] \tag{35}$$

Since, Δq_z is very small:

$$\frac{\bar{q}}{(s+\Delta q_z)A}\approx\frac{\bar{q}/A}{s}\qquad e^{-kL_p}=e^{-[(s+\Delta q_z)A/\bar{q}]L_p}\approx e^{-sL_p/\bar{v}}=e^{-t_0s} \tag{36}$$

Where, t_0 is the dead time or transport delay, the time needed for the fluid to reach the outlet from the pipeline inlet.An additional equation which state the density at the pipeline outlet is obtained by setting z = Lp in Eq. 24as follows:

$$D(L_p,s)=D_L(s)=D_0(s)e^{-kL_p}=D_0(s)e^{-t_0s} \tag{37}$$

Conclusively, the transfer function matrix representing the pipeline is:

$$\begin{bmatrix}P_L\\Q_L\\D_L\end{bmatrix}=\begin{bmatrix}\cosh(nL_P) & -Z_K\tanh(nL_P) & -\frac{Tv_0}{s}\left(\cosh(nL_P)-e^{-t_0s}\right)\\-\frac{1}{Z_K}\sinh(nL_P) & \cosh(nL_P) & -\frac{Tv_0}{Z_Ks}\sinh(nL_P)\\0 & 0 & e^{-t_0s}\end{bmatrix}\begin{bmatrix}P_0\\Q_0\\D_O\end{bmatrix} \tag{38}$$

Where $v_0=\bar{q}/A$. The linearized non-causal model can be rewritten in one of the following forms; Representation 10: $[P_L\ Q_0\ D_0]^T\rightarrow[P_0\ Q_L\ D_L]$:

$$\begin{bmatrix}P_0\\Q_L\\D_L\end{bmatrix}=\begin{bmatrix}\frac{1}{\cosh(nL_P)}Z_K\tanh(nL_P) & & \\\frac{Tv_0}{s}\left(1-\frac{1}{\cosh(nL_P)}-e^{-t_0s}\right)- & & \\\frac{1}{Z_K}\tanh(nL_P) & \frac{1}{\cosh(nL_P)} & \frac{Tv_0}{Z_Ks}\tanh(nL_P)e^{-t_0s}\\0 & 0 & e^{-t_0s}\end{bmatrix}\begin{bmatrix}P_L\\Q_0\\D_0\end{bmatrix} \tag{39}$$

Representation 2: $[P_0\ Q_L\ D_0]^T\rightarrow[P_L\ Q_0\ D_L]$:

$$\begin{bmatrix}P_L\\Q_0\\D_L\end{bmatrix}=\begin{bmatrix}\frac{1}{\cosh(nL_P)} & -Z_K\tanh(nL_P) & -\frac{Tv_0}{s}\left(\frac{1}{\cosh(nL_P)}-e^{-t_0s}\right)\\\frac{1}{Z_K}\tanh(nL_P) & \frac{1}{\cosh(nL_P)} & -\frac{Tv_0}{Z_Ks}\tanh(nL_P)\\0 & 0 & e^{-t_0s}\end{bmatrix}\begin{bmatrix}P_0\\Q_L\\D_0\end{bmatrix} \tag{40}$$

Representation 3: $[Q_0\ Q_L\ D_0]^T \rightarrow [P_0\ P_L\ D_L]^T$

$$\begin{bmatrix}P_0\\P_L\\D_L\end{bmatrix}=\begin{bmatrix}Z_K\coth(nL_P) & -Z_K\dfrac{1}{\sinh(nL_P)} & \dfrac{Tv_0}{s}\\ Z_K\dfrac{1}{\sinh(nL_P)} & -Z_K\coth(nL_P) & \dfrac{Tv_0}{s}e^{-t_0 s}\\ 0 & 0 & e^{-t_0 s}\end{bmatrix}\begin{bmatrix}Q_0\\Q_L\\D_0\end{bmatrix} \quad (41)$$

Representation 4: $[P_o\ P_L\ D_0]^T \rightarrow [P_0\ P_L\ D_L]^T$

$$\begin{bmatrix}Q_0\\Q_L\\D_L\end{bmatrix}=\begin{bmatrix}\dfrac{1}{Z_K}\coth(nL_P)-\dfrac{1}{Z_K}\dfrac{1}{\sinh(nL_P)}- & & \\ \dfrac{Tv_0}{Z_K s}\left(\coth(nL_P)-\dfrac{1}{\sinh(nL_P)}e^{-t_0 s}\right)\dfrac{1}{Z_K}\dfrac{1}{\sinh(nL_P)} & & \\ -\dfrac{1}{Z_K}\coth(nL_P)-\dfrac{Tv_0}{Z_K s}\left(\dfrac{1}{\sinh(nL_P)}-\coth(nL_P)e^{-t_0 s}\right) & & \\ 0 & 0 & e^{-t_0 s}\end{bmatrix}\begin{bmatrix}P_0\\P_L\\D_0\end{bmatrix} \quad (42)$$

Representations 1-4 can be approximated to rational transfer functions matrix by using Taylor series expansion as follows:

$$\frac{1}{\cosh(nL_p)}=\frac{1}{\cosh\left(L_p\sqrt{(Ls+R)Cs}\right)}\approx\frac{1}{\left(\frac{1}{2}L_p^2LC+\frac{1}{24}L_p^4R^2C^2\right)s^2+\frac{1}{2}L_p^2RCs+1} \quad (43)$$

$$\frac{1}{Z_K}\tanh(nL_p)=\sqrt{\frac{Cs}{Ls+R}}\tanh\left(L_p\sqrt{(Ls+R)Cs}\right)\approx\frac{\frac{1}{6}L_p^3RC^2s^2+L_pCs}{\left(\frac{1}{2}L_p^2LC+\frac{1}{24}L_p^4R^2C^2\right)s^2+\frac{1}{2}L_p^2RCs+1} \quad (44)$$

$$Z_K\tanh(nL_p)=\sqrt{\frac{Ls+R}{Cs}}\tanh\left(L_p\sqrt{(Ls+R)Cs}\right)\approx\frac{\frac{1}{3}L_p^3RLCs^2+\left(L_pL+\frac{1}{6}L_p^3R^2C\right)s+L_pR}{\left(\frac{1}{2}L_p^2LC+\frac{1}{24}L_p^4R^2C^2\right)s^2+\frac{1}{2}L_p^2RCs+1} \quad (45)$$

$$\frac{1}{Z_K}\coth(nL_p)=\sqrt{\frac{Cs}{Ls+R}}\coth\left(L_p\sqrt{(Ls+R)Cs}\right)\approx\frac{\left(\frac{1}{2}L_p^2LC+\frac{1}{24}L_p^4R^2C^2\right)s^2+\frac{1}{2}L_p^2RCs+1}{\frac{1}{3}L_p^3RLCs^2+\left(L_pL+\frac{1}{6}L_p^3R^2C\right)s+L_pR} \quad (46)$$

$$\frac{1}{Z_K\sinh(nL_p)}=\sqrt{\frac{Cs}{Ls+R}}\frac{1}{\sinh\left(L_p\sqrt{(Ls+R)Cs}\right)}\approx\frac{1}{\frac{1}{3}L_p^3RLCs^2+\left(L_pL+\frac{1}{6}L_p^3R^2C\right)s+L_pR} \quad (47)$$

Finally, at current study the linearized model in Eq. 40 was chosen as the second model to be implemented.

RESULTS AND DISCUSSION

Model validation: In order to verify the models responses are correct, their performances were compared against data of a three section real pipeline transporting LPG (Fig. 2). For modeling each the pipeline sections first the set of Eq. 10 and 11 shown at and then Eq. 40 were used. Pressure in station 1 and flow in Station 4 were fixed as boundary conditions. Table 1 shows the main parameters used at currents models.

Figure 3 and 4 show pressure and flow rate model error obtained with the discretized model, model 1. The maximum model errors gotten were about 3.8 and 7.1% for pressure and flow rate in station 4 and 1, respectively.

The transfer function model for each section, considering parameters in Table 1 had the form:

$$\begin{bmatrix}P_L\\V_0\\D_L\end{bmatrix}=\begin{bmatrix}G_{11}(s) & G_{12}(s) & G_{13}(s)\\ G_{21}(s) & G_{22}(s) & G_{23}(s)\\ 0 & 0 & G_{33}(s)\end{bmatrix}\begin{bmatrix}P_0\\V_L\\D_0\end{bmatrix} \quad (48)$$

Where for pipeline section 1:

Table 1: Model parameters

Variables	Symbol	Value	Units
Density	ρ	530	kg/m^3
Friction coefficient	λ	0.00965	
Fluid speed of sound	a	2490	m/sec
Pipelines lengths	Lp_1	10326	
	Lp_2	18560	m
	Lp_3	14980	
Pipelines diameter	Dp	0.4968748	m
Pipelines inclinations	α_3	-1.93*10^{-4}	
	α_2	3.77*10^{-4}	red
	α_3	5.74*10^{-3}	

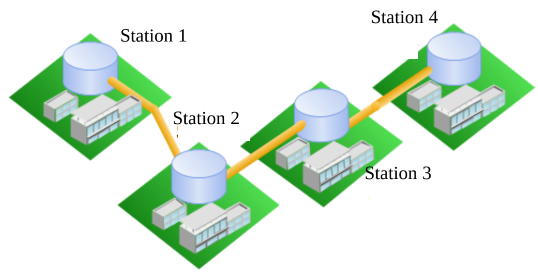

Fig. 2: Liquefied PetroLeum Gas (LPG) pipeline

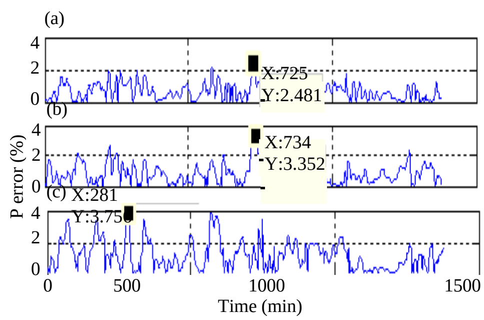

Fig. 3: Pressure model error-Model 1; a) Station 2; b) Station 3 and c) Station 4

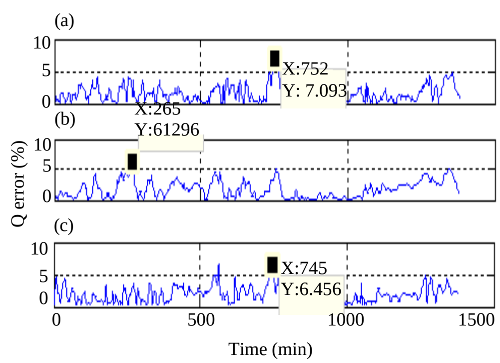

Fig. 4: Flow rate model error-Model 1; a) Station 1; b) Station 2 and c) Station 3

$$G_{11}(s)=\frac{1}{8.6s^2+0.2s+1}$$

$$G_{12}(s)=\frac{-9\times10^5s^2-5.5\times10^6s-1.6\times10^5}{8.6s^2+0.2s+1}$$

$$G_{13}(s)=-\frac{2.8\times10^{-2}}{s}\left(G_{11}(s)-e^{-7.1\times10^3s}\right)$$

$$G_{21}(s)=\frac{2.6\times10^{-7}s^2+3.1\times10^{-6}s}{8.6s^2+0.2s+1}$$

$$G_{22}(s)=G_{11}(s),\quad G_{33}(s)=e^{-7.1\times10^3s}$$

$$G_{23}(s)=\frac{-8.2\times10^{-10}s-1\times10^{-8}}{s^2+2.8\times10^{-2}s+0.1}$$

For pipeline section 2:

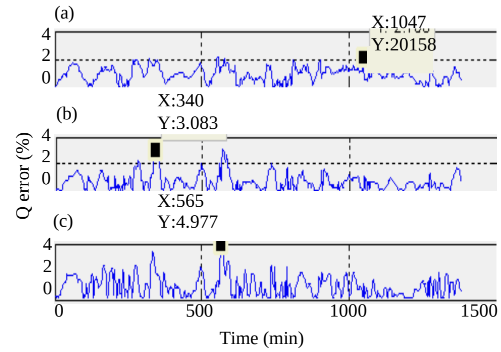

Fig. 5: Pressure model error; a) Model 1; b) Station 2 and c) Station 3 and Station 4

Table 2: Model errors

Model	Max pressure error (%)	Max flow rate error (%)
1	3.8	7.1
2	5.0	8.1

$$G_{12}(s)=\frac{-5.2\times10^6s^2-9.9\times10^6s-2.8\times10^5}{27.9s^2+0.8s+1}$$

$$G_{13}(s)=-\frac{3.6\times10^{-2}}{s}\left(G_{11}(s)-e^{-1.3\times10^4s}\right)$$

$$G_{21}(s)=\frac{1.5\times10^{-6}s^2+5.6\times10^{-6}s}{27.9s^2+0.8s+1}$$

$$G_{22}(s)=G_{11}(s),\quad G_{33}(s)=e^{-1.3\times10^4s}$$

$$G_{23}(s)=\frac{-1.9\times10^{-9}s-7.4\times10^{-9}}{s^2+2.8\times10^{-2}s+3.4\times10^{-2}}$$

And for pipeline section 3:

$$G_{11}(s)=\frac{1}{18.1s^2+0.5s+1}$$

$$G_{12}(s)=\frac{-2.8\times10^6s^2-8\times10^6s-2.3\times10^5}{18.1s^2+0.5s+1}$$

$$G_{13}(s)=-\frac{0.1}{s}\left(G_{11}(s)-e^{-10\times10^3s}\right)$$

$$G_{21}(s)=\frac{8\times10^{-7}s^2+4.6\times10^{-6}s}{18.1s^2+0.5s+1}$$

$$G_{22}(s)=G_{11}(s),\quad G_{33}(s)=e^{-10\times10^3s}$$

$$G_{23}(s)=\frac{-5\times10^{-9}s-2.9\times10^{-8}}{s^2+2.9\times10^{-2}s+5.5\times10^{-2}}$$

Figure 5 and 6 show pressure and flow rate model error obtained with the transfer functions model, model 2. For flow rate model error calculation, the relation Q = Av was considered. The maximum model errors gotten were about 5 and 8.1% for pressure and flow rate in Station 4 and 1, respectively.

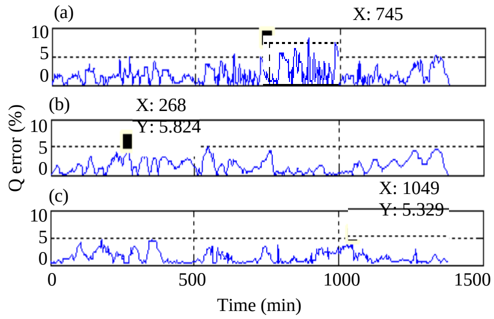

Fig. 6: Flow rate model error; a) Model 2; b) Station 1 and c) Station 2 and Station 3

CONCLUSION

The model error reached with both models. Having achieved well-fitting data between real process and models responses, it can be concluded that both model are describing the real system dynamics in a very accurate way. In fact as may be expected, Model 1 responses are more precise Than the obtained with model 2. Nevertheless, model 2 is easier to implement and less computational resources consumer.

On the other hand, since in the study of hydrocarbon transport, there have been implemented different control stratgies in order to optimize the pumping process, (Neuroth *et al*., 2000; Zhu *et al*., 2001) and to reduce downtime between transports of different substances (Gopalakrishnan *et al*., 2013) in control application, author suggests using model 2 for design (tuning) the control system and Model 1 to test the performance of the control system designed.

REFERENCES

Anderson, J.D., 1995. Computational Fluid Dynamics: The Basics with Applications. McGraw-Hill Education, New York, USA., ISBN: 9780070016859, Pages: 547.

Bird, R.B., W.E. Stewart and E.N. Lightfoot, 2002. Transport Phenomena. John Wiley & Sons Inc., New Jersey.

Blazic, S., D. Matko and G. Geiger, 2004. Simple model of a multi-batch driven pipeline. Math. Comput. Simulation, 64: 617-630.

Gopalakrishnan, A. and L.T. Biegler, 2013. Economic nonlinear model predictive control for periodic optimal operation of gas pipeline networks. Comput. Chem. Eng., 52: 90-99.

Herran, A., D.L.J.M. Cruz and D.B. Andres, 2010. A mathematical model for planning transportation of multiple petroleum products in a multi-pipeline system. Comput. Chem. Eng., 34: 401-413.

Jimenez, J., L. Torres, I. Rubio and M. Sanjuan, 2017. Auxiliary Signal Design and Liénard-Type Models for Identifying Pipeline Parameters. In: Modeling and Monitoring of Pipelines and Networks, Verde C. and L. Torres (Eds.). Springer, Berlin, Germany, ISBN: 978-3-319-55943-8, pp: 99-124.

Lurie, M., 2008. Modeling of Oil Product and Gas Pipeline Transportation. Wiley-Blackwell, Hoboken, New Jersey, USA., ISBN: 9783527408337, Pages: 214.

Matko, D., G. Geiger and W. Gregoritza, 2000. Pipeline simulation techniques. Math. Comput. Simulation, 52: 211-230.

Neuroth, M., P. MacConnell, F. Stronach and P. Vamplew, 2000. Improved modelling and control of oil and gas transport facility operations using artificial intelligence. Knowl. Based Syst., 13: 81-92.

Zhu, G.Y., M.A. Henson and L. Megan, 2001. Dynamic modeling and linear model predictive control of gas pipeline networks. J. Process. Control, 11: 129-148.