Journal of Engineering and Applied Sciences 13 (Special Issue 3): 3353-3357, 2018
ISSN: 1816-949X

Performance Analysis and Robust Control of a Ball and Beam System

[1]V.I. George, [1]Manu Varghese, [3]Ciji Pearl Kurian and [3]I. Thirunavukkarasu
[1]Department of Instrumentation and Control Engineering,
[2]Department of Electrical and Electronics Engineering,
[3]Department of Instrumentation and Control Engineering,
M.I.T Manipal, Manipal University Manipal, Udupi, Karnataka, India

Abstract: Robust control theory is useful when the plant may be stable, unstable, marginally stable, non-minimum phase or even certain extend nonlinear systems. In this study a ball and beam system is considered as an example with two poles at origin. Which is considered as unstable system. It is a special class of unstable system. A robust controller is designed to satisfy the robust performance criterion and the designed controller is compared with classical PID controller design. Also, the internal stability is checked by looking into the zeros of the characteristics equation.

Key words: Ball and beam system, performance, stability, controller, characteristics, poles

INTRODUCTION

Figure 1 shows the automatic control system which is used for robustness analysis. And is redrawn as generalised control system in LFT framework is shown in Fig. 2. Our aim is to design a robust controller controller, The signals w, z, y and u are in generally vector valued functions of time. Where w are all exogenous inputs (r-reference, d-disturbance, n-noise) z = signal we wish to control, y = output of all sensor and u = controlled input to the plant.

Performance of a control system is expressed in terms of the size of certain signals and is measured in one norm 2 norm or infinity norm. From the above block diagram (Doyle *et al*., 1992) there are three inputs and three outputs of interest. For internal stability there are nine transfer functions such as e/r (s), u/r (s), y/r (s), e/d (s), u/d (s), y/d (s), e/n (s), u/n (s), y/n (s). Sufficient condition for internal stability is the Infinity norm of the product of the plant and the controller is less than one at every frequency. In this study, we consider the analysis of a double integrator plant which is considered as the characteristic of an unstable open loop plant. That is its step response is exponentially increasing.

The robust performance criterion is for all frequencies the infinity norm of the product of a weighting function and the sensitivity function is less than one. That is the inequality has a nice graphical interpretation as shown in Fig. 3. For a multiplicative uncertainty model, the robust stability criterion is for all frequencies the infinity norm of the product of a weighting function and the complementary sensitivity function is less than one. The plant under consideration in this research is a typical (laboratory set up) ball and beam system is shown in Fig. 4 (Doyle *et al*., 1992).

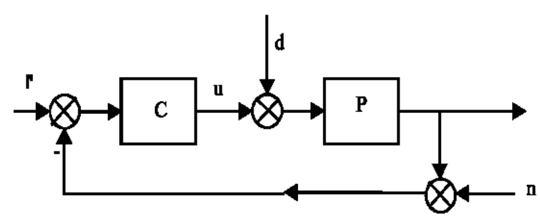

Fig. 1: Basic feedback system

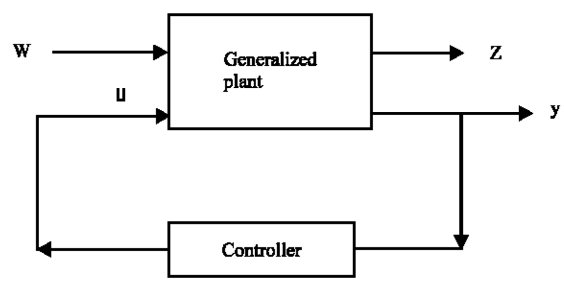

Fig. 2: Representation of control system in LFT framework

Corresponding Author: V.I. George, Department of Instrumentation and Control Engineering, M.I.T Manipal, Manipal University Manipal, Udupi, Karnataka, India

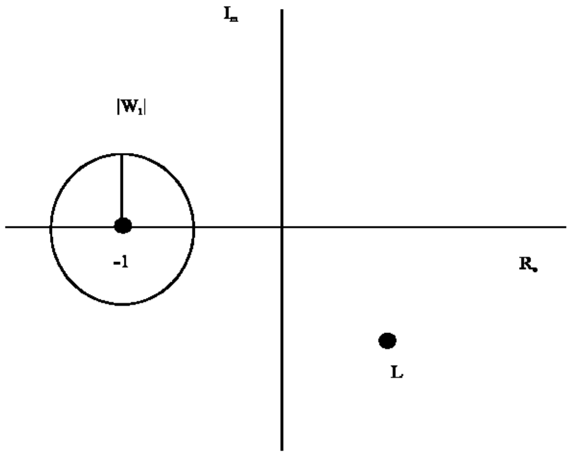

Fig. 3: For a multiplicative uncertainty model, the robust

Fig. 4: A typical ball and beam system

MATERIALS AND METHODS

Mathematical modelling: The net force acting on the system is equal to the sum of the translational motion (Virseda, 2004; GTC., 2006; Jose *et al*., 2015) of the ball and the force due to ball rotation Fig. 5. Figure 5 shows the dynamics of the system under consideration. Modelling and control of a ball and beam system is explained by Keshmiri *et al*. (2012) and Meenakshipriya and Kalpana *et al*. (2014). The parameters of the ball and beam system are defined as follows. Let the beam angle coordinate (α), Beam Length (L), Mass of the ball (m), Radius of the ball (R) and ball's moment of inertia (J) Gravitational acceleration (g) and Position of the ball. α neglecting frictional forces, the two forces influencing the motion of the ball. It can be derived the relation by linearizing for the ball position and the corresponding angle is given by the equation (Virseda, 2004):

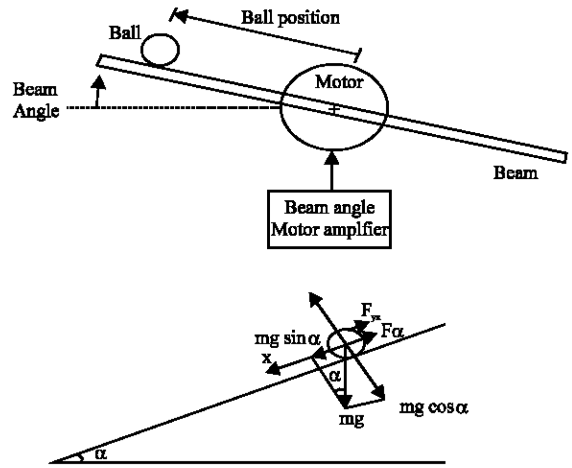

Fig. 5: Ball and beam schematic and Forces due to translation and rotation motion (Virseda, 2004)

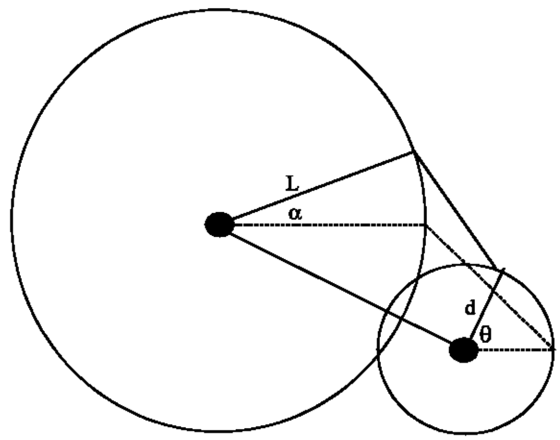

Fig. 6: Beam angle motor position of the system

$$\ddot{x} = \frac{5}{7} g \sin \alpha \tag{1}$$

$$\text{Sin } \alpha = \alpha \tag{2}$$

The diagramatic reprentation of the beam angle and motor position of a ball and beam system is shown in Fig. 5 and 6:

$$\alpha L = \theta d$$

Equation 2 and 3 in Eq. 1, we get $\ddot{x} \frac{5}{7} g \frac{d}{L} \theta$:

$$\ddot{x} \frac{5}{7} g \frac{d}{L} \theta \tag{4}$$

Physical parameters of ball and beam system, mass of the ball (m) = 0.011 kg, Radius of the ball (R) = 0.015 m, Acceleration due to gravity (g) = 9.8 m/sec^2. Length of the beam (L) = 0.4 m, Radius of the gear (d) = 0.04 m. Substituting given data above in above equation:

$$\frac{x(s)}{\theta(s)} = \frac{0.7}{s^2} = G(s)$$

RESULTS AND DISCUSSION

Simulation: The open loop response of the ball and beam (un compensated) is plotted using MATLAB is shown in Fig. 7 and closed loop response is pulsating in nature as shown in Fig. 8 and PID controlled system step response is given in Fig. 9. After tuning the PID controller using MATLAB we can see that the step response is improved and is shown in Fig. 10. Further, a robust control technique is applied to this unstable system and are illustrated below.

Furthermore by applying the robust control theory for the plant p = 0.7/s^2 and choosing a weighting function by trial and error method W_1 = 100/s+1. Do co-prime factorization of the plant and obtain the diophantine equation and prove Bezout identity NX+MY = 1 where p = N/M. We obtain N = 0.7/(s+1)^2, M = s^2/(s+1)^2 X = 1.428+4.285s/(s+1), Y = s+3/(s+1). Set the relative degree of the plant k = 2 and choose τ such that J = 1/(τs+1)^2. Various values of τ find the infinity norm of should be <1 is tabulated in Table 1. Calculate:

$$Q = N^{-1}J = \frac{(s+3)\,(s+1)}{0.7\,(10^{-3}s+1)^2}$$

Robust controller for an unstable system C = X+MQ/Y-NQ:

$$C = \frac{(7e\text{-}07)s^{12} + 0.001407s^{11} + +0.714s^{10} + 7.067s^9 + 31.7s^8 + 84.39s^7 + 147.5s^6 + 176.9s^5 + 147.3s^4 + 84.1s^3 + 31.52s^2 + 7s + 0.6997}{(4.9e\text{-}13)s^{12} + (1.965e\text{-}09)s^{11} + (2.47e\text{-}06)s^{10} + 0.001005s^9 + 0.009903s^8 + 0.0414s^7 + 0.09638s^6 + 0.1375s^5 + 0.1237s^4 + 0.06865s^3 + 0.02157s^2 + s + 0.00294}$$

Bode plot of the infinity norm of weighted sensitivity function at τ = 0.001 with its magnitude and phase plot is shown in Fig. 11. The step response of the plant using

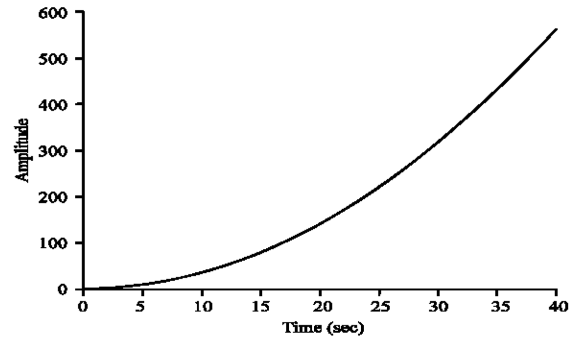

Fig. 7: Uncompensated system response (open loop)

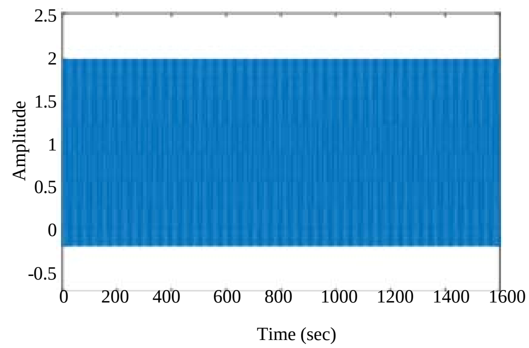

Fig. 8: Closed loop response (uncompensated)

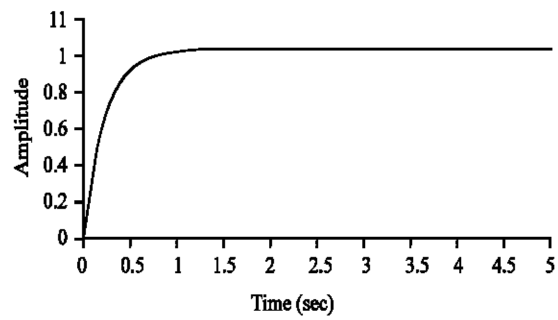

Fig. 9: Step response with PID controller; Tuned respnse, sys

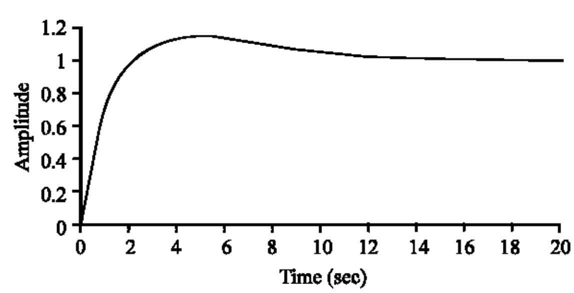

Fig. 10: Step response with optimal tuning of the PID controller; Tuned response, sys

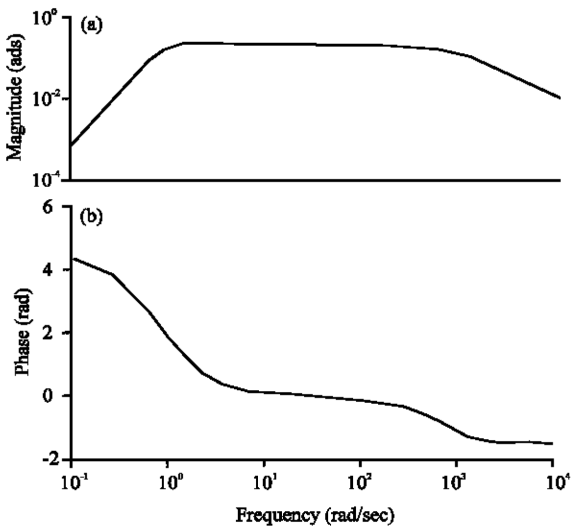

Fig. 11: Bode plot of $||W_1 MY (1\text{-}J)||_\infty$ when τ =0.001

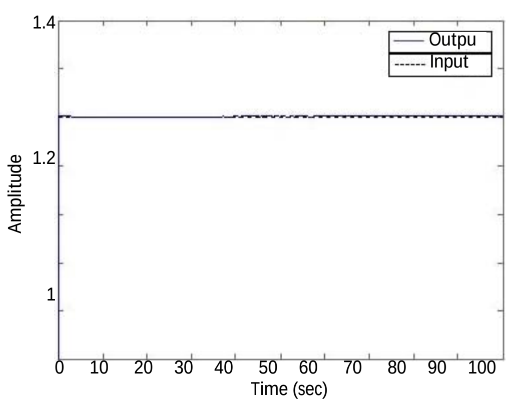

Fig. 12: Step response of the plant using robust controller

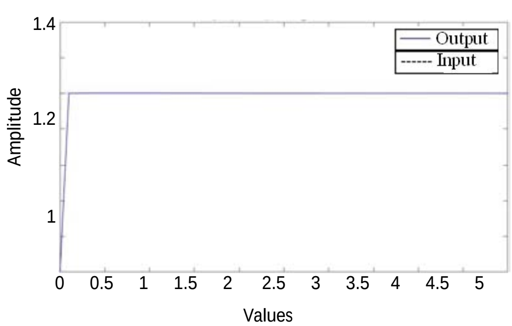

Fig. 13: Step response of the plant using robust controller when time duration is 5 sec

robust controller is shown in Fig. 12 and 13, the response is checked by applying a unit step input, shown in dotted line. The response is plotted in solid blue line which is seen to be tracking with the input closely. Figure 13 is a zoomed in image of Fig. 12 shown to understand the rising properties of the response during the initial 5 sec (Table 2). All poles of the characteristic equation is lying on the LHS of S plane the poles are given as:

Table 1: Infinity norm of weighted sensitivity function

τ	$\|\|W_1S\|\|_\infty = \|\|W_1MY\ (1\text{-}J)\|\|_\infty$
0.1	22.3171
0.01	2.32310
0.001	0.23250

Table 2: Performance comparison

Type of controller	Rise time tr (sec)	Settling time ts (sec)	Peak overshoot Mp (%)
PID Controller	1.49	12.2	15
Robust PID Controller	0.503	5.83	3.05
Robust Controller	improved	reduced	reduced

-1000.11271806611-0.112746138715977i
-999.887281933888+0.1126900029077133i
-999.887281933888-0.112690002907133i
-1.06192373129313+0.0177443182253563i
-1.06192373129313-0.0177443182253563i
-1.03332151765595+0.0481605711742385i
-1.03332151765595-0.0481605711742385i
-0.994023036020279+0.0583801780896844i
-0.994023036020279-0.0583801780896844i
-0.962580817336511+0.0471437266558632i
-0.962580817336511-0.0471437266558632i
-0.948150897694131+0.0174368338244773i
-0.948150897694131-0.0174368338244773i

CONCLUSION

The simulation results for a ball and beam experimental set up is discussed. Compare various type of controllers such as PID controller, robust PID controller and a robust controller and their step responses are compared and the results. The simulation results says the robust controller gives better time domain specifications. Even though the order of the controller high compare with other controllers but gives satisfactory step response characteristic.

LIMITATIONS

Instead of linearizing the plant the research can be done as nonlinear analysis method. The limitation of H infinity method is the order of the controller is very high. Also the selection of weighting function is crucial. A controller is designed for the unstable a ball and beam system, the designed controller is making the system internally stabilizing as all the poles are found to be lying on the LHS of S-plane. In conclusion this research assures both internal stability and performance of the closed loop system are achieved. It illustrates the comparative performance characteristics of PID controller robust PID controller and robust controller.

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