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Image Compression Using a Modified Principal Component Analysis Method

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Abstract: Principal Component Analysis (PCA) has received growing attention in its latent potential in image compression. However, the image reconstructed from PCA compressed data can be improved in terms of image quality and compression ratio. In this study, a modified PCA algorithm was considered. In this algorithm, the eigenvectors derived from the original image was used to reconstruct the compressed data. Performance evaluation show that PSNR and SSIM obtained for image compressed by the proposed modified PCA are significantly higher than the conventional PCA algorithm ($p<0.05$). The objective evaluation results were further confirmed by the visual inspection of the output images where less streaks and noise were found on image compressed by the proposed modified PCA at compression ratio as high as 90%.

Key words: Image compression, principal component analysis, quality metrics, subjective evaluation, image, obtained

INTRODUCTION

The goal of image compression is to reduce the number of bits used to represent an image in order to facilitate data transmission and storage. Transform based methods such as the Discrete Cosine Transform (DCT), Principal Component Analysis (PCA) and Discrete Wavelet Transform (DWT) are lossy compression methods commonly used in image compression. Due to its optimality of signal decorrelation and energy compaction, the PCA (also called the Karhunen-Loeve (KL) transform) achieves image compression by reducing the high dimensionality of the data inherent in an image while minimizing errors in the mean-square sense (Wang, 2012). In other words, PCA is a linear transformation that removes redundancy by decorrelating the data so that the information in an image can be represented more efficiently. It has been used in many pattern recognition (Webb and Keith, 2011; Baese and Schmid, 2014) and image processing (Ting *et al*., 2015; Costa and Fiori, 2001; Clausen and Wechsler, 2000) application as well as for medical image compression. Although, colour image compression using PCA is irreversible, it is being employed by Carevic and Caelli (1997) as an aided tool to decorrelate the spatial and spectral redundancy typically exhibited in a colour image. The research studies show that PCA and its variation (Taur and Tao, 1996; Bonab and Mofarreh, 2012; Pandey *et al*., 2011; Kumar *et al*., 2008; Lv and Zhao, 2005; Nowrozian and Hassanpour, 2014) can be successfully applied in image compression. However, the image quality can be severely affected when the compression ratio is high. In view of this, this study proposed a modified PCA algorithm that strives to maintain image quality even at a high compression ratio.

MATERIALS AND METHODS

Since, this study deal with image compression in particular, the description of the algorithm will be explained in connection with an image. Image can be represented by a matrix where each element along the rows and columns is the value of intensity at its specific location. The intention of applying PCA on an image is to reduce the dimensionality of the image in a way that the data has been compressed. Consider that PCA is applied on an image X with a size of $m\times n$, the output data can achieve a reduced size of $p\times m$. It is therefore meant that the number of dimension n is now reduced to p dimensions. PCA started with the elements of the original image minus the mean of each data row, $\bar{m}$:

$$X-\bar{m} = \bar{X} \quad (1)$$

The next step is crucial in PCA as to obtain the covariance matrix of $\bar{X}$. The covariance matrix measures the relationships between the dimensions as stated in Eq. 2:

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$$\mathrm{Cov}(\overline{X}) = \begin{bmatrix} s_1 & \mathrm{Cov}_{12} & \cdots & \mathrm{Cov}_{1n} \\ \mathrm{Cov}_{21} & s_2 & \cdots & \mathrm{Cov}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Cov}_{n1} & \mathrm{Cov}_{n2} & \cdots & s_n \end{bmatrix} \quad (2)$$

The diagonal elements in Eq. 2 are the variance of each dimension itself. It is obtained as shown in Eq. 3 where, N is the total number of elements in a dimension:

$$\sigma_j = \frac{\sum_{i=1}^{m}(X_{ij}-D_j)^2}{N-1} \quad (3)$$

The off-diagonal elements in Eq. 2 are the covariance and they are symmetrical, e.g., $\mathrm{Cov}_{21} = \mathrm{Cov}_{12}$. It is a measure of how a dimension deviate with respect to each other as formulated in Eq. 4:

$$\mathrm{Cov}_{ji} = \mathrm{Cov}_{ij} = \frac{\sum_{k=1}^{m}(X_{ki}-D_i)*(X_{kj}-D_j)}{N-1} \quad (4)$$

Next come in step is to obtain the eigenvectors of the covariance matrix that consists of the information of image characterization. An eigenvector of a matrix is a vector, when multiply to a transformation matrix or in this case covariance matrix, scaled from its original position without changing the direction:

$$A \cdot V = \lambda \cdot V \quad (5)$$

Where:

A = Transformation matrix in this case, covariance matrix
V = Eigenvector
λ = Eigenvalue

Eigenvector can only be found for square matrix and given a n×n square matrix that does have eigenvector, there will be n of them. The search for eigenvector of an image with a large data dimensions can be tedious but it can be done at ease with the help of any numerical computational software. Deriving from eigenvector, eigenvalue indicates the value by how far the original vector has been scaled and its value determine the rank of the eigenvectors in which eigenvector with the higher eigenvalue is the principal component of the image data. In MATLAB environment, princomp is the command used to compute the eigenvector of the input matrix from its covariance matrix. As shown in Eq. 6, the output of the command is a feature matrix, V that contains all principal component arranged according to the descending order of the Eigenvalues. It is hence suffice to say that the first column of the V matrix are the eigenvectors having the highest eigenvalues. p is the number of principal components selected in the feature matrix (Table 1):

Table 1: Mean for each dimension (column) in the mean-minus original image is obtained to form the covariance matrix

X	1	2	…	n
1	X_{11}	X_{12}	…	X_{1n}
2	X_{21}	X_{22}	…	X_{2n}
⋮	⋮	⋮	⋱	⋮
m	X_{m1}	X_{m2}	…	X_{mn}
Mean of each dimension	D_1	D_2	…	D_j

$$V = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1p} \\ V_{21} & V_{22} & \cdots & V_{11} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{np} \end{bmatrix}_{(n\times p)} \quad (6)$$

PCA deals with transforming the original data, so that, they are expressed in term of principal components. In this way, the compressed data, Y can achieve dimensionality reduction when less principal components are taken into account in Eq. 7. Eigenvectors are now used as the basic vectors for transformation:

$$Y = [\, V^T \times \overline{X}^T \,]_{(p\times m)} \quad (7)$$

No compression occurs if all principal components are selected for the transformation. The compression ratio increase when p reduces. The relationships between the compressed data and the principal components can be found in Eq. 8:

$$CR = 1-\frac{np}{mn} \quad (8)$$

It is important to note that the compressed data itself cannot visually represent the image without performing further steps. The compressed data, the feature matrix and the mean-minus matrix are again needed to reconstruct the image, g by performing the manipulation in Eq. 9:

$$g = (V\times Y)^T + \overline{m} \quad (9)$$

The image quality of the reconstructed image inherently depends on the principal component and thereby the compressed data, Y. With the goal of achieving higher image quality under the same compression rate, this study proposed a modified reconstructed scheme for a PCA algorithm (Fig. 1). He fact that feature matrix is one of the factors that contributes to the image quality in reconstruction initiates the idea

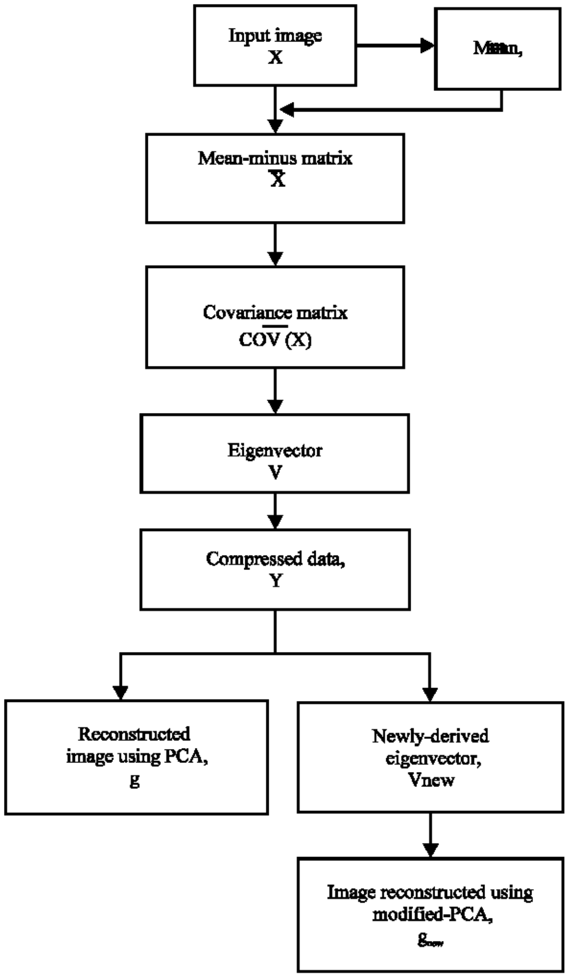

Fig. 1: Framework for the proposed modified-PCA algorithm

that employing a different feature matrix in reconstruction may help to improve image quality. Besides, the reconstructed image should faithfully represent the original image in any compression scheme. The new feature matrix, V_{new} is hence, formulated based on the original image, X as shown in Eq. 10:

$$V_{new} = (X-\overline{m})\times Y^{-1} \quad (10)$$

The compressed data is now ready to be reconstructed using V_{new}:

$$g_{new} = (V_{new}\times Y)^T+\overline{m} \quad (11)$$

RESULTS AND DISCUSSION

Algorithm implementation and performance testing in this study was done using MATLAB 2013a platform on a Dual Core 2.6 GHz PC. An uncompressed Lena image in TIFF format and with a size of 512×512 was used as the input image as shown in Fig. 2. Except for color to gray scale conversation no other pre-processing techniques have been performed on the image.

Fig. 2: The Lena image after gray scale

To evaluate the performance of the proposed modified-PCA algorithm, it is being compared with the existing PCA algorithm. The Lena image was compressed using both methods at an interval scale of compression rate from 10- 90% and the coding fidelity for both methods was compared using Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index (SSIM). PSNR is a commonly used metric that computes the mean-square error or similarity level between the compressed image and the original image. PSNR is simply converted from Mean Square Error (MSE) where the amount of distortion or error is determined from the difference between the pixels in the original image X and the output image g. Since, the image under test is a gray image, the dynamic range L in Eq. 12 is therefore, 255:

$$MSE(X, g) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}(X_{ij}-g_{ij})^2 \quad (12)$$

$$PSNR = 10\log_{10}\frac{L^2}{MSE} \quad (13)$$

Although, PSNR is extensively used in the literature, it is not sensitive to structural content in the image. For this reason to fairly evaluate the performance of the modified PCA algorithm in addition to PSNR, SSIM is used as a measure of similarity as a combination of three

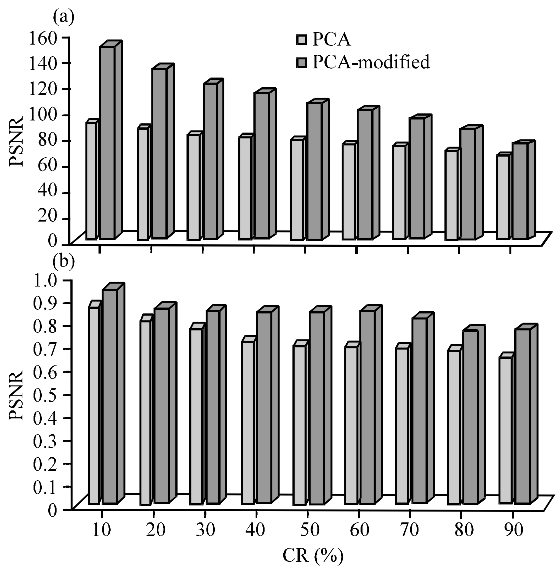

Fig. 3: Performance results of PCA and modified-PCA algorithm for different compression rate using: a) PSNR and b) SSIM

factors: the similarity of the luminance l(X,g), the similarity of the contrast c(X,g) and the similarity of the structures s(X,g) (Nowrozian and Hassanpour, 2014):

$$SSIM(X, g) = l(X, g)\times c(X, g)\times s(X, g) = \left(\frac{2\mu_X\mu_g+C_1}{\mu_X^2+\mu_g^2+C_1}\right)\times\left(\frac{2s_Xs_g+C_2}{s_X^2+s_g^2+C_2}\right)\times\left(\frac{s_{Xg}+C_3}{s_Xs_g+C_3}\right) \quad (14)$$

Where:

μ_x and μ_g = The local sample means of X
g, σ_x and σ_g = Correlation coefficient of X and g
C_1, C_2 and C_3 = Constants that prevent a null denominator

Statistical analysis was performed using SPSS V. 21.0. The differences in mean PSNR and mean SSIM for PCA and modified PCA algorithm was estimated using a Student-t-test. A value of $p<0.005$ was considered statistically significant.

As shown in Fig. 3a, b, the proposed modified-PCA algorithm obtained better PSNR and SSIM than the PCA algorithm at different compression rate. It can also be seen from Fig. 4a, b that the mean PSNR and SSIM are significant higher for images compressed using proposed method. Image compressed at compression rate of 10, 70 and 90% were shown in Fig. 5 to illustrate the visual perspective of the compressed images. The full image collection compressed at every compression rate is available by request. At CR = 10%, no visible distortion were seen on both images except some barely visible vertical line streaks on the image compressed using PCA algorithm. However, at CR = 70%, there is a clear distinction on image fidelity where image compressed using modified-PCA algorithm maintain artifacts-free while image compressed using PCA algorithm contains visible vertical line streak across the image due to the lack of information carried by the image. At the highest compression rate done in this study which is 90%, both images no longer hold their image integrity as compared to the original Lena image. In comparison, although the image compressed by modified-PCA algorithm does not exhibit visible streak lines as shown in image compressed by PCA algorithm, it is still generally blurred out and smudge with few horizontal streaks.

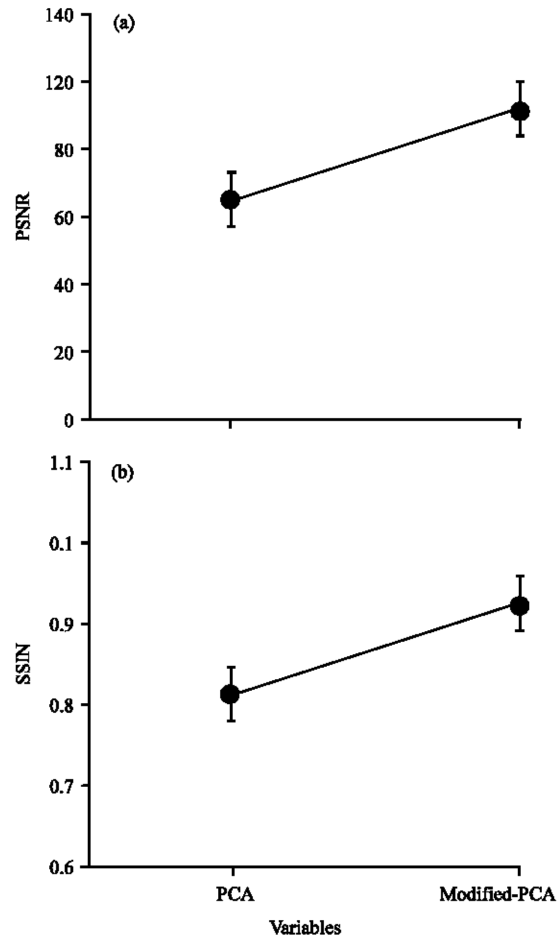

Fig. 4: Comparison of average fidelity metrics value obtained for both methods: a) PSNR and b) SSIM. There were a significant increase in values ($p<0.05$)

Fig. 5: Comparison of Lena image compressed and reconstructed by both algorithms; a-c) PCA algorithm at CR = 10, 70 and 90%, respectively; d-f) PCA-modified algorithm at CR = 10, 70 and 90, respectively

CONCLUSION

In this study, a new approach to compress image using PCA is considered. The novelty of the method lies in reconstructing the images using the eigenvectors derived from the original image. Despite its simplicity and fast computational process, image compressed by PCA algorithm often yield undesirable image degradation. Hence, the main motivation of this study comes from improving the image quality of the existing PCA method and as the name suggests, PCA-modified algorithm is developed based on the basis of PCA algorithm. By using this PCA-modified method, the compressed image captures more information about the original image at the same compression rate as the PCA method. The memory ratio for the compressed data and the computational time are the same for both methods.

Two aspects of measuring the coding efficiency for comparison between the PCA algorithm and proposed method were used. The performance evaluation by using these two metrics along with statistical analysis showed that performance of the proposed modified-PCA algorithm offers substantial improved performance compared to the PCA algorithm. Based on the results demonstrated from this study, further validation can be performed by using different datasets with comparison to reviewer's subjective rating. This may open up the opportunities to improve medical image compression and face recognition applications.

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